IOSR Journal of Engineering (IOSRJEN) www.iosrjen.org
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 08, Issue 11 (November. 2018), ||V (I) || PP 89-99

Heat and Mass Transfer Effect in Melting Process of Boundary Layer Stagnation Point Flow of Micropolar Fluid towards a Stretching/Shrinking Surface

Dr. Mamta Goyal[1], Gyanchand Gurjar[2]
Associate ProffessorDepartment Of Mathematical Sciences
University of Rajasthan, jaipur, Rajasthan, India
Department Of Mathematical Sciences
University of Rajasthan, Jaipur, Rajasthan, India
Corresponding Author:Dr. Mamta Goyal

Abstract: The present investigation deals with the study of fluid flow mass and heat transfer characteristics in the melting process towards a stretching/shrinking surface. The governing equations representing fluid flow have been transformed into nonlinear ordinary differential equations using similarity transformation. The equations thus obtained have been solved numerically using Runge–Kutta forth order method with shooting technique. The effects of the magnetic parameters on the fluid flow mass and heat transfer characteristics have been illustrated graphically and discussed in detail. Significant changes were observed in the fluid flow mass and heat transfer with respect to magnetic parameter.

Key words: Melting process, Stagnation point, Heat absorption/generation, Stretching Surface, micropolar fluid, MHD, heat transfer.

Date of Submission: 11-11-2018 Date of acceptance: 22-11-2018

I. INTRODUCTION

Micropolar fluids are fluids with microstructurebelonging to a class of non-Newtonian fluids with non-symmetrical stress tensor,physically; they represent fluids consisting of randomly oriented particles suspended in a viscousmedium,where the deformation of the particle is ignored. The allure model of micropolar fluids are come from the fact that it is both a significant and a simple generalization of the classical Navier stokes model. The common theory of micropolar fluids was introduced by Eringen[1].This theory is expected to be useful in analysing the behaviour of non-Newtonian fluids. Eringen[2] and[3] enlarge the theory of thermo-micropolar fluids and derived the constitutive law for fluids with microstructure.Ishak et al. [4] considered steady laminar magnetohydrodynamic(MHD) boundary layer flow past a wedge with constant surface heat flux which is immersed in the presence of variable magnetic field.Chamka[5] has studied the mixed convection flow near the stagnation point of a vertical semi-infinite permeable surface in the presence of magnetic field.

Motivated by the above mentioned investigations, we consider the problem of hydrodynamics stagnation point flow toward aporous stretching/shrinking surface with boundary condition.Boundary layer growth has been considered by Chawala[6].Recently, a similarity solution for boundary layer stagnation flow is being reported by Ebert[7]. Rajgopal et al. [8]studied a boundarylayer flow non-Newtonian over a stretching sheet with a unvarying free stream.Two dimensional boundary layer flow caused by moving plate or a stretching sheet is of interest in manufacture of sheeting material through an extrusion process.The problem of stretching sheet has been of great use in engineering studies.Ishak et al. [9] studied the flow and heat transfer over a stretching sheet.Kelson and desseaux[10] studied the effect of surface condition on the micropolar fluid flow driven by a porous stretching sheet.Tien and Yen[11] investigated the effect of melting on forced convection heat transfer between a melting body and surrounding fluid. Pavlow[12] investigated the boundary layer of an electrically conducting fluid due to a stretching of a plane elastic surface in the presence of a uniform transverse magnetic field.Goyal et al. [13] studied stagnation point flow of MHD micropolar fluid in the presence of melting process and heat absorption/ generation.Melting effect of MHD micropolar fluid over a radiative vertical surface towards stagnation point investigated by Goyal et al. [14].

Heat and mass transfer over a continuously stretched surface have considerable recognition in present. This branch from various possible engineering and metallurgical uses such as a hot rolling, wire drawing, metal and plastic extrusion, crystal growing etc. Y.J. kim[15-17] studied unsteady convection flow of micropolar flow over a vertical moving plate in a porous medium and Heat and mass transfer in MHD micropolar fluids past a vertical plate embedded in a porous flow over a vertical moving plate in a porous medium. Tien and vafai[18] studied the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer Heat generation and thermal radiation effects over a stretching sheet in a micropolar fluid was investigated by gnaneswara[18]. Khan et al. [19]studied heat and mass transfer in the third grade nano-fluid flow over a convectively heated stretching permeable surface.

To the best our knowledge this problem in melting process of MHD micropolar fluid toward the stagnation point with mass transfer and heat generation/absorption over stretching/shrinking sheet has not be consider before, so that the result are new.

In the present work, we consider the micropolar fluid flow towards the stagnation point over the stretching/ shrinking sheet with heat and mass transfer in the presence of melting process.A uniform constant transverse magnetic fieldB_0 taken in to account such that the magnetic Reynolds number is assumed to small so that the induced magnetic field is negligible. The non-linear partial differential equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically by Runge-Kutta fourth order with the shooting technique. The effect of different governing parameters on the velocity,temperature,angular velocity,concentration, skin friction coefficient are shown in the figures and analysed in detail.

NOMENCLATURE

a ,c Constant (m^{-1})
B_0 Magnetic field
B Magnetic parameter
C_f Skin-friction coefficient
C_p Specific heat at constant pressure ($Jkg^{-1}K^{-1}$)
C_s Heat capacity of solid surface
Ec Eckert Number
h Dimensionless angular velocity
j Microinertia density (m^2)
K Micropolar or material parameter
K Thermal conductivity of the fluid ($Wm^{-1}K^{-1}$)
n Constant
Nu Nusselt number
Pr Prandtl number
T Temperature (K)
T_s Temperature of the solid medium
u, v Dimensionless velocities along x and y direction respectively
x ,y Axial and perpendicular co-ordinate (m)
Q_oCoefficient of heat absorption
Re_xReynolds number
εStretching parameter
f Dimensionless stream function

Greek symbols

ΨStream function
α Thermal diffusivity
γ Spin-gradient viscosity (N s)
μ Dynamic viscosity (Pa s)
σ Electrical conductivity of the fluid
θ Dimensionless temperature
ρ Density
ν Kinematic viscosity
κ Vortex viscosity
Ω Component of microrotation (rad s^{-1})
λLatent heat of the fluid
τ_wSurface heat flux
ηNon dimensionless distance

Subscripts

M Condition at the melting surface
∞ free stream condition.

s Solid medium

Superscripts

′Derivative with respect to η

II. MATHEMATICAL FORMULATION

The geometrical model of the problem has been given along with flow configuration and coordinate system (see Fig. 1). The system deals with two dimensional stagnation point steady flow of micropolar fluid towards a stretching surface with heat absorption/generation with presence of viscous dissipation and subject to a constant transverse magnetic fieldB_0.The magnetic Reynolds number is assumed to small so that the induced magnetic field is negligible. The velocity of the external flow is $u_e(x) = ax$ and the velocity of the stretching surface is$u_w(x) = cx$, where are$a$and$c$ are positive constants, and x is the coordinate measured along the surface. It is also assumed that the temperature of the melting surface and free stream condition is T_M andT_∞, where T_∞>T_M In addition, the temperature of the solid medium far from the interface is constant and is denoted by T_s where T_s < T_M . The viscous dissipation has assumed to be negligible. Under these assumptions, the governing equations representingflow are as follows:

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (1)$$

Equation of linear momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{(\mu + k)}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial \omega}{\partial y} - \frac{\sigma}{\rho}B_0^2 U_w \quad \ldots (2)$$

Equation of angular momentum

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2} - \frac{\kappa}{\rho j}\left(2\omega + \frac{\partial \omega}{\partial y}\right) \quad \ldots. (3)$$

Equation of Energy

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial y^2} + Q^*(T - T_\infty) \quad \ldots (4)$$

Equation of Mass Transfer

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \ldots (5)$$

The boundary conditions suggested by the physics of the problem are:

$$u = U_w(x) = -ax; \quad v = v_w \omega = -m\frac{\partial u}{\partial y}; \quad T = T_W; \quad C = C_w at y = 0 \ldots (6)$$

$$u \to 0; \quad \omega \to 0 \quad T \to T_w C \to C_\infty as y \to \infty \ldots (7)$$

Here u and v are the velocity component along the x and y axis, respectively. Further, $\mu$is dynamic viscosity, κ is vortex viscosity, σ is electrical conductivity of the fluid, $\rho$is fluid density, T is fluid temperature, j is micro inertia density, ω is microrotation, γ is spin gradient viscosity, α is thermal diffusivity, k is the thermal conductivity, λ is the latent heat of the fluid and C_s is the heat capacity of solid surface. We note that n is a constant such 0≤ n ≤1. The case when n = 0, is called strong concentration which indicates that no microrotation near the wall. In case n = 0.5, it indicates that the vanishing of anti-symmetric part of the stress tensor and denote weak concentration and case n =1 is used for the modelling of turbulent boundary layer flows. $\gamma = (\mu + \frac{K}{2})\, j = \mu(1 + \frac{K}{2})\, j$, where $K = \frac{\kappa}{\mu}$ is the micro polar or material parameter and $j = \frac{\upsilon}{a}$ as reference length. The total spin ω reduces to the angular velocity.

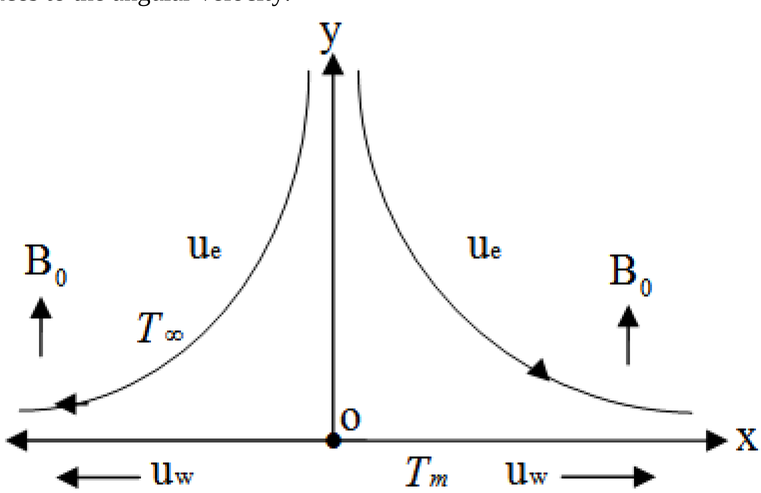

Figure 1 Flow Geometry

3. Problem solution

Equation (2) – (5) can be transform into a set of nonlinear ordinary differential equation by using the following similarity variables:

$\gamma = (\mu + \kappa)\, j = \mu\left(1 + \frac{K}{2}\right) j, \qquad K = \frac{\kappa}{\mu}, \qquad j = \frac{\upsilon}{a},$

$$\Psi = (a\upsilon)^{\frac{1}{2}} x f(\eta); \qquad \omega = xa\left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} h(\eta), \quad \ldots\ldots (7)$$

$$\theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \qquad \eta = \left(\frac{a}{\upsilon}\right)^{\frac{1}{2}} y; \qquad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad \ldots (8)$$

The transformed ordinary differential equations are:

$$(1 + K) f''' + f f'' + 1 - f'^2 + K h' + B(1 - f') = 0 \quad \ldots. (9)$$

$$\left(1 + \frac{K}{2}\right) h'' + f h' - f' h + K(2h + f'') = 0 \ldots (10)$$

$$\theta'' + Pr(f\theta' + H\theta) = 0 \quad \ldots (10)$$

$$\phi'' + Sc(f\phi') = 0 \quad \ldots (11)$$

The boundary conditions (5) and (6) becomes

$$f'(0) = \varepsilon, \quad h(0) = -n f''(0), \quad Pr f(0) + M\theta'(0) = 0, \quad \theta(0) = 0, \phi(0) = 1 \ldots (12)$$

$$f'(\infty) \to 1, \quad h(\infty) \to 0, \quad \theta(\infty) \to 1, \phi \to \infty \ldots. (13)$$

Where prime denote differentiation with respect to η and $Pr = \frac{\upsilon}{\alpha}$ is Prandlt number, $\varepsilon = \frac{c}{a}$is the stretching ($\varepsilon > o$) parameter, M is the dimensionless melting parameter, B is magnetic parameter, H is heat absorption parameter and Eckert number which are defined as

$$M = \frac{C_p(T_\infty - T_m)}{\lambda + C_s(T_m - T_s)}, \qquad B = \frac{\sigma B_0^2}{a\rho}, \qquad H = \frac{Q_0}{a\rho C_p}, Ec = \frac{u_e^2(x)}{c_p(T_m - T_\infty)} \ldots (14)$$

The physical parameter of interest is the skin friction coefficient C_f , local Couple stress coefficient C_m and the Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2} \ldots. (15)$$

$$C_m = \frac{C_w}{x\rho u_e^2} \ldots.. (16)$$

$$Nu_x = \frac{x q_w}{K(T_\infty - T_m)} \ldots. (17)$$

Where $\tau_w, C_w\, and\, q_w$ are the surface shear stress, the local couple stress and the surface heat flux respectively, which are given by

$$\tau_w = (\mu + K)\left(\frac{\partial u}{\partial y}\right)_{y=0} \ldots. (18)$$

$$C_w = \gamma\left(\frac{\partial u}{\partial y}\right)_{y=0} \ldots (19)$$

$$q_w = K\left(\frac{\partial u}{\partial y}\right)_{y=0} \ldots (20)$$

Hence using (6), we get

$$Re_x^{\frac{1}{2}} C_f = [1 + (1 - n)K] f''(0) \quad \ldots (21)$$

$$Re_x C_m = \left(1 + \frac{K}{2}\right) h'(0) \quad \ldots (22)$$

$$Re_x^{-\frac{1}{2}} Nu_x = -\theta'(0) \quad \ldots (23)$$

Where $Re_x = u_e(x)\frac{x}{\upsilon}$ is the local Reynolds number.

III. NUMERICAL SOLUTION

To solve transformed equation (8) to (11) with reference to boundary conditions (12)-(13) as an initial value problem, the initial boundary conditions of $f''(0), h'(0), \theta'(0)$ and $\phi'(0)$ are chosen and Runge-Kutta fourth order method is applied to get solution and calculated value of $f(\eta), h(\eta), \theta(\eta)$ and $\varphi(\eta)$.

at $\eta = \eta_\infty$, where η_∞ is sufficient large value of η are compared with the given boundary conditions $f'(\eta_\infty) = 1, h(\eta_\infty) = 0$ and $\theta(\eta_\infty) = 1$. The missing values of $f''(0), h'(0), \theta'(0)$, and $\varphi'(0)$ for some values of the heat absorption\generation parameter H, magnetic parameter B, melting parameter M, micropolar parameter

K, and the stretching parameter ε are adjusted by shooting method, while the Prandtl number Pr = 1 unity is fixed and we take $n = 0.5$ for weak concentration. We use MATLAB computer programming for different values of step size $\Delta\eta$ and found that there is a negligible, change in the velocity, temperature, local Nusselt number and skin friction coefficient for values of$\Delta\eta > 0.001$. Therefore in present paper we have set step- size $\Delta\eta = 0.001$.

IV. RESULTS AND DISCUSSION

In order to get physical insight into the problem, the numerical calculations for the velocity, micro-rotation and temperature profilesfor various values of the parameter have been carried out. The effects of the main controlling parameters as they appear in the governing equations are discussed in the current section. In this study, entire numerical calculations have been performed with ε = 0.5, n = 0.5 and Pr =1 while Sc, M, K and Hare varied over ranges, which are listed in the figure legends. In order to validate the numerical result obtained they are found to be in a good agreement to previously published paper. The velocity profile u is plotted in **Figure 1** for different values of the Prandtl number when m = 1, H = 0.2, K = 1 and M = 0.5 figure exhibits that when we increase the Prandtl number the velocity profile $f'(\eta)$increase.**Figure2** encounter that the effect of micropolar parameter k on velocity profile $f'(\eta)$when other parameter arem=1, H = 0.2,e=0.75, K = 1 and M = 0.5.Figure evident that increasing the values of the micropolar parameterthe velocity profile decrease. **Figure 3**show that the variation of melting parameter m it is clear from graph velocity$f'(\eta)$ decrease with increase of m. Similarly the same effect show that the variation of magnetic parameter M by **Figure 4**. **Figure 5** shows the variation of Heat source the velocity profile is decreases.The effect of Prandtl number Pr on temperature profile is presented in **Figure 6** and **Figure 7** show that the variation of micropolar parameter when M = 0.5, H = 0.1, m = 1 and K takes various value K = 0, 1, 2. It is evident that temperature profile decrease as K increase, but there is slightly decrease in temperature profile of fluid as increasing the value of micropolar parameter K.**Figure 8**reveals that the effect of melting parameter which shows increment in temperature profile $\theta(\eta)$ due to increasing the value of melting parameter m.

Figure 9 show that the effect of variation of magnetic field then the temperature profile is decrease.The effects of heat absorption parameter H on temperature profile is represented in**Figure 10**. From **Figure 10** we have found that the temperatureprofiles increase as the heat absorption parameter H increase. Again **Figure 11 represent that** angular velocity for different values of Prandtl number.**Figure 12** show that the different value of magnetic field M then the angular velocity profile is decreases and **Figure 13** represent the effect of heat absorption/generation on angular velocity profile whereas angular velocity profile decrease when heat absorption parameter increase on other hand when heat generation parameter increase then angular velocity profile is also increase these result can be explained by the fact that heat absorption parameter is sink and heat generation parameter is source which is evident by **Figure 13**. For different values of the Schmidt number Sc, the translational velocity and microrotation profiles are plotted in Figure 14. It is obvious that the effect of increasing values of Sc result in a decreasing velocity distribution across the boundary layer.

Finally From **Figure 1 to Figure 14** it can be easily seen that far field boundary condition (11) are satisfied asymptotically and it is verifies the accuracy of numerical method scheme used.

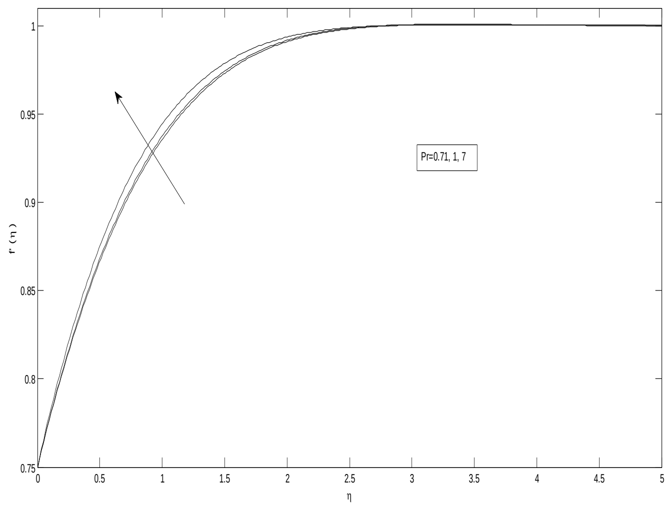

Figure 1 velocity profiles for different values of Prandlt number Pr while M= 0.5, m=1, e=0.75, k=1 ,H=0.2

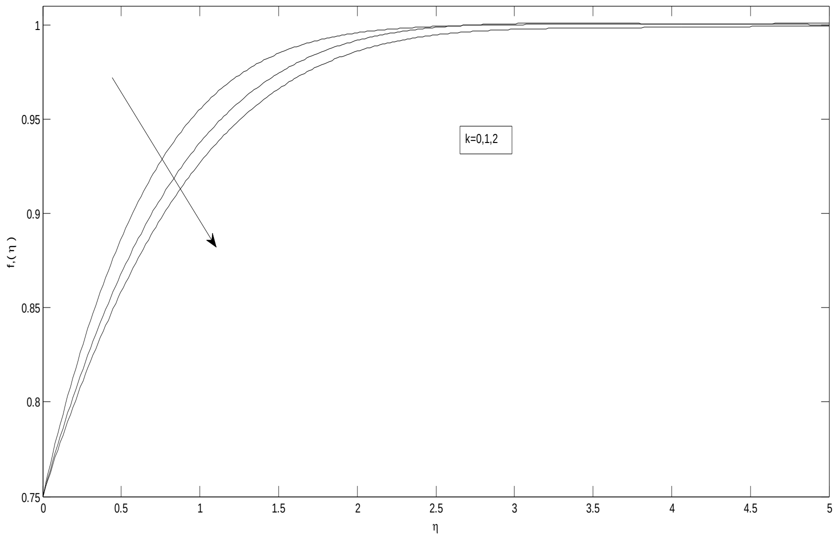

Figure 2 velocity profiles for different values of Micropolar Parameter K while M= 0.5, m=1, e=0.75, Pr=1, H=0.1

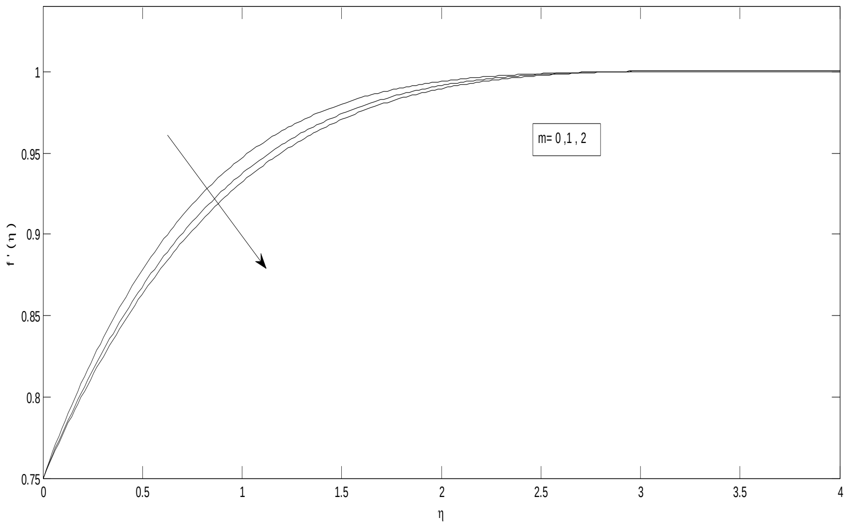

Figure 3 velocity profiles for different values of Melting parameter m while M= 0.5, e=0.75, k=1, Pr=1, H=0.1

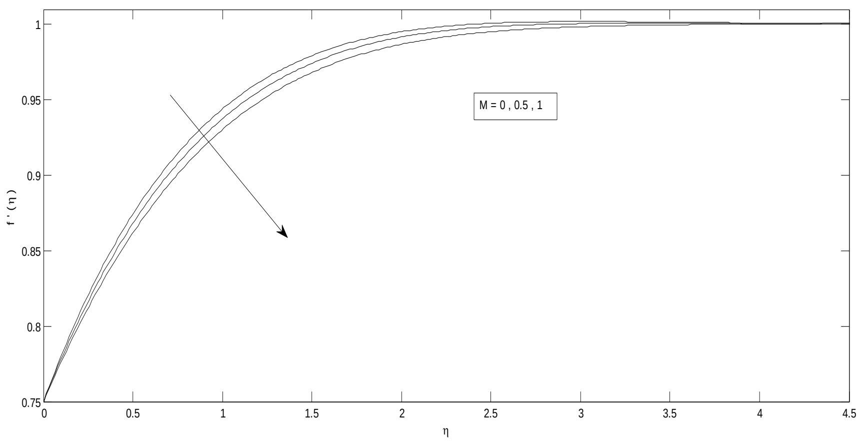

Figure 4 velocity profiles for different values of Magnetic field M while k=1, m=1, Pr=1, H=0.1, e=0.75

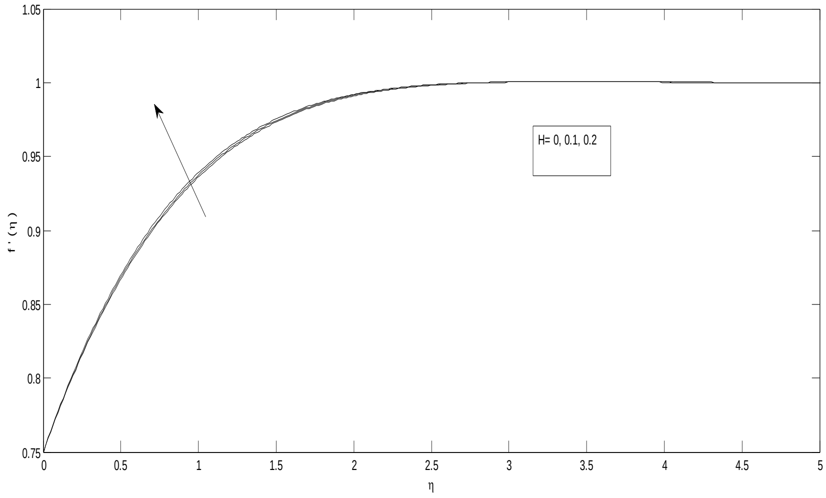

Figure 5 velocity profiles for different values of Heat Source while k=1, m=1, Pr=1, M=0.5, e=0.75

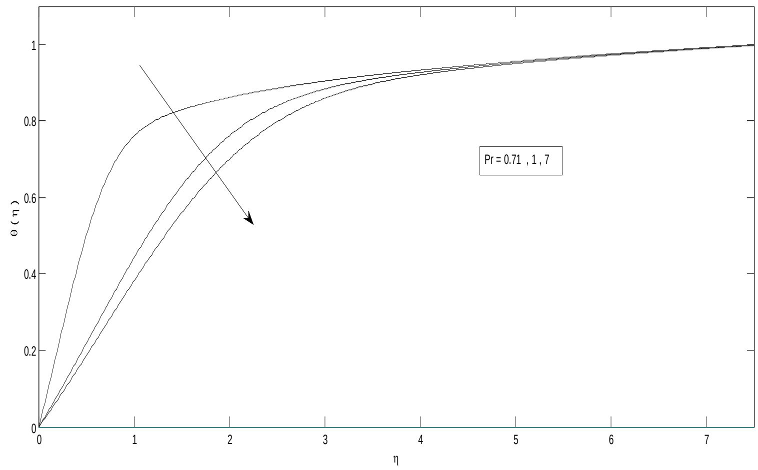

Figure 6 Temperature profiles for different values of Prandtl number Pr while M= 0.5, m=1, e=0.75, k=1

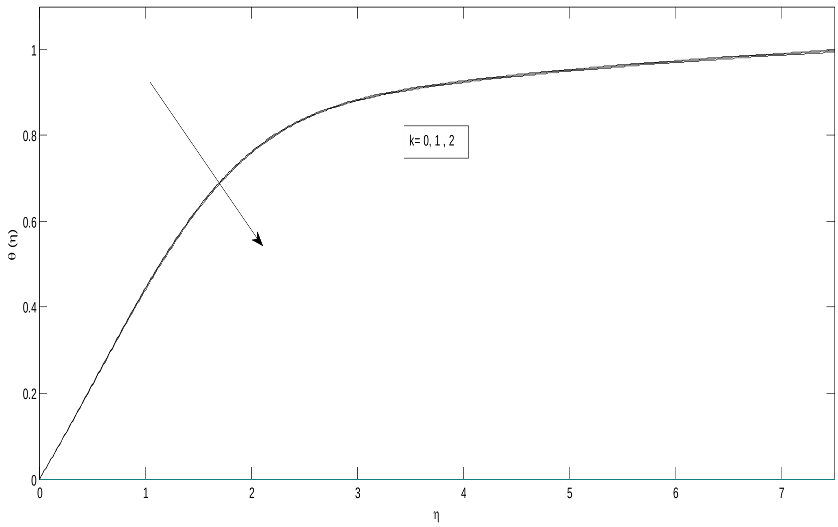

Figure 7 Temperature profiles for different values of Micropolar parameter k while M= 0.5, m=1, e=0.75, Pr=1, H=0.1

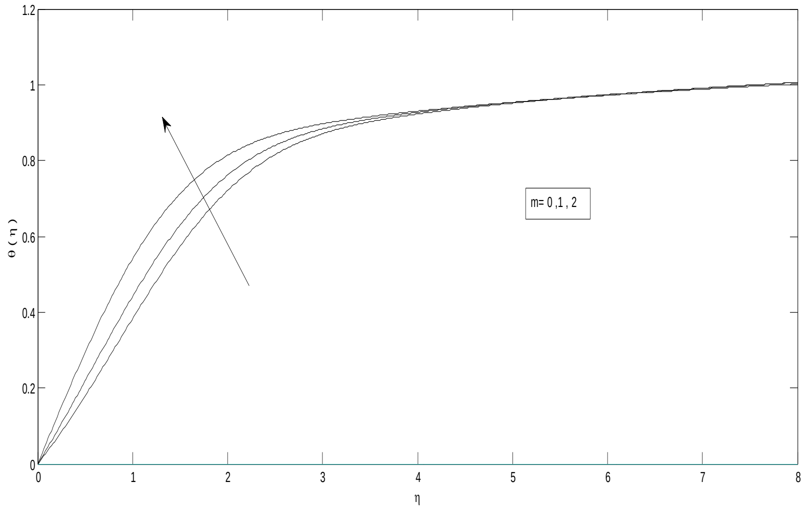

Figure 8 Temperature profiles for different values of Melting parameter m while M= 0.5, e=0.75, k=1, Pr=1, H=0.1

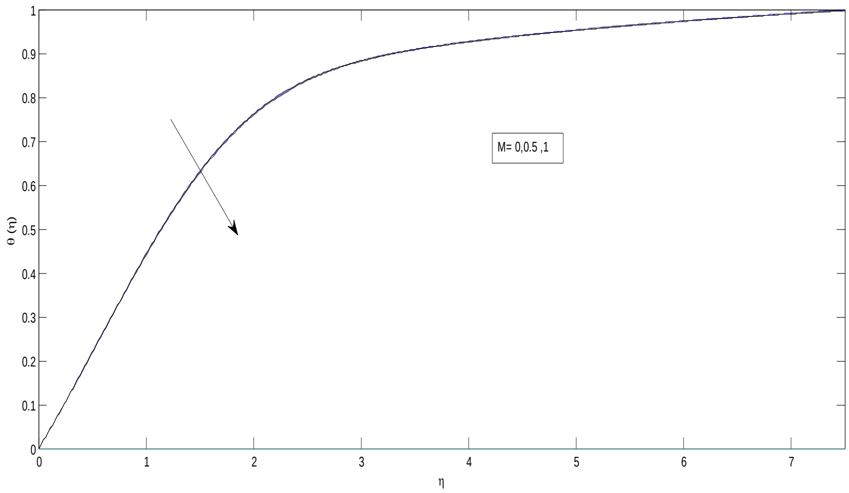

Figure 9 Temperature profiles for different values of Magnetic field M while k=1, m=1, Pr=1, H=0.1, e=0.75

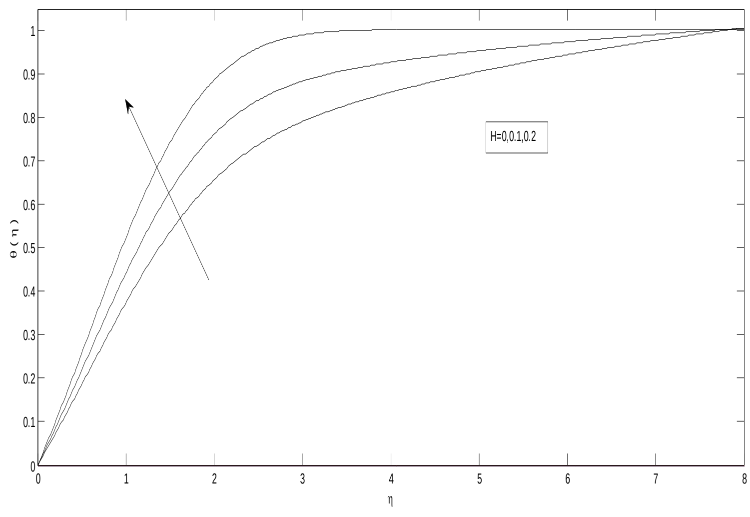

Figure 10 Temperature profiles for different values of Heat Source while k=1, m=1, Pr=1, M=0.5, e=0.75

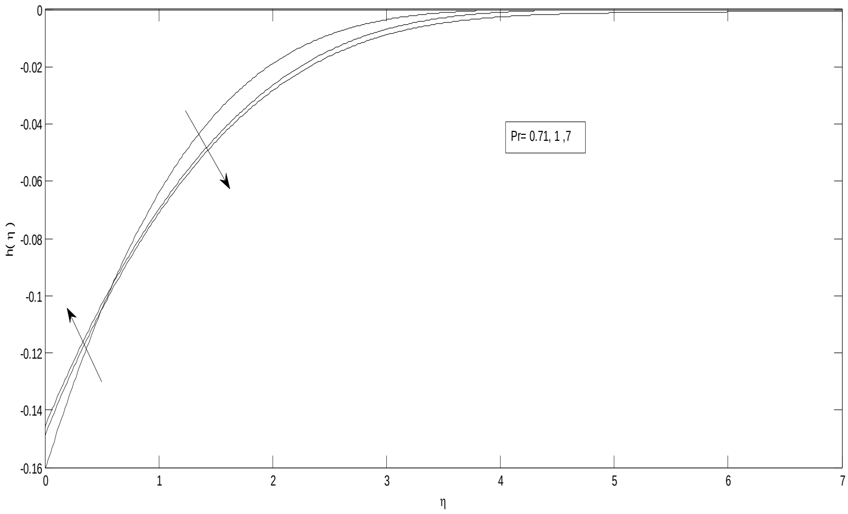

Figure 11 Angular velocity profiles for different values of Prandtl number Pr while M= 0.5, m=1, e=0.75, k=1

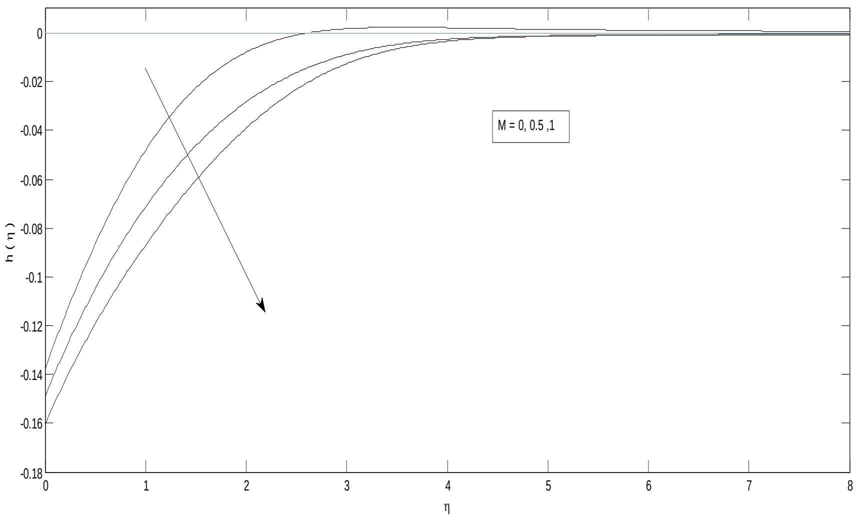

Figure 12Angular velocity profiles for different values of Magnetic field Mwhile m=1, e=0.75, k=1, Pr=1, H=0.1

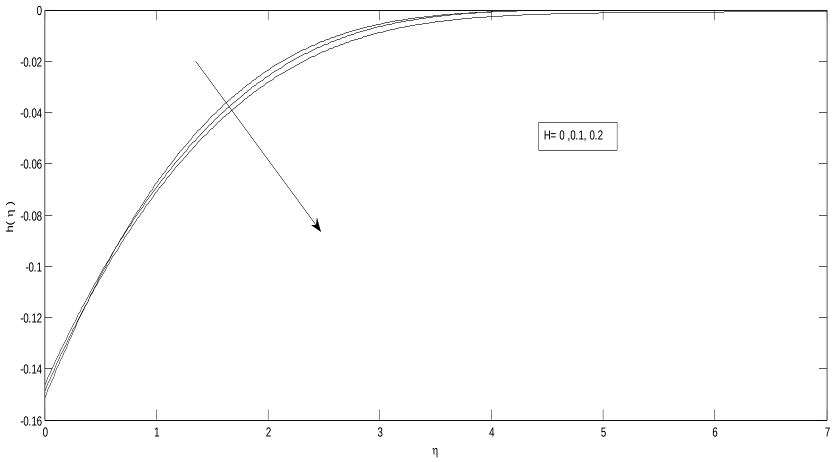

Figure 13 Angular velocity profiles for different values of Heat Source while k=1, m=1, Pr=1, M=0.5, e=0.75

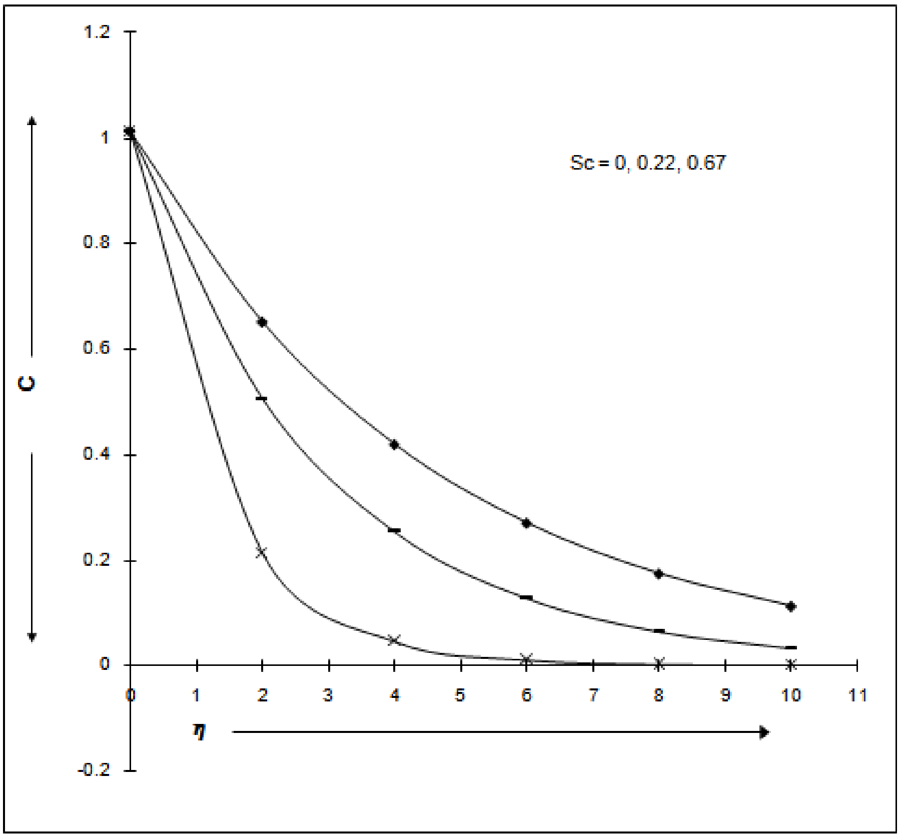

Figure 14 Concentration profiles for different values of Schmidt Number Sc while k=1, m=1, Pr=1, M=0.5, e=0.75

REFERENCES

[1]. A.C. Eringen simple micropolar fluids. Int. J. Eng. Sci. 2 , 205-217(1964)

[2]. A.C. Eringen, Theory of micropolar fluids.J. Math Mech. 16, 1-18 (1966)

[3]. A.C. Erigen, Theory of Thermomicrofluids. J. Math Anal appl. 38, 480-496(1972)

[4]. Ishak A, yocob NA, Pop I, Melting heat transfer in boundary layer stagnation-point flow toward a stretching/shrinking sheet in a micropolar fluid. Heat Mass transfer 47,16-21(2011)

[5]. A.J.Chamka, Hydromagnetic mixed convection stagnation flow with suction and blowing, Int. comm. Heat mass transfer 25,417-426(1998).

[6]. S.S. Chawla, Boundary layer growth. Int. J. Eng. Sci. 10, 981(1972)

[7]. F. Ebert, A similarity solution for boundary layer stagnation flow, Che. Eng. J. 5, 85(1973)

[8]. K.R. Rajagopal, A.S.Gupta,A non-similar boundary layer on a stretching sheet in a non-Newtonian fluid with uniform free stream. Journal of Mathematical and Physical Science, Vol.21 no.2,PP 189-2009 (1987).

[9]. A. Ishak, R Nazar,I. Pop, Heat transfer over a stretching surface with variable heat flux in micro polar fluid.PhyLett A 372,559-561(2008).

[10]. A.Desseaux and N.A Kelson,Flow of micropolar fluid boundary by stretching sheet.ANZIAM Journal, Vol. 42, pp 532-560 (2000).

[11]. C.I. Tien, K Vafai, Convective and radiative heat transfer in porous media, Ad. In Applied mechanics, Vol. 27, pp 225-281(1989).

[12]. K.B. Pavlov, Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a surface, Magneto Hydrodynamic 4, 146-147(1974).

[13]. .M. Goyal, V.Tailor, R. Yadav, Stagnation point flow of MHD presence of melting process and Heat Absorption/generation. Global Journal of Pure and Applied Mathematics, Volume 13,pp.4889-4907(2017)

[14]. M. Goyal, V. Tailor,Melting effect of MHD Micropolar fluid flow over a radiative vertical surface towards stagnation point**,**International organization of Scientific Research, ISSN 2250-3021, Issue-8, Vol.-8, pp.70- 83, August 2018.

[15]. Y.J. Kim,Unsteady convection flow of micropolar fluid past a vertical plate embedded in a porous medium.Acta.Mech.148, 105-116(2001).
[16]. Y.J Kim, Heat and mass transfer in MHD micropolar flow over a vertical moving plate in a porous medium. Trans. Porous Media, 56, 17-37 (2004).
[17]. Y.J.Kim, J.C.Lee, Analytical studies on MHD oscillatory flow of a micropolar fluid over a vertical porous plate, Surface and Coating Technology 171, 187-193 (2003).
[18]. C.I.Tien , K Vafai , Convective and radiative heat transfer in Applied Mechanics 27,225-281 (1989).
[19]. M.GnaneswaraRaddy, Heat generation and thermal radiation effect over a stretching sheet in a micropolar fliud , ISRN Thermodynamics , Vol. 2012 pp 6(2012).
[20]. W.A. Khan, J.R. Cullam and O.D.Makinde, Combined heat and mass of third-grade nanofliud over a convective – heated stretching permeable surface, Can. J. Chem. Eng. 93(10), 1880-1888 (2015).

Dr. Mamta Goyal. "Heat and Mass Transfer Effect in Melting Process of Boundary Layer Stagnation Point Flow of Micropolar Fluid towards a Stretching/Shrinking Surface."IOSR Journal of Engineering (IOSRJEN), vol. 08, no. 11, 2018, pp. 89-99.