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# Pathway Integral Operator Associated With Mainardi Functions

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**Abstract**: In this paper, we obtain some result of the pathway fractional integral operator associated with Mainardi function and its exponential form. Further, some relevant connection of our main results are also be considered.

**Keywords**: Exponential form, Mainardi function , Pathway fractional integral operator, Riemann- Liouville fractional integral operator, Wright function.

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## I. INTRODUCTION

The Mainardi function is one of the most powerful special function and it is a particular case of wright function . The application of this function have extensively used in a large number of areas of physical and applied sciences.

Very first ,we will begin with the wright function of second kind that is defined by E.M. Wright , which is as follows:

$$W_{p,q}(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!\Gamma(pk+q)}, p > -1, q\epsilon c, z\epsilon c. \quad (1.1)$$

If we set p = - η and q = 1- η , then equation (1.1) is reduce to Mainardi function given by Mainardi [1] which as

$$\mathrm{M}(z,\eta) = M_\eta(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{z^k}{\Gamma(-\eta(k+1)+1)}, \quad \eta \,\epsilon\, C, R(\eta) > 0, z \,\epsilon\, C. \quad (1.2)$$

If put $z = e^z$ in equation (1.2) we get Exponential form of Mainardi function as

$$\mathrm{M}^\eta(e^z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{e^{zk}}{\Gamma(-\eta(k+1)+1)}, \eta \,\epsilon\, C, R(\eta) > 0, e^z \epsilon\, C. \quad (1.3)$$

Next we required the concept of pathway fractional integral operator . This operator is related to pathway model , various fractional integral operators and special function. Pathway fractional integral operator are introduced in the paper of Nair [6] and defined in the following way

$$(P_{0+}^{(\mu,\lambda)} f)x = x^\mu \int_0^{\left(\frac{x}{\alpha(1-\lambda)}\right)} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{(1-\lambda)}} f(\tau)d\tau \quad (1.4)$$

Where $f \in$ L(a,b), { L(a,b) is a lebesgue measurable real and complex valued function}
μ ϵ C , R(μ)>0 , α > 0 & pathway parameter λ <1 .

For pathway model , we use the concept of Mathai [2], Mathai and Haubold [3,4] . If the pathway parameter λ → 1_ then equation (1.4) is convert to Laplace integral transform.

**Remark 1** If λ = 0 , α = 1 and μ = μ -1 then pathway fractional integral operator in equation (1.4) transform to the Riemann – Liouville fractional integral operator as follows:

$$\left(P_{0+}^{\mu-1} f\right)x = \int_0^x (x-\tau)^{\mu-1} f(\tau)d\tau = \Gamma(\mu) \left(I_{0+}^{\mu} f\right)x .$$

## II. MAIN RESULTS

In this section we derive the relation between pathway fractional integral operator and Mainardi functions from (1.2).

**Theorem 2.1** Suppose that η ϵ C , R(η) > 0 and $P_{0+}^{(\mu,\lambda)}$ be the pathway fractional integral operator then there holds the relation

$$P_{0+}^{(\mu,\lambda)} M^\eta(x) = \frac{\Gamma(k+1)\,\Gamma(\frac{\mu}{1-\lambda}+1)}{\Gamma\left(\frac{\mu}{1-\lambda}+k+2\right)\; [\alpha(1-\lambda)]^{k+1}}\, x^{\mu+1}\, M^\eta(x) \quad (2.1)$$

Proof :- If we derive equation (2.1) then express $M^\eta(x)$ by using equation (1.2) and apply equation (1.4) we have

$P_{0+}^{(\mu,\lambda)} M^{\eta}(z) = x^{\mu} \int_0^{\frac{x}{\alpha(1-\lambda)}} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{1-\lambda}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1}$

Now interchanging the order of integral and summation, which is valid by uniform convergence of the involved series with the given conditions , we get

$$= x^{\mu} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1} \int_0^{\frac{x}{\alpha(1-\lambda)}} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{1-\lambda}} \tau^k d\tau$$

Put $\frac{\alpha(1-\lambda)\tau}{x} = v$

$$= x^{\mu} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1)} \frac{x^{k+1}}{[\alpha(1-\lambda)]^{k+1}} \int_0^1 (1-v)^{\frac{\mu}{1-\lambda}} v^k \qquad (2.2)$$

Now by using the Beta function , we have

$$= x^{\mu} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1)} \frac{x^{k+1}}{[\alpha(1-\lambda)]^{k+1}} \beta\left(\frac{\mu}{1-\lambda} + 1, k+1\right)$$

$$= x^{\mu+1} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^k}{\Gamma(-\eta(k+1)+1)} \frac{1}{[\alpha(1-\lambda)]^{k+1}} \frac{\Gamma(k+1)\,\Gamma(\frac{\mu}{1-\lambda}+1)}{\Gamma(\frac{\mu}{1-\lambda}+k+2)}$$

$$= x^{\mu+1} M^{\eta}(x) \frac{1}{[\alpha(1-\lambda)]^{k+1}} \frac{\Gamma(k+1)\,\Gamma(\frac{\mu}{1-\lambda}+1)}{\Gamma(\frac{\mu}{1-\lambda}+k+2)} .$$

Which completes the required proof of theorem (2.1).

**Corollary (2.1)** If we take pathway parameter λ=0 , α = 1 , μ = μ-1 in equation (2.1) then we get

$$P_{0+}^{(\mu-1)} M^{\eta}(x) = \frac{\Gamma(\mu)\Gamma(k+1)}{\Gamma(\mu+k+1)} x^{\mu} M^{\eta}(x) = I_x^{\mu} M^{\eta}(x)\,\Gamma(\mu)$$

(2.3)

i.e. the equation (2.3) show that relation between Mainardi function $M^{\eta}(x)$ with Riemann- Liouville fractional integral operator $I_x^{\mu} f(x)$ and Pathway fractional integral operator.

## III. PATHWAY INTEGRAL OPERATOR ASSOCIATED WITH MAINARDI FUNCTION FOR EXPONENTIAL FORM

**Theorem 3.1** Suppose that η ϵ C , R(η) > 0 and $P_{0+}^{(\mu,\lambda)}$ be the pathway fractional integral operator then there holds the relation

$$P_{0+}^{(\mu,\lambda)} M^{\eta}(e^x) = \frac{\Gamma(s+1)\,\Gamma(\frac{\mu}{1-\lambda}+1)}{\Gamma\left(\frac{\mu}{1-\lambda}+s+2\right)[\alpha(1-\lambda)]^{s+1}} x^{\mu+1} M^{\eta}(e^x). \qquad (3.1)$$

Proof :- If we derive equation (3 .1) then express $M^{\eta}(e^x)$ by using equation (1.3) and apply equation (1.4) we have

$$P_{0+}^{(\mu,\lambda)} M^{\eta}(e^x) = x^{\mu} \int_0^{\frac{x}{\alpha(1-\lambda)}} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{1-\lambda}} M^{\eta}(e^{\tau}) d\tau.$$

$$= x^{\mu} \int_0^{\frac{x}{\alpha(1-\lambda)}} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{1-\lambda}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{e^{\tau k}}{\Gamma(-\eta(k+1)+1}.$$

$$= x^{\mu} \int_0^{\frac{x}{\alpha(1-\lambda)}} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{1-\lambda}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1} \sum_{s=0}^{\infty} \frac{(\tau k)^s}{s!}.$$

Now interchanging the order of integral and summation, which is valid by uniform convergence of the involved series with the given conditions , we get

$$= x^{\mu} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1} \sum_{s=0}^{\infty} \frac{k^s}{s!} \int_0^{\frac{x}{\alpha(1-\lambda)}} \left[1 - \frac{\alpha(1-\lambda)\tau}{x}\right]^{\frac{\mu}{1-\lambda}} \tau^s d\tau.$$

Put $\frac{\alpha(1-\lambda)\tau}{x} = v$ in R.H.S.

$$= x^{\mu} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1} \sum_{s=0}^{\infty} \frac{k^s}{s!} \frac{x^{s+1}}{[\alpha(1-\lambda)]^{s+1}} \int_0^1 (1-v)^{\frac{\mu}{1-\lambda}} v^s dv.$$

Now by using the Beta function , we have

$$= x^{\mu} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1} \sum_{s=0}^{\infty} \frac{k^s}{s!} \frac{x^{s+1}}{[\alpha(1-\lambda)]^{s+1}} \beta\left(\frac{\mu}{1-\lambda} + 1, s+1\right).$$

$$= x^{\mu+1} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{1}{\Gamma(-\eta(k+1)+1} \sum_{s=0}^{\infty} \frac{(kx)^s}{s!} \frac{1}{[\alpha(1-\lambda)]^{s+1}} \frac{\Gamma\left(1+\frac{\mu}{1-\lambda}\right)\Gamma(s+1)}{\Gamma(\frac{\mu}{1-\lambda}+s+2)}.$$

$$= \frac{x^{\mu+1}}{\alpha(1-\lambda)^{s+1}} \frac{\Gamma(s+1)\,\Gamma(1+\frac{\mu}{1-\lambda})}{\Gamma(\frac{\mu}{1-\lambda}+s+2)} M^{\eta}(e^x).$$

Which completes the required proof of theorem (3.1).

**Corollary (3.1)** If we take pathway parameter λ=0 , α = 1 , μ = μ-1 in equation (3.1) then we get known result of Mohd. Farman Ali , Manoj Sharma , Renu Jain [5]

$$P_{0+}^{(\mu-1)} M^{\eta}(e^x) = \frac{\Gamma(\mu)\,\Gamma(s+1)}{\Gamma(\mu+s+1)} x^{\mu} M^{\eta}(e^x) = \Gamma(\mu)\, I_x^{\mu} M^{\eta}(e^x) . \qquad (3.2)$$

i.e. the equation (3.2) show that relation between Mainardi function for exponential form $M^{\eta}(e^{x})$ with Riemann- Liouville fractional integral operator $I_x^{\mu}\ f(x)$ and Pathway fractional integral operator.

## IV. CONCLUSION

In this paper , We have established relation between Mainardi function and pathway fractional integral operator. And it can be easily seen that special case of Pathway fractional integral operator with $\lambda$=0 , $\alpha$ = 1 , $\mu$ = $\mu$-1 reduce to Riemann- Liouville fractional integral operator associated with mainardi functions as earlier proved [5].

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