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Skolem Mean Labeling Of Four Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 + a_2 + a_3 - 1 \le b \le a_1 + a_2 + a_3 + 1$

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Abstract: A graph G = (V, E) with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by $f^*(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from the set {2, 3, . . . , p}. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 - 1 \le b \le a_1 + a_2 + a_3 + 1$. (or $|b - a_1 - a_2 - a_3| \le 1$).

Keywords: *Skolem mean graph, skolem mean labeling, star graphs*

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I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [3]. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. A graph with p vertices and q edges is called a (p, q) graph. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 - 1 \le b \le a_1 + a_2 + a_3 + 1$.

1. Skolem mean labeling

Definition 1.1: A graph G is a non empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively. $|V(G)| = q$ is called the size of G, we say that u and v are adjacent and that u and v are incident with e.

Definition 1.2: A vertex labelling of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). Similarly, an edge labelling of a graph G is an assignment of labels to the edges of G that induces for each vertex v a label depending on the edge labels incident on it. Total labelling involves a function from the vertices and edges to some set of labels.

Definition 1.3: A graph G with p vertices and q edges is called a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0, 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edge labels are distinct. The labeling f is called a mean labeling of G.

Definition 1.4: A graph G = (V,E) with p vertices and q edges is said to be skolem mean if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from 2, 3, . . . , p. f is called a skolem mean labeling of G. A graph G = (V, E) with p vertices and q edges is said to be a **skolem mean graph** if there exists a function f from the vertex set of G to

{1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by

$$f^*(e=uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even}\\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd}\end{cases}$$

the resulting edges get distinct labels from the set {2, 3, . . . , p}.

Theorem 2.1: The four star $G=K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1+a_2+a_3-1 \le b \le a_1+a_2+a_3+1$.

Proof: Let $A_i=\sum_{k=1}^{i} a_k$. That is, $A_1=a_1$; $A_2=a_1+a_2$ and $A_3=a_1+a_2+a_3$.

Consider the graph $G=K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$. Let $V=\bigcup_{k=1}^{4} V_k$ be the vertex set of G where $V_k=\left\{v_{k,i}:0 \le i \le a_k\right\}$ for $1 \le k \le 3$ and $V_4=\left\{v_{4,i}:0 \le i \le b\right\}$. Let $E=\bigcup_{k=1}^{4} E_k$ be the edge set of G where $E_k=\left\{v_{k,0}v_{k,i}:0 \le i \le a_k\right\}$ for $1 \le k \le 3$ and $E_4=\left\{v_{4,0}v_{4,i}:0 \le i \le b\right\}$.

The condition $a_1+a_2+a_3-1 \le b \le a_1+a_2+a_3+1 \Rightarrow A_3-1 \le b \le A_3+1$

That is, there are three cases viz. $b=A_3-1$, $b=A_3$ and $b=A_3+1$. .

Let us prove in each of the three cases the graph G is a skolem mean graph.

Case 1: Let $b=A_3+1$. G has $A_3+b+4=2A_3+5$ vertices and $A_3+b=2A_3+1$ edges.

The vertex labeling $f:V \to \{1,2,3,\ldots,A_3+b+4=2A_3+5\}$ is defined as follows:

$f(v_{1,i})=2i+3 \qquad 1 \le i \le a_1$

$f(v_{1,0})=1$; $\quad f(v_{2,0})=2$; $\quad f(v_{3,0})=3$; $\quad f(v_{2,i})=2A_1+2i+3 \qquad 1 \le i \le a_2$

$f(v_{4,0})=A_3+b+4=2A_3+5 \qquad f(v_{3,i})=2A_2+2i+3 \qquad 1 \le i \le a_3$

$f(v_{4,i})=2i+2 \qquad 1 \le i \le b=A_3+1$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $2+i$ for $1 \le i \le a_1$ (edge labels are $3,4,\ldots,a_1+2=A_1+2$), $v_{2,0}v_{2,i}$ is A_1+3+i for $1 \le i \le a_2$ (edge labels are $A_1+4, A_1+5,\ldots,A_1+3+a_2=A_2+3$), $v_{3,0}v_{3,i}$ is A_2+3+i for $1 \le i \le a_3$ (edge labels are $A_2+4, A_2+5,\ldots,A_2+3+a_3=A_3+3$), $v_{4,0}v_{4,i}$ is A_3+4+i for $1 \le i \le b=A_3+1$ (edge labels are $A_3+5, A_3+6,\ldots, A_3+4+A_3+1=2A_3+5$)

These induced edge labels of graph G are distinct.

Hence G is a skolem mean graph.

Example:

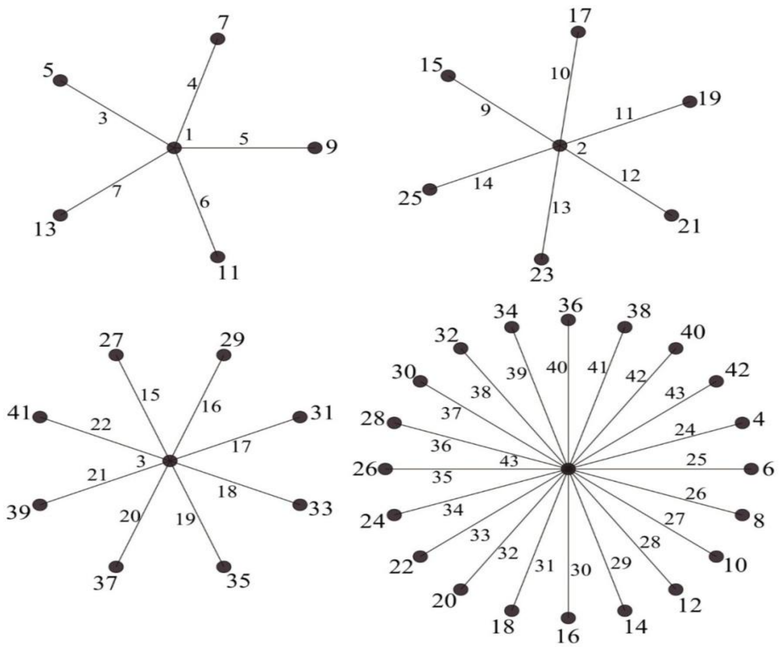

Figure $\left(K_{1,5} \cup K_{1,6} \cup K_{1,8} \cup K_{1,20}\right)$

Case 2: Let $b = A_3$. G has $A_3 + b + 4 = 2A_3 + 4$ vertices and $A_3 + b = 2A_3$ edges.

The vertex labeling $f : V \rightarrow \{1, 2, \ldots, A_3 + b + 4 = 2A_3 + 4\}$ is defined as follows:

$f(v_{1,0}) = 1;\quad f(v_{2,0}) = 2;\quad f(v_{3,0}) = 3;$

$f(v_{4,0}) = A_3 + b + 4 = 2A_3 + 4$

$f(v_{1,i}) = 2i + 3 \qquad 1 \le i \le a_1$

$f(v_{2,i}) = 2A_1 + 2i + 3 \qquad 1 \le i \le a_2$

$f(v_{3,i}) = 2A_2 + 2i + 3 \qquad 1 \le i \le a_3$

$f(v_{4,i}) = 2i + 2 \qquad 1 \le i \le b = A_3$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $2 + i$ for $1 \le i \le a_1$ (edge labels are $3, 4, \ldots, a_1 + 2 = A_1 + 2$), $v_{2,0}v_{2,i}$ is $A_1 + 3 + i$ for $1 \le i \le a_2$ (edge labels are $A_1 + 4, A_1 + 5, \ldots, A_1 + 3 + a_2 = A_2 + 3$), $v_{3,0}v_{3,i}$ is $A_2 + 3 + i$ for $1 \le i \le a_3$ (edge labels are $A_2 + 4, A_2 + 5, \ldots, A_2 + 3 + a_3 = A_3 + 3$), $v_{4,0}v_{4,i}$ is $A_3 + 3 + i$ for $1 \le i \le b = A_3$ (edge labels are $A_3 + 4, A_3 + 5, \ldots, 2A_3 + 3$).

These induced edge labels of graph G are distinct.

Hence G is a skolem mean graph.

Example:

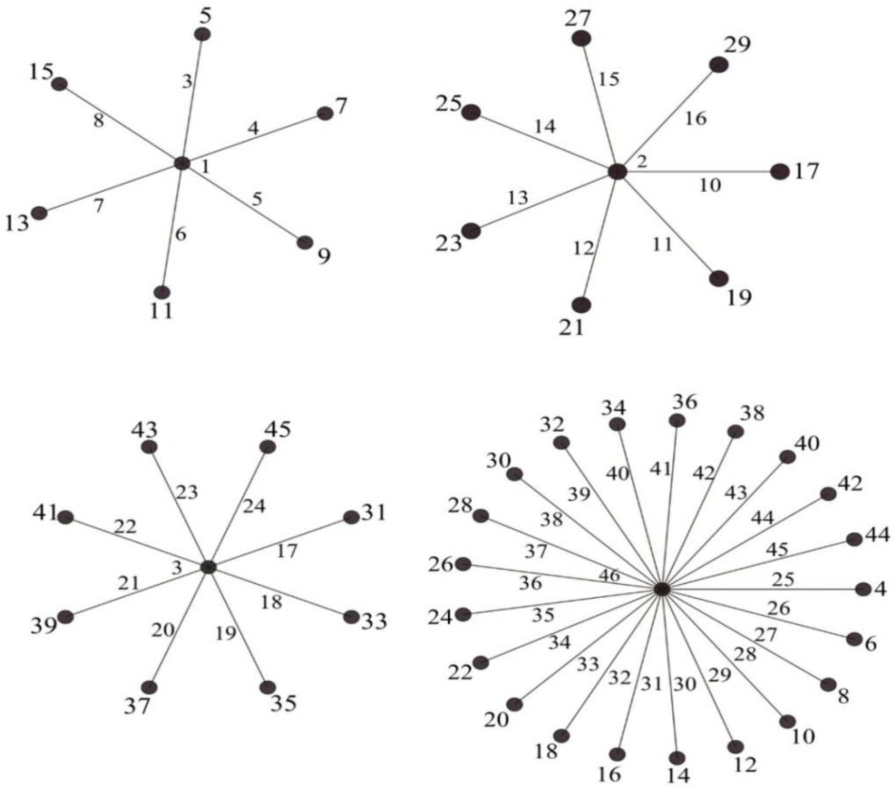

Figure $\left(K_{1,6} \cup K_{1,7} \cup K_{1,8} \cup K_{1,21}\right)$

Case 3: Let $b = A_3 - 1$. G has $A_3 + b + 4 = 2A_3 + 3$ vertices and $A_3 + b = 2A_3 - 1$ edges.

The vertex labeling $f : V \rightarrow \{1, 2, \ldots, A_3 + b + 4 = 2A_3 + 3\}$ is defined as follows:

$f(v_{1,0}) = 1; \quad f(v_{2,0}) = 2; \quad f(v_{3,0}) = 3;$

$f(v_{4,0}) = A_3 + b + 4 = 2A_3 + 3$

$f(v_{1,i}) = 2i + 2 \qquad 1 \le i \le a_1$

$f(v_{2,i}) = 2A_1 + 2i + 2 \qquad 1 \le i \le a_2$

$f(v_{3,i}) = 2A_2 + 2i + 2 \qquad 1 \le i \le a_3$

$f(v_{4,i}) = 2i + 3 \qquad 1 \le i \le b = A_3 - 1$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $2 + i$ for $1 \le i \le a_1$ (edge labels are $3, 4, \ldots, a_1 + 2 = A_1 + 2$), $v_{2,0}v_{2,i}$ is $A_1 + 2 + i$ for $1 \le i \le a_2$ (edge labels are $A_1 + 3, A_1 + 4, \ldots, A_1 + 2 + a_2 = A_2 + 2$), $v_{3,0}v_{3,i}$ is $A_2 + 3 + i$ for $1 \le i \le a_3$ (edge labels are $A_2 + 4, A_2 + 5, \ldots, A_2 + 3 + a_3 = A_3 + 3$), $v_{4,0}v_{4,i}$ is $A_3 + 3 + i$ for $1 \le i \le b = A_3 - 1$ (edge labels are $A_3 + 4, A_3 + 5, \ldots, 2A_3 + 2$).

These induced edge labels of graph G are distinct.

Hence G is a skolem mean graph.

Example:

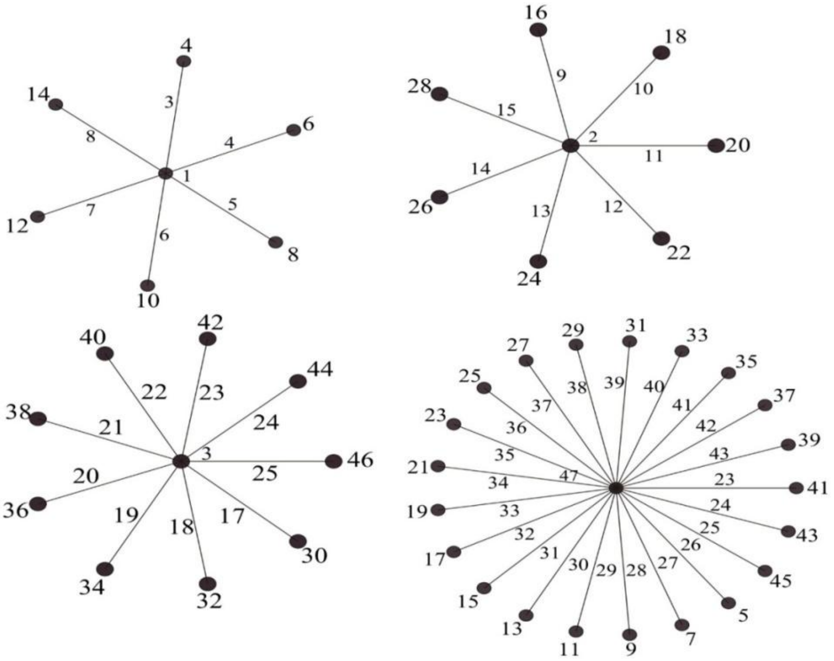

Figure $\left(K_{1,6} \cup K_{1,7} \cup K_{1,9} \cup K_{1,21}\right)$

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