IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 09, Issue 5 (May. 2019), ||S (I) || PP 42-52

A Solution for Orthotropic Rectangular Thin Plate on Elastic Foundation with Horizontal Resistance Based On a Modified Function

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Abstract: Pavement is one of the main engineering of airport, which is usually assumed to be a foundation plate model. Generally speaking, most of the plates and common composite slabs on elastic foundations can be regarded as orthotropic rectangular thin plates. Considering the horizontal friction between the foundation soil and the thin plates, the concept reality of engineering and science can be better reflected. Firstly, the governing differential equations and the boundary conditions for the static and dynamic behavior of a rectangular thin plate on Winkler foundation are derived in this paper. Then, for the problem of orthotropic rectangular thin plates with four free edges on Winkler foundation, this paper improves a deflection function based on the existing research, which satisfies all the boundary conditions on four free edges and the conditions at four free corner points. Finally, Galerkin method is used to solve the bending, steady-state vibration, free vibration and stability problems of a orthotropic rectangular thin plate on elastic foundation. Numerical examples show that the method is in good agreement with the results of other literatures. The influence of various parameters on the deflection of foundation plate is discussed. This paper provides a theoretical basis for pavement structure.
Key Words: orthotropic thin plate, horizontal resistance, Winkler foundation, a modified function, Galerkin method.

Date of Submission: 06-05-2019 Date of acceptance: 20-05-2019

I. INTRODUCTION

The airport pavement can be divided into two major categories: cement concrete and asphalt pavement , namely rigid and flexible pavement [1]. Structural mechanics calculation is based on the theory of elastic foundation plates and the theory of elastic layered system respectively [2]. For the static and dynamic analysis of airport cement concrete pavement plates, due to the complexity of the objects and the difficulty of mathematical calculation, the common methods for simplifying the process are as follows: (1) the pavement plate and foundation are simplified into a viscoelastic structure with a degree of freedom; (2) the pavement plate can be simplified into a beam on elastic foundation; (3) the airport pavement is treated as an elastic rectangular thin plate and the foundation is regarded as a viscoelastic half-space body [3]. In this paper, a simple model is adopted: the pavement is regarded as a orthotropic rectangular thin plate and the foundation is regarded as Winkler foundation [4]. Rectangular plates on elastic foundation is a classical mechanical model. The elastic analysis of the interaction between soil and plates has a great significance not only to structural engineering and geotechnical engineering [5][6][7], but also to engineering problems like dock floor, floating airport at sea, chemical containers and test-bed [8].

Firstly, about the mechanical analysis of foundation plate under dynamic load, dynamic stiffness of concrete pavement was analyzed [9]; the vibration of rectangular plate on viscoelastic foundation under impact load was analyzed [10]; the dynamic response of airport pavements subjected to a moving aircraft load was investigated [11]. Secondly, on the issue of horizontal resistance between the components and elastic foundation, a new idea when there is friction between the plate and elastic foundation was put forward [12]; nonlinear behavior of finite length beams on Winkler foundation with horizontal resistance was investigated [13]; the influence of resistance at the bottom of the plate on the vibration of foundation plate under moving load was discussed [14]; a theoretical study on the axisymmetric bending of a circular plate on Winkler foundation considering tangential friction was conducted [15]. Thirdly, in regard to orthotropic rectangular thin plate, orthotropic rectangular plates in detail was analyzed [16]; the general analytical solutions for transverse vibration of orthotropic rectangular thin plate was calculated [17]; free vibration of orthotropic rectangular

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plates on double-parametric elastic foundation was solved [18]; the bending of anisotropic rectangular medium-thickness plate with four free edges was studied [19].

In the current research, the orthotropy of the plate and the horizontal friction between the foundation soil and the foundation plate are rarely considered simultaneously. Besides, it is difficult to construct a solution that satisfies not only the governing differential equation of plate bending, but also the boundary conditions on four free edges and the conditions at four free corner points. In addition, most of the literatures are about the static bending of plates on elastic foundations , and little work has been done on vibration, stability or dynamic response. Based on the analysis of Winkler elastic foundation model, considering the orthotropic anisotropy of a rectangular thin plate and the horizontal friction between foundation soil and base plate at the same time, the governing differential equation of orthotropic rectangular thin plate on Winkler elastic foundation including the effects of horizontal friction is derived according to the force analysis of micro-element. On the basis of previous research, a new deflection function is improved, which satisfies all the boundary conditions on four free edges and the conditions at four free corner points [8]. Finally, Solutions for bending, steady-state vibration, free vibration and stability problems of a orthotropic rectangular thin plate on elastic foundation with the horizontal frictional resistance are derived by using Galerkin method. In the numerical examples, effects of various parameters on the deflection of foundation plate are analyzed.

II. Mathematical Formulation

2.1 Orthotropic Rectangular Thin Plate

In discussing the problem of anisotropic thin plates, attention should also be paid to both thin plates made of anisotropic materials and thin plates that are anisotropic due to structural reasons. The theory of small deflection bending of thin plates on elastic foundations is based on some assumptions [20][21]. In addition, horizontal friction between the thin plate and the foundation soil is proportional to the strain in the corresponding direction.

As shown in Fig.1, the middle surface of the thin plate is $x-y$ plane, and the size is a,b and the thickness is h. $q(x,y)$ represents the transverse load and $F(x,y)$ represents the reaction force of the foundation.

Although the constitutive relationship of orthotropic plates is different from that of isotropic plates, the geometric relationship remains consistent, namely

$$\varepsilon_x=\frac{\partial u}{\partial x}=-z\frac{\partial^2 w}{\partial x^2}\ ,\ \varepsilon_y=\frac{\partial v}{\partial y}=-z\frac{\partial^2 w}{\partial y^2}\ ,\ \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=-2z\frac{\partial^2 w}{\partial x\partial y} \tag{1}$$

where the displacement components in the direction of corrdinate x,y,z axis is u,v and w; and the strain components is $\varepsilon_x,\varepsilon_y$ and γ_{xy}.

The constitutive relation is

$$\begin{cases}\sigma_x=-\dfrac{z}{1-\mu_1\mu_2}\left(E_1\dfrac{\partial^2 w}{\partial x^2}+\mu_1 E_2\dfrac{\partial^2 w}{\partial y^2}\right)\\ \sigma_y=-\dfrac{z}{1-\mu_1\mu_2}\left(E_2\dfrac{\partial^2 w}{\partial y^2}+\mu_2 E_1\dfrac{\partial^2 w}{\partial x^2}\right)\ ,\ \ \tau_{xy}=-2Gz\dfrac{\partial^2 w}{\partial x\partial y}\end{cases} \tag{2}$$

in which σ_x,σ_y and τ_{xy} are the stress components in the cross section. E_1 and E_2 are the elastic modulus of the direction x and y. μ_1 and μ_2 are the poisson's ratio, and G is the shear modulus. The above mentioned are the five elastic constants of orthotropic rectangular thin plate, which satisfy the following relationship

$$\frac{\mu_1}{\mu_2}=\frac{E_1}{E_2} \tag{3}$$

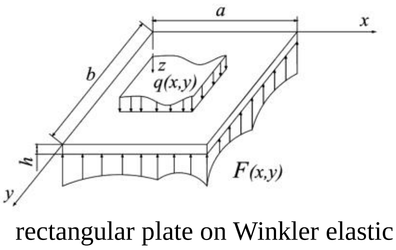

Fig.1. rectangular plate on Winkler elastic foundation

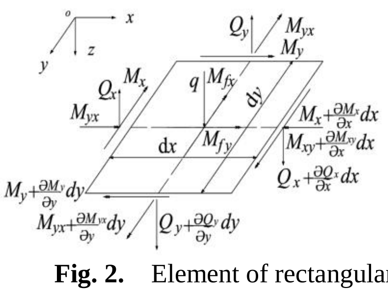

Fig. 2. Element of rectangular plate

The internal forces of plates, including bending moment M_x, M_y, the torque M_{xy} and comprehensive shear force V_x, V_y, can be expressed as the deformation w of a thin plate.

$$\begin{cases} M_x = -D_x\left(\frac{\partial^2 w}{\partial x^2} + \mu_2 \frac{\partial^2 w}{\partial y^2}\right) \\ M_y = -D_y\left(\frac{\partial^2 w}{\partial y^2} + \mu_1 \frac{\partial^2 w}{\partial x^2}\right),\ M_{xy} = -2D_{xy}\frac{\partial^2 w}{\partial x \partial y} \end{cases} \quad (4)$$

$$\begin{cases} V_x = -D_x \frac{\partial^3 w}{\partial x^3} - \left(H + 2D_{xy}\right)\frac{\partial^3 w}{\partial x \partial y^2} \\ V_y = -D_y \frac{\partial^3 w}{\partial y^3} - \left(H + 2D_{xy}\right)\frac{\partial^3 w}{\partial x^2 \partial y} \end{cases} \quad (5)$$

where D_x and D_y are the bending stiffness in the direction x and y; H is the equivalent stiffness; D_{xy} is the torsional stiffness of the thin plate in the main direction of elasticity [22].

$$\begin{cases} D_x = \frac{E_1 h^3}{12(1-\mu_1\mu_2)},\ D_y = \frac{E_2 h^3}{12(1-\mu_1\mu_2)} \\ D_{xy} = \frac{Gh^3}{12},\ H = D_x\mu_2 + 2D_{xy} \end{cases} \quad (6)$$

2.2 Bending problem

As shown in Fig.2, the transverse load, the internal force of the thin plate and the moment generated by the frictional resistance are drawn on the middle plane $dxdy$ of a small rectangular plate element.

According to the equilibrium condition, the equation along the z-axis is

$$\left(Q_x + \frac{\partial Q_x}{\partial x}dx\right)dy - Q_x dy + \left(Q_y + \frac{\partial Q_y}{\partial y}dy\right)dx - Q_y dx + qdxdy - p = 0 \quad (7)$$

After simplifying the process and dividing $dxdy$ out, the following equation is obtained.

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q - p = 0 \quad (8)$$

where Q_x and Q_y represent the shear force, and p represents reaction force from the foundation, assuming that k_v is the coefficient of foundation reaction.

$$p = k_v w(x,y) \quad (9)$$

The sum of the moments taken by a straight line parallel to the y-axis through the center of the plate element is equal to zero.

$$(M_x + \frac{\partial M_x}{\partial x}dx)dy - M_x dy + (M_{yx} + \frac{\partial M_{yx}}{\partial y}dy)dx - M_{yx}dx - (Q_x + \frac{\partial Q_x}{\partial x}dx)\frac{dxdy}{2} - Q_x\frac{dxdy}{2} - M_{fx} = 0 \quad (10)$$

After simplifying the process and dividing $dxdy$ out, the following equation is obtained.

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = m_{fx} \quad (11)$$

The sum of the moments taken by a straight line parallel to the x-axis through the center of the plate element is equal to zero.

$$(M_y + \frac{\partial M_y}{\partial y}dy)dx - M_y dx + (M_{xy} + \frac{\partial M_{xy}}{\partial x}dx)dy - M_{xy}dy - (Q_y + \frac{\partial Q_y}{\partial y}dy)\frac{dxdy}{2} - Q_y\frac{dxdy}{2} - M_{fy} = 0 \quad (12)$$

After simplifying the process and dividing $dxdy$ out, the following equation is obtained

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = m_{fy} \quad (13)$$

where M_{fx} and M_{fy} are the moments along the x-axis and the y-axis caused by horizontal friction.

Besides, m_{fx} and m_{fy} are moment per unit area, assuming that k_x and k_y are horizontal friction coefficients.

$$M_{fx} = m_{fx} dxdy \ , \ M_{fy} = m_{fy} dxdy \tag{14}$$

$$m_{fx} = -\frac{h^2}{4} k_x \frac{\partial w}{\partial x} \ , \ m_{fy} = -\frac{h^2}{4} k_y \frac{\partial w}{\partial y} \tag{15}$$

Substituting (11) and (13) into (8), the governing differential equation of a thin plate on elastic foundation is obtained

$$D_x \frac{\partial^4 w}{\partial x^4} + 2(2D_{xy} + D_1)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + k_v w - \frac{h^2}{4} k_x \frac{\partial^2 w}{\partial x^2} - \frac{h^2}{4} k_y \frac{\partial^2 w}{\partial y^2} = q \tag{16}$$

The reason to discard the force equilibrium equations in the plate plane is to simplify the complex practical problem, when establishing the governing equations accounting for frictional effects.

2.3 Steady-state vibration

When an orthotropic rectangular thin plate on Winkler elastic foundation is subjected to the harmonic loads, whose frequency is θ and amplitude is $q(x,y)$, the deformation amplitude $W(x,y)$ satisfies the following governing differential equation without considering the additional mass m_s of the foundation to the plate (omitting the time factor).

$$D_x \frac{\partial^4 W}{\partial x^4} + 2(2D_{xy} + D_1)\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} + k_v W - \frac{h^2}{4} k_x \frac{\partial^2 W}{\partial x^2} - \frac{h^2}{4} k_y \frac{\partial^2 W}{\partial y^2} - \rho h \theta^2 W = q(x,y) \tag{17}$$

When we consider the impact of the additional mass m_s of the foundation soil to the plate body, the following governing equation is satisfied (omitting the time factor).

$$D_x \frac{\partial^4 W}{\partial x^4} + 2(2D_{xy} + D_1)\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} + k_v W - \frac{h^2}{4} k_x \frac{\partial^2 W}{\partial x^2} - \frac{h^2}{4} k_y \frac{\partial^2 W}{\partial y^2} - (\rho h + m_s)\theta^2 W = q(x,y) \tag{18}$$

In the process of vibration, the foundation soil has additional mass m_s to the plate as explained in the references [21][23] and the specific expression can be obtained.

2.4 Free vibration

According to the elastic thin plate theory and vibration theory, Galerkin method is used to analyze the frequency of a thin plate on Winkler foundation in lateral free vibration. Without considering the additional mass of the foundation soil to the plate body, the following governing differential equation is satisfied (omitting the time factor).

$$D_x \frac{\partial^4 W}{\partial x^4} + 2(2D_{xy} + D_1)\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} + k_v W - \frac{h^2}{4} k_x \frac{\partial^2 W}{\partial x^2} - \frac{h^2}{4} k_y \frac{\partial^2 W}{\partial y^2} - \rho h \theta^2 W = 0 \tag{19}$$

When we consider the impact of the additional mass of the foundation soil to the plate body, the following governing equation is satisfied (omitting the time factor).

$$D_x \frac{\partial^4 W}{\partial x^4} + 2(2D_{xy} + D_1)\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} + k_v W - \frac{h^2}{4} k_x \frac{\partial^2 W}{\partial x^2} - \frac{h^2}{4} k_y \frac{\partial^2 W}{\partial y^2} - (\rho h + m_s)\theta^2 W = 0 \tag{20}$$

where m_s represents the additional mass of the foundation soil to the thin plate, and the expression is the same as the steady-state vibration.

2.5 Stability problem

The governing differential equation for the stability of orthotropic rectangular thin plate on elastic foundation with horizontal friction, subjected to uniform pressure $T_y/T_x = r$ along the the middle of the plate and shear load T_{xy} around the plate, is given as

$$D_x \frac{\partial^4 w}{\partial x^4} + 2(2D_{xy} + D_1)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + k_v w - \frac{h^2}{4} k_x \frac{\partial^2 w}{\partial x^2} - \frac{h^2}{4} k_y \frac{\partial^2 w}{\partial y^2} = T_x \frac{\partial^2 w}{\partial x^2} + T_y \frac{\partial^2 w}{\partial y^2} + T_{xy} \frac{\partial^2 w}{\partial x \partial y} \tag{21}$$

2.6 Boundary conditions

The four sides of the rectangular thin plate are free edges. The boundary conditions on four free edges and the conditions at four free corner points are expressed by the deflection of the rectangular plate as follows

At $x=0$ and $x=a$:

$$M_x = 0 \ , \ V_x = 0 \qquad (22)$$

At $y=0$ and $y=b$:

$$M_y = 0 \ , \ V_y = 0 \qquad (23)$$

For the intersection of two free boundaries, there is also the following corner condition

$$\frac{\partial^2 w}{\partial x \partial y} = 0 \qquad (24)$$

III. SOLUTION METHOD

3.1 Improved deflection function

Based on the analysis and modification of the deflection function in the literature, a form suitable for an orthotropic rectangular thin plates on elastic foundation can be obtained. Moreover, the improved deflection function satisfies all the boundary conditions on four free edges and the conditions at four free corner points [8].

$$W = c_0 + c_1 x + c_2 y + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn} \varphi_{mn} \qquad (25)$$

where

$$\varphi_{mn} = \cos\alpha_m x \cos\beta_n y - \left(1+\frac{\beta_n^2}{\mu_1 \alpha_m^2}\right)\cos\alpha_m x - \left(1+\frac{\alpha_m^2}{\mu_2 \beta_n^2}\right)\cos\beta_n y - \frac{\beta_n^2}{2\mu_1}x^2 - \frac{\alpha_m^2}{2\mu_2}y^2 \qquad (26)$$

$$\alpha_m = \frac{2m\pi}{a}, \beta_n = \frac{2n\pi}{b}, m, n = 1, 2, 3\cdots$$

3.2 Bending problem

The deflection function is substituted into the bending governing equation of orthotropic rectangular thin plates on Winkler foundation, and the residual error R_1 is obtained. Reduce the residual error and make the weighted residual be equal to zero over the entire plate.

$$\int_0^a \int_0^b R_1 \cdot \phi_{mn}(x,y) dx dy = 0 \ , \ m, n = 0, 1, 2, ... \qquad (27)$$

where

$$\phi_{mn}(x,y) = 1 \ , \ \phi_{mn}(x,y) = x \ , \ \phi_{mn}(x,y) = y \ , \ \phi_{mn}(x,y) = \varphi_{mn} \qquad (28)$$

The following formula can be obtained by sorting it out

$$H_1 \cdot X_1 = Q_1 \qquad (29)$$

where H_1 is a symmetric matrix; X_1 is a one-dimensional array of undetermined coefficients in the assumed deflection function; and Q_1 is a one-dimensional array formed by load.

3.3 Steady-state vibration

The deflection function is substituted into the steady-state vibration governing equation of orthotropic rectangular thin plates on Winkler foundation, and the residual error R_2 is obtained.

When the additional mass of the foundation to the plate is not considered, the expression is as follows.

$$R_2 = R_1 - \left(\rho h \theta^2\right) R_4 \qquad (30)$$

Considering the additional mass of the foundation to the plate body, the expression is as follows.

$$R_2 = R_1 - \left[(\rho h + m_s)\theta^2\right] R_4 \qquad (31)$$

where

$$R_4 = c_0 + c_1 x + c_2 y + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn} \varphi_{mn} \qquad (32)$$

Reduce the residual error and make the weighted residual be equal to zero over the entire plate.

$$\int_0^a\int_0^b R_2\cdot \phi_{mn}(x,y)dxdy = 0 \ , \ m,n = 0,1,2,... \qquad (33)$$

The following formula can be obtained by sorting it out

$$H_2 \cdot X_2 = Q_2 \qquad (34)$$

where H_2 is a symmetric matrix; X_2 is a one-dimensional array of undetermined coefficients in the assumed deflection function; and Q_2 is a one-dimensional array formed by loading.

3.4 Free vibration

The deflection function is substituted into the free vibration governing equation of orthotropic rectangular thin plates on Winkler foundation, and the residual error R_3 is obtained.

When the additional mass of the foundation to the plate is not considered, the expression is as follows.

$$R_3 = R_1 - \left(\rho h\theta^2\right)R_4 \qquad (35)$$

Considering the additional mass of the foundation to the plate body, the expression is as follows.

$$R_3 = R_1 - \left[(\rho h + m_s)\theta^2\right]R_4 \qquad (36)$$

where

$$R_4 = c_0 + c_1x + c_2y + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}\varphi_{mn} \qquad (37)$$

Reduce the residual error and the following formula can be obtained by sorting it out

$$H_3 \cdot X_3 = 0 \qquad (38)$$

where X_3 is the vibration amplitude vector of the plate which can not be all zero. If all of them were zero, the plate could not vibrate [23][24]. Therefore, in order to obtain non-zero solution, the determinant of the coefficients must be equal to zero. And the frequency equation is obtained. After expansion, a higher order algebraic equation is obtained, with θ considered as an unknown quantity. On the other hand, the free vibration problem can also be transformed into a matrix eigenvalue problem to solve the free vibration frequency θ.

3.5 Stability problem

It is assumed that there is uniform pull T_x, T_y along the coordinate direction x=0,x=a, y=0,y=b and suppose $T_y/T_x = r$, T_{xy}=0. The deflection function is substituted into the stability governing equation of orthotropic rectangular thin plates on Winkler foundation, and the residual error R_5 is obtained.

$$R_5 = R_1 - T_x R_6 \qquad (39)$$

where R_6 is an intermediate variable in mathematical operations.

The following formula can be obtained by sorting it out

$$H_4 \cdot X_4 = 0 \qquad (40)$$

where H_4 and X_4 are similar to the above. The critical load T_x of rectangular thin plate can be solved by using the zero determinant and transforming it into matrix eigenvalue.

IV. NUMERICAL EXAMPLES

4.1 Numerical calculations

Example 1: A square isotropic plate is on an elastic foundation with a free perimeter, whose side length is a. The plate is subjected to the concentrated load P at the center $\left(x = a/2, y = a/2\right)$ and the Poisson's ratio is $\mu = 0.167$. $ka^4/D = 10^4$. Calculate the deformation and internal force of the foundation plate.

This example is an calculation example in the reference [22][25]. The rectangular thin plate is regarded as the simplest case of isotropic without considering the friction resistance between the plate body and the foundation soil. Fig. 3 is the deflection diagram of rectangular thin plate on Winkler foundation based on this method. Fig. 4

is the bending moment diagram. The trend of the diagrams is in line with the reality.

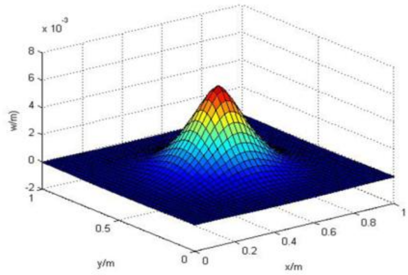

Fig. 3. Deflection distribution of rectangular plate

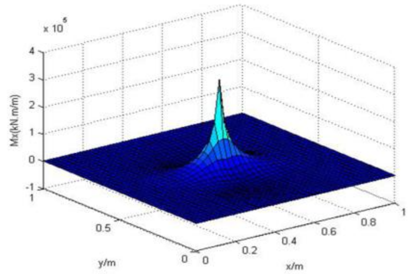

Fig. 4. Bending moment of rectangular plate

In Table 1, the first row is the exact solution of rectangular thin plate on Winkler elastic foundation [22][25]; and the second row is the result of this paper, where the bending problem of orthotropic rectangular thin plate on Winkler foundation including the effects of horizontal friction resistance degenerates into the bending problem of an isotropic thin plate on Winkler foundation without horizontal friction resistance. The two results are in good agreement, which proves the correctness of the computational program and the validity of the improved deflection in solving the problem of a rectangular plate on elastic foundation with four free edges.

Table 1 Deflection of Example 1 ($\left(Pa^2/D\right)\times10^{-3}$)

x /m	y/m				
	0.500	0.625	0.750	0.875	1.000
0.500	1.25	0.66	0.18	0.00	-0.06
	1.245	0.648	0.171	0.000	-0.028
0.625	0.66	0.41	0.12	-0.01	-0.05
	0.648	0.406	0.111	-0.008	-0.027
0.750	0.18	0.12	0.03	-0.02	-0.04
	0.171	0.111	0.020	-0.019	-0.023
0.875	0.00	-0.01	-0.02	-0.02	-0.02
	0.000	-0.008	-0.019	-0.020	-0.016
1.000	-0.06	-0.05	-0.04	-0.02	-0.01
	-0.028	-0.027	-0.023	-0.016	-0.014

Example 2: A orthotropic square plate is on an elastic foundation with a free perimeter, whose side length is $a = 20m$. And the thickness and stiffness parameters are: $h = 1.0\,m$,

$D_x = 1.7122\times10^6\,\text{kN}\cdot\text{m}$, $D_y = 2.5683\times10^6\,\text{kN}\cdot\text{m}$, $D_1 = 0.3244\times10^6\,\text{kN}\cdot\text{m}$, $D_{xy} = 8.62069\times10^6\,\text{kN}\cdot\text{m}$. The coefficient of foundation bed is $k = 1.0\times10^5\,\text{kN}\cdot\text{m}$. The plate is subjected to a concentrated load at the center. $P = 5\text{KN}$.

Table 2 Deflection of rectangular plate of Example 2 (mm)

m	10	20	30	40	50	referen ce
w_A	1.346	1.362	1.365	1.366	1.368	1.3-1.5
w_B	0.256	0.258	0.259	0.259	0.259	0.1-0.4

This example is an calculation example in the reference [26]. The rectangular is regarded as oothotropic case without considering the horizontal friction between the plate and the soil. Table 2 shows the calculation results of this paper, in which the orthotropic rectangular thin plate on Winkler elastic foundation considering horizontal friction is degenerated into the case in the literature. The coordinates of point A and point

B are $(10,10)$, $(10,15)$ respectively. In the area of plate under load, especially the central position, the deformation of the thin plate calculated in this paper are similar to those obtained by natural element method and finite element method in the reference (Zeng and Deng, 2008). The data in Table 2 shows that the convergence speed is quite fast. The reliability of the analysis results is proved again.

Example 3: Consider the bending of orthotropic rectangular thin plate on Winkler foundation including the effects of horizontal resistance under transverse loads, based on the refined deflection function. The elastic modulus, shear modulus, Poisson's ratio, and so on are as follows:

$$E_1 = 1.96\times10^4\,\text{MPa},\; E_2 = 3.92\times10^4\,\text{MPa},\; a = 1\text{m},\; G = 8.3976\times10^3\,\text{MPa},\; \mu_1 = 0.167,\; \mu_2 = 0.334,\; h = 0.04\text{m}$$

$$k_v = 1.5\times10^5\,\text{kN}\cdot\text{m}^{-3},\; k_x = 2.0\times10^4\,\text{kN}\cdot\text{m}^{-3},\; k_y = 2.0\times10^4\,\text{kN}\cdot\text{m}^{-3},\; P = 98\text{kN},\; q = 9.8\text{kN}\cdot\text{m}^{-2}$$

Fig. 5 is the deflection diagram of orthotropic thin plate on Winkler foundation with horizontal friction under transverse load. The trend of the diagram is in line with the reality. Table 3 shows the deflection of the rectangular thin plate in Example 3.

Table 3: Deflection of rectangular plate of Example 3 (mm)

x /m	y/m 0.500	0.625	0.750	0.875	1.000
0.500	2.452	1.933	1.265	0.827	0.703
0.625	1.774	1.522	1.053	0.708	0.610
0.750	0.959	0.856	0.630	0.441	0.390
0.875	0.461	0.415	0.304	0.199	0.172
1.000	0.329	0.296	0.212	0.120	0.082

It can be seen from the comparison between the Fig. 3 and the Fig. 5 that the deformations of orthotropic rectangular thin plates are not the same in two coordinate axes, which is different from the deflection distribution of isotropic rectangular plates under transverse load. The main reason is that the physical property parameters of orthotropic rectangular thin plates are different in the directions of two coordinate axes. But the physical property parameters of isotropic rectangular thin plates are identical in the directions of two coordinate axes.

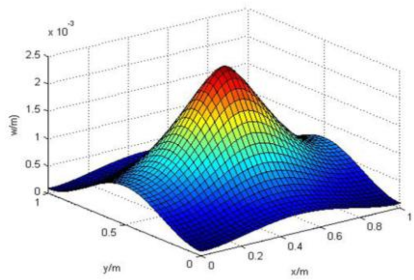

Fig. 5. Deflection distribution of rectangular plate

Example 4: Regardless of the additional mass of the foundation to the plate, the deflection amplitude of orthotropic rectangular thin plate on Winkler foundation considering the horizontal resistance under steady-state vibration will be calculated when other conditions remain unchanged in case 3. The density of the plate is $\rho_s = 3000\,\text{kg}/\text{m}^3$; and the frequency of the load is $\theta = 100\,\text{rad}/\text{s}$. Table 4 shows the deflection amplitude of rectangular thin plate under steady-state vibration in Example 4.

Table 4 Deflection amplitude of plate of Example 4 (mm)

x /m	y/m 0.500	0.625	0.750	0.875	1.000
0.500	2.546	2.024	1.347	0.902	0.776
0.625	1.860	1.605	1.129	0.777	0.677
0.750	1.027	0.921	0.690	0.496	0.444
0.875	0.513	0.465	0.350	0.241	0.213
1.000	0.376	0.341	0.254	0.158	0.119

4.2 impact of parameters

Under the same other conditions, only the physical property constant of the orthotropic rectangular thin plate are changed to investigate their effects on the bending behavior of the plate.

As can be seen from the Fig. 6, when the elastic modulus increases, the deflection of the thin plate decrease. The main reason is that when the elastic modulus increases, the stiffness of the thin plate increases, so the deformation near the load decreases. At the same time, the warpage at the boundary increases due to the stiffness enhancement of the plate, which shows that the deformation at the boundary increases. In addition, as can be seen from Fig. 7, the influence of Poisson's ratio on the deflection of thin plates is relatively small.

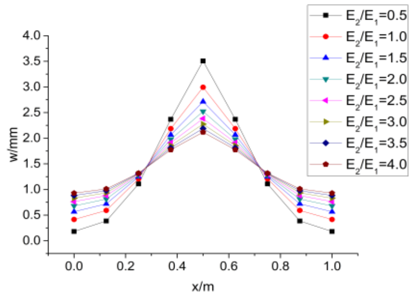

Fig. 6. Deflection of plate with different elastic modulus

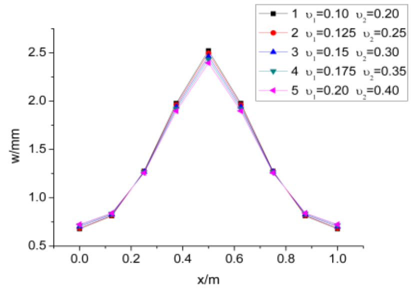

Fig. 7. Deflection of plate with different Poisson's ratio

Under the same other conditions, only the horizontal friction coefficients between the plate and the soil are changed to research their influence on the bending characteristics of a thin plate.

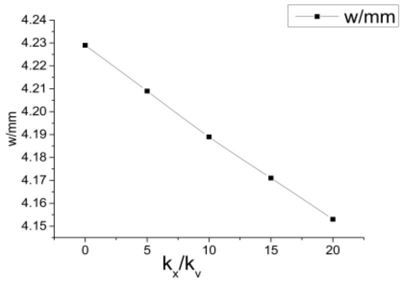

Fig. 8. Deflection of plate with different friction of A group

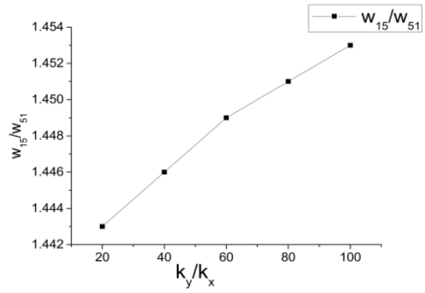

Fig. 9. Deflection of plate with different friction of B group

As shown in Figs., Fig. 8 compares the deflection of the center of the rectangular thin plate under different k_x/k_v. And Fig. 9 compares the deflection ratio of the midpoint of the boundary of the rectangular thin plate under different k_y/k_x. w_{15} and w_{51} are the midpoints of the edge $x=1$ and edge $y=1$ of rectangular thin plate.

It can be seen from the Fig. 8 that when the horizontal friction coefficients become larger, the deflection of the thin plate becomes smaller. Moreover, it can be seen from the Fig. 9 that when the ratio of horizontal friction coefficients in two directions becomes larger, the deflection difference of thin plates in two directions becomes larger.

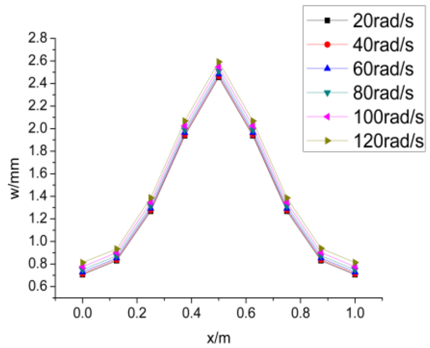

Fig. 10. Deflection of plate with different excitation frequency

Under the same other conditions, only the excitation frequency θ of the steady-state vibration of the plate is changed, and the influence of the frequency on the deflection amplitude and bending moment amplitude of the thin plate is investigated, regardless of the additional mass of the foundation to the plate. It can be seen from the Fig. 10 that when the excitation frequency increases, the deflection amplitude also increases.

V.CONCLUSIONS

The problem of orthotropic rectangular thin plates with free edges on Winkler foundation considering horizontal friction is studied in this paper. The conclusions are as follows :

1. The modified deflection function satisfies all the boundary conditions on four free edges of orthotropic rectangular thin plate on elastic foundation and the conditions at four free corner points. Based on this, Galerkin method can be used to solve the orthotropic rectangular thin plate with four free edges on Winkler elastic foundation .
2. Rectangular thin plates are supported on elastic foundation, which were previously assumed to be in smooth contact with the soil. In consideration of horizontal friction between the plate and the soil, the governing differential equations of orthotropic rectangular thin plate with four free edges on Winkler elastic foundation are derived and established, including bending, steady-state vibration, free vibration and stability.
3. The program is simple. The results are in good agreement with other literatures. The deformations of an orthotropic rectangular thin plate are different in the direction of the two coordinate axes, even if the load is centrally symmetric. The main reason is that the physical parameters of orthotropic rectangular plates are not the same in two coordinate axes.
4. The numerical results show that many factors can affect the bending and vibration of a thin plate on elastic foundation. For example, the larger horizontal friction between the plate and the soil can not be ignored. The elastic modulus of orthotropic thin plate directly affects the mechanical behavior of foundation plate under transverse load more than Possion's ratio does. In stesdy-state vibration, the excitation frequency is positively correlated with the deformation amplitude and bending moment amplitude.
5. The improved foundation model and deflection function have important theoretical significance and wide application prospect in practice. On this basis, we can continue to study the dynamic response of thick plate ,non-linear problems and the mechanical behavior of rigid pavement on visco-elastic Winkler foundation under aircraft dynamic loads. The research results can provide theoretical basis for static and dynamic response analysis, the thickness design of plates, parameter identification and quality evaluation of airport pavement structure.

ACKNOWLEDGMENTS

The study described in this paper was financially by the National Natural Science Foundation of China under Grant 51278420 and the Natural Science Foundation of Shaanxi Province under Grant 2017JM5021.

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IOSR Journal of Engineering (IOSRJEN) is UGC approved Journal with Sl. No. 3240, Journal no. 48995.

Feng Yue "A Solution for Orthotropic Rectangular Thin Plate on Elastic Foundation with Horizontal Resistance Based On a Modified Function." IOSR Journal of Engineering (IOSRJEN), vol. 09, no. 05, 2019, pp. 42-52.