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Optimization Methods to Develop Bounds for Reliability using Normal Distribution

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Abstract: In this paper, we proposed an alternative method to obtain the reliability of a system. This study deals with the analysis reliability bounds. Here, we consider that a life time of a system is normally distributed with prior normal distribution, the compound distribution is also a normal distribution. We developed a methodology to find the GLB and LUB for basic and updated basic distribution. The method enable us to determine the reliability and unreliability for each stage in the process of distribution development. Due to cost & time saving these reliability bounds are more useful than any other method.

Keywords: Greatest Lower Bound, Least Upper Bound, Basic Distribution, Updated Distribution, Average Sample Number.

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I. INTRODUCTION

Each product has its different life time and the situation when the products fails due to its poor quality or the time spending at above average life time of the product. There is a need to analyzing the reliability with regard guarantee & loyalty (i.e. quality). Quality of a product can be controlled through Statistical Quality Control (SQC). The reliability has a good connection with statistical quality control. Due to this connection, the life testing of a product has been analyzing under the branch of reliability engineering. The electronic items and machine equipment has been tested under reliability theory. Most of the cases, when we care interested to analyzing the life of the product. Reliability is the probability that the system will survive for the time interval, (t_0, $t_0 + t$). In the life testing experiments a lot of products are subject to tests for their life. The lower and upper bounds of the reliability provide the quality limits of these products. The lower bound of the life reliability of the product have been discussed by Dao – thein and Mossound (1974), Kapur and Lambersan (1977) and Mischke (1991) under using Bienuyme – Chebycher Theorem. .

Later, Chang (1995) provided the reliability bounds for the stress – strength model. Group (2007) highlights the connection between reliability and quality. A vest literature has been provided by Hor& Seal (2017), Khan & King (2013) and Safdor& Ahmed (2014) about reliability bounds and the characteristics of the distributions. However,Hamson (1964) developed the bounds for non – parametric distributions and discussed the lower and upper bounds with their confidence limits in case of normal distribution. Later Bhattacharya and Johnson (1994) provided a method to estimate the reliability of a system. Bienayme – chebyshev theorem and Johnson’s theorem helps to set up the bounds for a system to sustain its reliability. In this study, we have been used lower and upper bounds to estimate the reliability of a system rather than many other complicated estimates.

If we provide the average life and variance of a component, It enable us to find the lower and upper bounds of the reliability of the components. Further, it is very difficult to put a system on life testing for a desired time because it requires more money and time. Sometimes the system is costly to use on testing. We want the constant (or desired) quality of the system but practically it is not possible, so quality varies and behave like random variable Bayesian and Posterior analysis would a good literature on this issue. Kapur and Lamberson (1977), Lawless (1982) and Sinha (1986) gave their concerned for the literature.

The prior and predictive experiments are compounded. Basically, the compound distribution tells patterns of the basic random variable with certain assumptions and variation can be ignored in the parameters.

This method provides a trade – off between time and cost of testing of a system. These bounds helps to estimates the behavior of system with compound normal distribution. We assumed that, the random variable denotes the life time of a product follows normal distribution with the probability density function (p.d.f)

$$f_1(x, \mu, \sigma_1) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_1^2}(x-\mu)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(I)$$

$-\infty < x, \mu < \infty, \sigma_1 > 0$

Where, μ & σ_1 are the parameters, which represents the quality standard of the product.

$$E(x) = \mu, \; V(x) = \sigma_1^2$$

(i) Now, the process mean (μ) behaves like random variable) because the quality cannot be treated as constant. It has known conjugate prior distribution as normal with p.d.f

$$f_2(\mu, \theta, \sigma_2) = \frac{1}{\sigma_2\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_2^2}(\mu - \theta)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(II)$$

$-\infty < \mu, \theta < \infty, \sigma_2 > 0$

Here the standard deviation σ_2 is assumed to be known.

$$E(\mu) = \theta, \; V(\mu) = \sigma_2^2$$

(ii) The variation in μ may be ignored if we consider the compound distribution of x with equation (I) & (II) . Then the probability density function will be as follows:

$$f(x, \theta, \sigma) = \int_{-\infty}^{\infty} f_1(x, \mu, \sigma_1).f_2(\mu, \theta, \sigma_2)\, d\mu$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(x - \theta)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(III)$$

$-\infty < x, \theta < \infty, \sigma > 0$

The compound distribution of x in equation (III) is $x \sim N(\theta, \sigma^2)$

$$E(x) = \theta, \; V(x) = \sigma_1^2 + \sigma_2^2$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

(iii) Further, let $x = (x_1, x_2, \ldots..x_n)$ be random sample of size n from the normal population $N(\mu, \sigma_1)$. Then the posterior distribution of μ given x will be:

$$\pi(\mu / x) = \frac{1}{\delta\sqrt{2\pi}} \exp.\left[-\frac{1}{2\delta^2}(\mu - \lambda)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(IV)$$

$-\infty < \mu, \lambda < \infty, \delta > 0$

(iv) Finally, from (I) & (IV), the predictive (or / compound) distribution of x can be obtained as follows:

$$g(x / x) = \int_{-\infty}^{\infty} f(x, \mu, \sigma_1)\pi(\mu / x)\, d\mu$$

$$= \frac{1}{\beta\sqrt{2\pi}} \exp\left[-\frac{1}{2\beta^2}(x - \lambda)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(V)$$

$-\infty < x, \lambda < \infty, \beta > 0$

With $E(x) = \lambda$, $V(x) = \sigma_1^2 + \delta^2$

$$\beta^2 = \sigma_1^2 + \delta^2$$

This is also a normal distribution as $N(\lambda, \beta^2)$.

This may be concluded as that we have basic distribution and updated distribution to be used in the following two conditions:

(i) The basic distribution equations (I) & (II) will be used to develop bounds of the reliability function when their respective parameters μ & θ are assumed as constants.

(ii) The updated basic and predictive distributions equations (III) & (IV) will be used for developing bounds for reliability when their respective parameters μ & θ are assumed as random variables.

II. RELIABILITY AND UNRELIABILITY BOUNDS DEVELOPMENT FOR THE GIVEN SYSTEM

(a) Distribution Free Bounds
(b) Actual Distribution Bounds
(c) Updated Distribution Bounds

(a) <u>Distribution Free Bounds</u>: Let us consider y be the non – negative continuous random variable which represents the lifetime of the system. Reliability function is the probability of the system that system will servive at least (t) time such that:

R (t) = P (Y > t) ……………………….(1)

with E (x) = μ_y , V (x) = σ_y^2

Now from equation (2.1)

R (t) = P (Y > t)

= P [(Y – μ_y) > (t – μ_y)]

= P (X > Y)

where, X = Y – μ_y , x= t – μ_y

and E (X) = E (Y – μ_y) = 0 and V (X) = V (Y – μ_y) = V (Y) = σ_y^2

usingChebychev's Inequality , we have

R (t) = P (Y > t) = P (X > x) < $\dfrac{\sigma_y^2}{\sigma_y^2 + x^2}$

$$R\ (t) = \frac{\sigma_y^2}{\sigma_y^2 + (t-\mu_y)^2} \quad \text{for } x > 0,\ t > \mu_y \ \ldots\ldots\ldots\ldots(2)$$

Similarly, R (t) = R (y > t) = P (x > x) ≥ $\dfrac{x^2}{\sigma_y^2 + x^2}$

$$R\ (t) = \frac{(t-\mu_y)^2}{\sigma_y^2 + (t-\mu_y)^2} \quad \text{for } x < 0,\ t < \mu_y \ \ldots\ldots\ldots\ldots(3)$$

From the equation, we may obtain reliability functions as

R (t) ≤ $\dfrac{\sigma_y^2}{\sigma_y^2 + (t-\mu_y)^2}$ provides the lower upper bound (LUB) when x > 0 or t >$\mu_y$such that target time (t) is greater than mean life time (μ_y) . If, t tends μ_y than R (t) approach to 1.

On the other hand, R (t) ≥ $\dfrac{(t-\mu_y)^2}{\sigma_y^2 + (t-\mu_y)^2}$, it provides the greatest lower bound (GLB) when x < 0 or t <μ_y such that when target time (t) approaches to zero (0). Unreliability may be denote as $\bar{R}$ (t) = 1 – R (t). It is the probability of a system fails before time (t).

$$\bar{R}\ (t) = 1 - \frac{\sigma_y^2}{\sigma_y^2 + (t-\mu_y)^2}$$

$$\bar{R}\ (t) \leq \frac{(t-\mu_y)^2}{\sigma_y^2 + (t-\mu_y)^2} \quad \ldots\ldots\ldots\ldots(4)$$

Similarly,
$$\bar{R}\ (t) \geq \frac{\sigma_y^2}{\sigma_y^2 + (t-\mu_y)^2} \quad \ldots\ldots\ldots\ldots(5)$$

So, it is noticeable reliability and unreliability bounds depends on the mean and variance of a sample of the life time distribution. Hence, the mean and variance are enough to estimate these desired parameters & their bounds.

(b) <u>Actual Distribution Bounds</u> : The reliability and unreliability bounds for normal and normal life time distribution when the parameters are considered to be constant .

(i) Normal distribution N (μ, λ, σ_1^2) bounds

Reliability R (t) = P (y > t)

$$= \int_t^\infty f_1(x, \mu, \sigma_1)\, dx$$

$$= \int_t^{\infty} \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma_1^2}\right) dx$$

$$= 1 - \phi\left(\frac{t-\mu}{\sigma_1}\right)$$

Unreliability $\overline{R}$ (t)s= $\int_{-\infty}^{t} f_1(x, \mu, \sigma_1)$ dx

$$= \int_{-\infty}^{t} \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma_1^2}\right) dx$$

$$= \phi\left(\frac{t-\mu}{\sigma_1}\right)$$

Bounds for the reliability for t > μ , the GLB & LUB

$$0 \le R(t) \le \frac{\sigma_1^2}{\sigma_1^2 + (t-\mu)^2} \quad \ldots\ldots(6)$$

For t < μ, the GLB & LUB

$$\frac{(t-\mu)^2}{(t-\mu)^2 + \sigma_1^2} \le R(t) \le 1$$

Similarly bounds for unreliability for t > μ

$$\frac{(t-\mu)^2}{(t-\mu)^2 + \sigma_2^2} \le \overline{R}(t) \le 1$$

For t <μ , greatest lower bound & lowest upper bond are

$$0 \le \overline{R}(t) \le \frac{\sigma_1^2}{\sigma_1^2 + (t-\mu)^2} \quad \ldots\ldots(7)$$

(ii) Similarly, bounds for N (θ , σ_2^2)

Reliability R (t) $= 1 - \phi\left(\frac{t-\theta)}{\sigma_2}\right)$

Unreliability $\overline{R}$ (t) = $\phi\left(\frac{t-\theta)}{\sigma_2}\right)$

Bounds for the reliability for t > θ, the GLB & ULB

$$0 \le R(t) \le \frac{\sigma_2^2}{\sigma_2^2 + (t-\theta)^2} \quad \ldots\ldots(8)$$

For t < θ, the GLB & LUB

$$\frac{(t-\theta)^2}{(t-\theta)^2 + \sigma_2^2} \le R(t) \le 1$$

Similarly for unreliability for t > θ, the GLB & ULB

$$\frac{(t-\theta)^2}{(t-\theta)^2 + \sigma_2^2} \le \overline{R}(t) \le 1$$

For t < θ, GLB & LUB are

$$0 \le \overline{R}(t) \le \frac{\sigma_2^2}{\sigma_2^2 + (t-\theta)^2} \qquad \text{......(9)}$$

(c) <u>Bounds for updated normal distribution</u>:

Reliability $R(t) = \int_t^\infty f_2(\mu,\lambda,\delta)\, dx$

$$= \int_t^\infty \frac{1}{\delta\sqrt{2\pi}} \exp\left(\frac{-(x-\lambda)^2}{2\delta^2}\right) dx \qquad \text{......(10)}$$

and unreliability, $\overline{R}(t)$ is

$$\overline{R}(t) = \int_{-\infty}^t \frac{1}{\delta\sqrt{2\pi}} \exp\left(\frac{-(x-\lambda)^2}{2\delta^2}\right) dx \qquad \text{......(11)}$$

May be written as

$$R(t) = 1 - \phi\left(\frac{t-\lambda}{\delta}\right)$$

$$\overline{R}(t) = \phi\left(\frac{t-\lambda}{\delta}\right)$$

Thus , the bounds for reliability , R (t) for $t > \lambda$, the greatest lower bound and lowest upper bound are :

$$0 \le R(t) \le \frac{\delta^2}{\delta^2 + (t-\lambda)^2}$$

for $t < \lambda$, the greatest lower bound and lowest upper bound for R (t) are

$$\left(\frac{(t-\lambda)^2}{(t-\lambda)^2 + \delta^2}\right) \le R(t) \le 1$$

Similarly, bounds of unreliability, $\overline{R}(t)$, for $t > \lambda$, the GLB and LUB are :

$$\left(\frac{(t-\lambda)^2}{(t-\lambda)^2 + \delta^2}\right) \le \overline{R}(t) \le 1 \qquad \text{......(12)}$$

for $t < \lambda$, the GLB and LUB for $\overline{R}(t)$ are : $0 \le \overline{R}(t) \le \frac{\delta^2}{\delta^2 + (t-\lambda)^2}$

Similarly, for $N(\mu, \sigma_1)$ updated normal distribution reliability,

$$R(t) = \int_t^\infty f_3(x,\mu,\sigma_1)\, dx$$

$$R(t) = 1 - \phi\left(\frac{t-\mu}{\sigma_1}\right)$$

Unreliability $\overline{R}(t) = \phi\left(\frac{t-\mu}{\sigma_1}\right)$

The bounds for R (t) for $t > \mu$

GLB & LUB are, $0 \le R(t) = \frac{\sigma_1^2}{\sigma_1^2 (t-\mu)^2}$

for $t < \mu$, GLB & LUB are , $\frac{(t-\mu)^2}{\sigma_1^2 (t-\mu)^2} \le R(t) \le 1$ ......(13)

Similarly, bounds for the unreliability, $\overline{R}(t)$, for $t > \mu$

GLB & LUB are, $$\frac{(t-\mu)^2}{\sigma_1^2(t-\mu)^2} \leq \overline{R}(t) \leq 1 \qquad \text{.................................(14)}$$

for $t < \mu$, the GLB and LUB for $\overline{R}$ (t) are

$$0 \leq \overline{R}(t) \leq \frac{\sigma_1^2}{\sigma_1^2(t-\mu)^2}$$

III. AVERAGE SAMPLE NUMBER FOR BOUNDS

Life time distribution contains n observations on testing and r < n such that the experiment is terminated if failures are less than these (n) observations before time (t) w.r.t. cost. Some authors like Sharma &Bhutani (1991) provide a trade– off between cost and time during life testing experiments.

The greatest lower bound for $\overline{R}$ (t) with compose to proportion $\left(\frac{r}{n}\right)$

$$\overline{R}(t) = \frac{r}{n}$$

Efficiency (E) of the censored sample for estimate average life time to complete sample is given by:

$$E = \frac{\sigma_y^2 / n}{\sigma_y^2 / r}$$

Now, the cost criteria for experiment is given ass

$$C_n = C_1 + C_2 n$$

$$C_t = C_1 + C_3 t$$

Where C_1 = Overhead cost

C_2 = Cost per unit of n

C_3 = cost per unit of t

if $C_n = C_t$

then $$\frac{C_3}{C_2} = \frac{n}{t}$$

So, the sample size may be estimate with C2 and C3 for desired target time (t).

IV. CONCLUSION

The estimation of reliability for updated distribution (normal and normal) are uniformly lower than the basic distributions (normal and normal) and their bounds also behave the same. The unreliability estimates are uniformly higher (or lower) than their GLB (or LUB). For sample size, if we increase time (t) then sample size also increases with cost per unit of (t) and decrease with cost per unit of n. The extended work is under processhowever this work can be used for other distributions also.

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