IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 07, Issue 05 (May. 2017), ||V1|| PP 58-65

Bondage Number of a Butterfly Graph

[1] Dr . P.Vijaya Jyothi [2]Dr Indrani Pramod Kelkar
[1]*Assistant Proffesor in Mathematics, Dept of S&H, N.B.K.R.I.T, Vidhyanar ,SPSN, Andhra Pradesh, India,*
[2]*Assistant Vice President, Academics, GATEFORUM, Hyderabad,*

Abstract: Domination Theory is an important branch of Graph Theory that has wide range of applications to various branches of Science and Technology. A new family of graphs called Butterfly Graphs is introduced and study of its parameters is under progress. Butterfly Graphs are undirected graphs and are widely used in interconnection networks.
The bondage number b(G) of a graph G is the minimum cardinality among all sets $S \subset E(G)$ such that $\gamma(G - S) > \gamma(G)$.
In this paper bondage numbers of butterfly graph are studied for various values of n=2, 3, 4, then bondage number of BF(n) is generalized.

Keywords: *Butterfly graph, Domination set, bondage number*
Subject Classification: 68R10

I. INTRODUCTION

A concept connected to domination numbers, called bondage number of a graph was studied by Fink, Jacobson, Kinch and Roberts [5]. The bondage number b(G) of a graph G is the minimum cardinality among all sets $S \subset E(G)$ such that $\gamma(G - S) > \gamma(G)$.
Thus the bondage number of G is the minimum cardinality among all subsets of edges whose removal will render every minimum dominating set in G a non-dominating set in the resultant spanning subgraph. Since the domination number of every spanning subgraph of a non empty graph G is at least γ (G), the bondage number is well defined.
In this chapter various bondage numbers of butterfly graph are studied. Further bondage number and some bounds on them are discussed.
For results on dominating sets and domination number of BF(N) required to study bondage number we refer Chapters 4, 5 of [8].
The cases n = 2, 3, 4 are considered separately and then the result is generalized by using Recursive Construction 2 for all values of $n > 4$.

II. BONDAGE NUMBER OF A BUTTERFLY GRAPH

Lemma 1 : The bondage number of BF(2) is 2.
Proof : Consider the butterfly graph BF(2). The domination number of BF (2) is 2 (Lemma 4.5, Chapter 4 of [8]). In BF(2) every vertex in one level is adjacent to 3 vertices of the other level. So by the definition of edges in BF(2), observe that a vertex (0; s) of L_0 is adjacent to a vertex (0; r) of L_1, for s, r = 0, 1, 2, 3, where $s + r \neq 3$. Let e be an edge joining the vertices (0; s) and (0; r) such that $s + r \neq 3$. Consider the graph BF(2) \ {e}. Let D = {(0; m_1), (1; m_2) / $m_1 + m_2 = 3$ }. The four possibilities for D are given below.

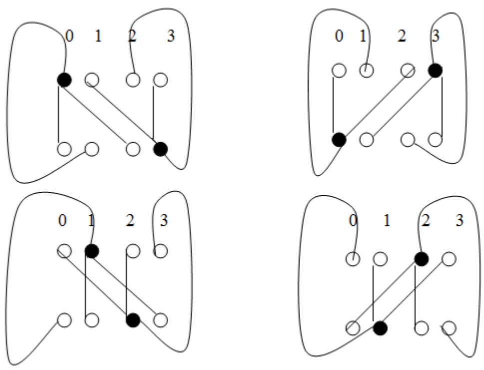

Fig. 1 BF(2)

From the above possibilities , it can be verified that D given above also becomes a dominating set of BF(2) \ e of cardinality 2 for any selection of edge e. Therefore γ (BF(2) \ e) = 2 = γ (BF(2)). That is , deletion of a single edge from BF(2) does not alter the domination number. Now the deletion of two edges from BF(2) increases the domination number is shown below.

Let e_1, e_2 be any two winged edges of BF(2), given by e_1 = {(o; r), (1; s)} , e_2 = {(o; t_1), (1; t_2)}, where s + r $\neq$ 3, $t_1 + t_2 \neq 3$. Let F = { e_1, e_2 }. Consider the graph BF(2) \ F . Without loss of generality take r = 0, s = 1 and t_1 = 3 and t_2 = 2. Then e_1 = { (0 ; 0), (1; 1)}, e_2 = {(0; 3), (1; 2)}. Then for all possible choices of D given above, it can be verified that no D can dominate one of the end vertices of either e_1 or e_2 .

Consider the dominating set D = {(0; 0), (1; 3) }. It is obvious that, this set can not dominate the vertex (1, 1) , as e_1 is the only edge joining (0; 0) and (1; 1). Therefore , adjoin (1; 1) to D, so that D becomes D = {(0; 0), (1; 3), (1; 1)} and D dominates all vertices of BF(2) \ F. Further this set D is minimum. That is γ(BF(2) \ F) = 3 and hence γ (BF(2) \ F) > γ (BF(2)).

For all possible values of r , s and t_1 , t_2 such that s + r $\neq$ 3, $t_1 + t_2 \neq 3$, any of the above mentioned dominating sets can not dominate one of the end vertices of e_1 or e_2 .

Hence for all possible choices of e_1, e_2, γ (BF(2) \ F) > γ (BF(2)). Therefore b (BF(2)) = 2. □

Lemma 2 : The bondage number of BF(3) is 4.

Proof : Consider the graph BF(3). It is known that γ (BF(3)) = 6 (Lemma 4.6, Chapter 4 of [8]). Let F be a set of edges incident on a vertex, say (1; r), r = 0,1, 2,…7. Consider the graph BF(3)\ F. This vertex becomes isolated in BF(3) \ F. Then | F | = 4 , since the degree of every vertex in BF(n) is 4. It is observed in Chapter 2 that there are six triangles in BF(3) and each triangle contributes a vertex to a dominating set of BF(3). Thus to dominate the remaining 11 vertices in the left copy of BF(3) \ (1; r) , 3 vertices are needed and to dominate 12 vertices in the right copy of BF(3) \ (1; r) , again 3 vertices are needed . Thus these 6 vertices dominate all the vertices of BF(3) \ (1; r). So along with the vertex (1; r), a dominating set of BF(3) \ F has minimum cardinality 7.

Thus γ(BF(3) \ F) $\neq$ γ(BF(3)).

Now we claim that removal of any 3 edges from BF(3) does not increase the domination number of BF(3). This is proved by taking the edges in $K_{2,2}$ s, between L_0 , L_1 and L_1 ,L_2.

Case 1 : Let F = {{ (0; r), (1; r) }, { (0; r), (1; r + 2) },{ (0; s), (1; s)} where r = 0,1 and $\left| r - s \right| = 1$. Consider the graph BF(3) \ F. The selection of vertices for domination in BF(3) \ F is as follows. Let D_1 denote a dominating set of BF(3) \ F in the left copy.

Consider a $K_{2,2}$ between L_0 and L_1 incident on four vertices **(**0; r), (0; r + 2), (1; r), (1; r + 2) in the left copy for r = 0, 1. Since the edge incident on two vertices (0; r) , (1; r) and the edge incident on two vertices (0; r) , (1; r + 2) in the first $K_{2,2}$ are deleted , select the vertex (0; r + 2) or (1; r) into D_1 . First select a vertex of L_0 , say (0; r + 2) into D_1 . This vertex dominates two vertices (1; r+2) , (1; r) of L_1 and one vertex (2; r+2) of L_2 .

Consider another $K_{2,2}$ between L_0 and L_1 incident on four vertices **(**0; s), (0; s + 2), (1; s), (1; s + 2) in the left copy, where $\left| r - s \right| = 1$. The edge incident on two vertices (0; s) , (1; s) in the second $K_{2,2}$ is deleted . So select the vertex (0; s + 2) or (1; s + 2) into D_1 . As a vertex from L_0 is already taken into D_1 , we select (1; s + 2) into D_1 . This vertex dominates two vertices (0; s + 2) , (0; s) of L_0 and one vertex (2; s + 2) of L_2 .

Then D_1 = {(0; r + 2), (1; s + 2)} = {(0; r_1), (1; s_1)} , say. That is r + 2 = r_1 and s + 2 = s_1 .Now select a vertex from L_2 into D_1 . Then select the vertex (2; t_1) in L_2 such that $\left| t_1 - s_1 \right| = 2$ or 1. Thus D_1 = {(0; r_1),(1; s_1), (2; t_1)}.

For the case r = 0 , s = 1, we get D_1 = {(0; 2), (1; 3), (2; t_1)} where $\left| t_1 - 3 \right| = 2$ or 1 . We take t_1 = 1. So D_1 = {(0; 2), (1; 3), (2; 1)}.

The following figure illustrates the selection of vertices into D_1 .

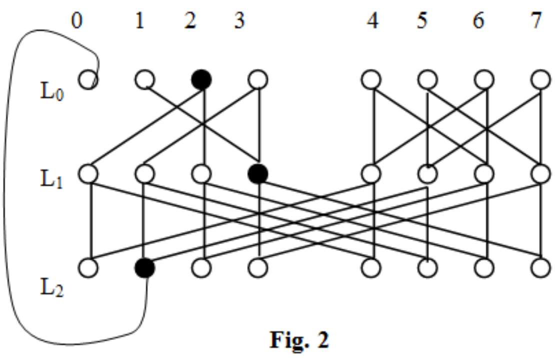

Fig. 2

From the figure, observe that the vertices in D_1, dominate all the vertices except (2; r) = (2; 0) in the left copy of BF(3) \ F .

The mirror image of D_1 in the right copy, denoted by D_2 contains the vertices {(0; r_2), (1; s_2), (2; t_2) } where $r_1 + r_2 = 7$, $s_1 + s_2 = 7$, $t_1 + t_2 = 7$. Then as per the above values of r_1 , s_1 , t_1 , D_2 = {(0; 5), (1; 4), (2; 6) } .

The following figure illustrates the selection of vertices into D_2 .

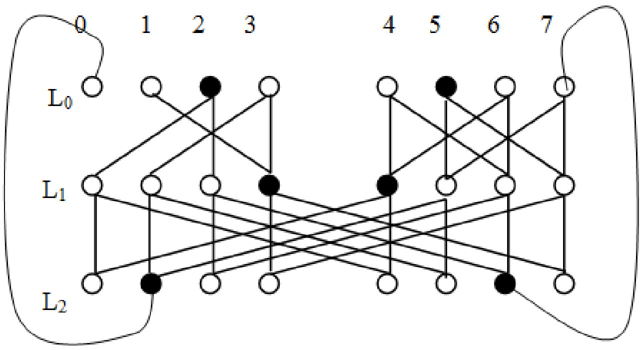

Fig. 3

It can be verified from the figure that D_2 dominates all the vertices in the right copy of BF(3) \ F except the vertex (2; t_2+1) = (2; 7).

Now the undominated vertex (2; r) = (2; 0) in the left copy is dominated by the selected vertex (1; 4) = (1; s_2) and the undominated vertex (2; 7) = (2; t_2+1) in the right copy is dominated by the selected vertex (1; 3) = (1; s_1) .

Let D = $D_1 \cup D_2$. Then all vertices of BF(3) \ F are dominated by the 6 selected vertices in D. Therefore γ (BF(3)) = γ (BF(3) \ F) .

For other choices of edges in these two $K_{2,2}$ s, it can be shown that the domination number for BF(3) and BF(3) \ F is unaltered. Similar is the case if $K_{2,2}$ s are taken between L_0 and L_1 in the right copy .

Case 2 : Let F = {{ (1; r^1), (2; r^1) }, { (1; r^1), (2; r^1+2^2) },{ (1; s^1), (2; s^1)} where r^1 = 0, 1, 2, 3 , $\left| r^1 - s^1 \right|$ = 1 . Consider the graph BF(3) \ F. The selection of vertices for domination in BF(3) \ F is as follows. Let D_1 denote a dominating set of BF(3) \ F in the left copy of BF(3).

Consider a $K_{2,2}$ between L_1 and L_2 incident on four vertices (1; r^1), (1; $r^1 + 2^2$), (2; r^1), (2; $r^1 + 2^2$) for r^1 = 0 , 1, 2, 3. Since the edge incident on two vertices (1; r^1) , (2; $r^1 + 2^2$) and the edge incident on two vertices (1; r^1), (2; r^1) in the first $K_{2,2}$ are deleted , select the vertex (1; $r^1 + 2^2$) or (2; r^1) into D_1 . First select a vertex of L_2 , say (2; r^1) into D_1 . This vertex dominates two vertices (1; $r^1 + 2^2$) of L_1 and one vertex (0; r^1) of L_0 .

Consider another $K_{2,2}$ between L_1 and L_2 incident on four vertices (1; s^1), (1; $s^1 + 2^2$), (2; s^1), (2; $s^1 + 2^2$) , where $\left| r^1 - s^1 \right|$ = 1 . Again, since the edge incident on two vertices (1; s^1), (2; s^1) in the second $K_{2,2}$ is deleted , select the vertex (1; s^1) or (2; $s^1 + 2^2$) into D_1 . As a vertex from L_2 is already taken into D_1 , select (1; s^1) of L_1 into D_1 . This vertex dominates 2 vertices (2; s^1+2^2) of L_2 and one vertex (0; s^1) of L_0 . Then D_1 = {(2; r^1) , (1; s^1) } .

Next select a vertex from L_0 into D_1. Let this vertex be (0; t^1) in L_0 such that $\left| t^1 - s^1 \right|$ = 1. Thus D_1 = {(2; r^1), (1; s^1) , (0; t^1)}.

For the values of $r^1 = 0$, $s^1 = 1$, D_1 = {(2; 0), (1; 1), (0; t^1)} where $\left| t^1 - s^1 \right|$ = 1 . So $t^1 = 2$ and D_1 = {(2; 0), (1; 1), (0; 2)} .

The following figure illustrates the selection of vertices into D_1 .

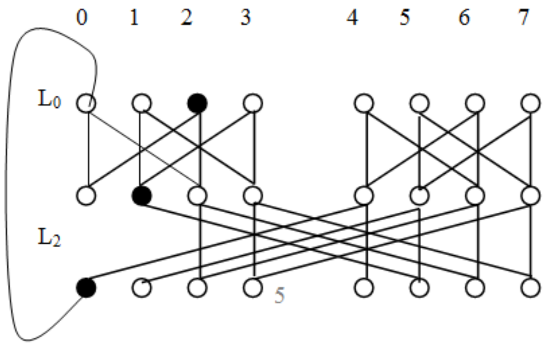

Fig. 4

From the figure it can be verified that the vertices in D_1, dominate all the vertices except $(0; t^1 + 1) = (1;3)$ in the left copy of BF (3) \ F . As in Case 1 , the mirror image of D_1 in the right copy, denoted by D_2 contains the vertices { $(2; r_1^1)$, $(1; s_1^1)$, $(0; t_1^1)$ } where $r^1 + r_1^1 = 7$, $s^1 + s_1^1 = 7$, $t^1 + t_1^1 = 7$. Then as per the above values of r^1 , s^1 , t^1 , D_2 becomes {(2; 7), (1; 6), (0; 5) }.

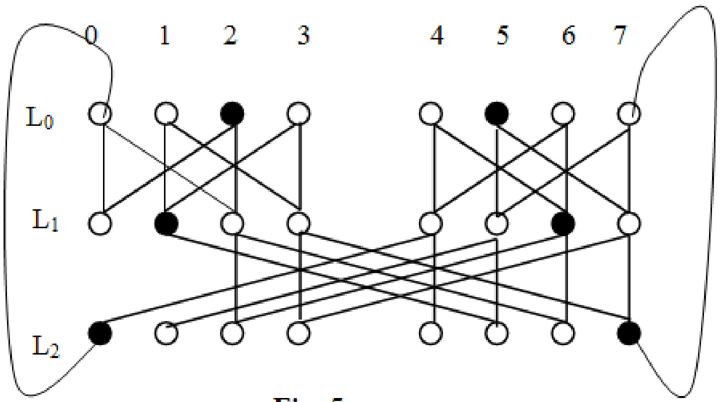

Fig. 5

Again it can be verified from the figure that D_2 dominates all the vertices in the right copy of BF(3) \ F except the vertex $(1; s_1^1 + 3) = (1; 4)$. Now the undominated vertex (1; 3) in the left copy is dominated by the selected vertex (2; 7) and the undominated vertex (1; 4) in the right copy is dominated by the selected vertex (2; 0) . Let $D = D_1 \cup D_2$. Thus all vertices of BF(3) \ F are dominated by the 6 selected vertices in D. Therefore γ (BF(3)) = γ (BF(3) \ F) .
For other choices of edges in these two $K_{2,2}$ s, for $r^1 = 4, 5, 6, 7$, it can be shown that the domination number of BF(3) and BF(3) \ F is unaltered.
Similarly for any choice of three edges in BF(3) between L_2 and L_0 , it can be shown that the domination number of BF(3) and BF(3) \ F is unaltered.
Thus b(BF(3)) = 4. □

Lemma 3 : The bondage number of BF(4) is 4.

Proof : Consider the graph BF(4). Then γ(BF(4)) = 16 (Lemma 4.7, Chapter 4 of [8]). Consider a set F = $\{e_1, e_2, e_3, e_4\}$ of edges in BF(4). Suppose these 4 edges in F are incident on a single vertex say (k; m). Then removal of these 4 edges from BF(4) makes (k; m) isolated in BF(4) \ F. Hence this vertex is to be included into every dominating set of BF(4) \ F.
Let D_1 denote a dominating set of BF(4). From Recursive Construction 1, we know that BF(4) has two copies of BF(3) and a level L_4 with 16 vertices. Each of this copy includes 8 vertices into any dominating set of BF(4). Suppose (k; m) belongs to BF(3) in the left copy of BF(4).
Now to dominate the remaining vertices of the left copy of BF(4), it is must to include 8 vertices into D_1. Also 8 vertices from the right copy of BF(4) are to be included into D_1. Now these 16 vertices in D_1 will dominate all the vertices of BF(4), except (k; m), since it is isolated.
Thus cardinality of a dominating set of BF(4) \ F becomes 16 + 1 = 17. This is the minimum cardinality, as any set of cardinality 16 cannot dominate BF(4) \ F.
Thus γ(BF(4) \ F) > γ(BF(4)).

Hence $b(BF(4)) \geq 4$.
Now we claim that removal of any 3 edges from BF(4) does not increase the domination number of BF(4). This is proved by taking the edges in $K_{2,2}$ s, between L_0, L_1 and L_1, L_2.

Case 1 : Let $F = \{\{(0; r), (1; r)\}, \{(0; s), (1; s)\}, \{(0; s), (0; s + 2)\}\}$, where r = 0, 1, $|r - s| = 1$.
Consider the graph BF(4) \ F. The selection of vertices for domination in BF(4) \ F is as follows. Let D_1 denote a dominating set of BF(4) \ F.
Consider a $K_{2,2}$ between L_0 and L_1 incident on four vertices (0; r), (0; r + 2), (1; r), (1; r + 2) in the left copy where r = 0, 1. Since the edge incident on vertices (0; r), (1; r) in the first $K_{2,2}$ is deleted, select the vertex (0; r+2) or (1; r + 2) into D_1. First select a vertexof L_1, say (1; r+2) into D_1. This vertex dominates two vertices (0; r + 2), (0; r) of L_0 and one vertex (2; r + 2) of L_2.
Consider another $K_{2,2}$ between L_0 and L_1 incident on four vertices (0; s), (0; s + 2), (1; s), (1; s + 2) in the left copy, where $|r - s| = 1$. Again the edge incident on two vertices (0; s), (1; s) and the edge incident on two vertices (1; s), (0; s + 2) in the second $K_{2,2}$ are deleted, either the vertex (1; s + 2) or (0; s + 2) is to be selected into D_1. Select the vertex (1; s + 2) into D_1. This vertex dominates two vertices (0; s + 2), (0; s) of L_0 and one vertex (2; s + 2) of L_2.
Then $D_1 = \{(1; r + 2), (1; s + 2)\}$.
It is observed that there are four $K_{2,2}$ s between L_0 and L_1 in the left copy of BF(4). As the next $K_{2,2}$ s are mirror image of the first $K_{2,2}$ s in the left copy, we select the mirror image of D_1 in these $K_{2,2}$ s. So D_1 becomes $D_1 = \{(1; r + 2), (1; s + 2), (1; r_1), (1; s_1)\}$ where $r_1 = r + 2^2$, $s_1 = s + 2^2$.
Here $(1; r_1)$, $(1; s_1)$ dominate the vertices $(0; r_1)$, $(0; r_1 + 2)$ and $(0; s_1)$, $(0; s_1 + 2)$ respectively. Thus all vertices of L_0 in the left copy are dominated.
Consider four $K_{2,2}$ s between L_1 and L_2 incident on vertices $\{(1; r_2), (1; r_2 + 2^2), (2; r), (2; r_2 + 2^2)\}$, $\{(1; s_2), (1; s_2 + 2^2), (2; s_2), (2; s_2 + 2^2)\}$, $\{(1; p), (1; p + 2^2), (2; p), (2; p + 2^2)\}$, $\{(1; q), (1; q + 2^2), (2; q), (2; q + 2^2)\}$ respectively, where $r_2 \neq s_2 \neq p \neq q$ and $r_2 = 0, 1, 2, \ldots .7$ in the left copy of BF(4) \ F. Then select the vertices $(2; r_2)$, $(2; s_2)$, (2; p), (2; q) of L_2 into D_1, as the vertices in L_1 are already selected into D_1 and any dominating set of BF(4) \ F contains vertices from two consecutive levels.
Now $D_1 = \{(1; r + 2), (1; s + 2), (1; r_1), (1; s_1), (2; r_2), (2; s_2), (2; p), (2; q)\}$.
Here $(2; r_2)$ dominates $(1; r_2)$, $(1; r_2 + 2^2)$, $(2; s_2)$ dominates $(1; s_2)$, $(1; s_2 + 2^2)$, (2; p) dominates (1; p), $(1; p + 2^2)$, (2; q) dominates (1; q), $(1; q + 2^2)$. Thus four vertices of L_1 are dominated in the left copy of BF(4) \ F. Now the selected vertices in L_1 into D_1 viz., (1; r + 2), (1; s + 2), $(1; r_1) = (1; r + 2^2)$ $(1; s_1) = (1; s + 2^2)$ dominate respectively the vertices $(2; r + 2 + 2^2)$, $(2; s + 2 + 2^2)$, $(2; r_1) = (2; r + 2^2)$, $(2; s_1) = (2; s + 2^2)$ of L_2. Thus including the selected vertices of L_2, all the vertices of L_2 in the left copy of BF(4) \ F are dominated.
The selected vertices in L_2 will dominate the vertices $(3; r_2)$, $(3; s_2)$, (3; p), (3; q) of L_3 in the left copy of BF(4) \ F and also $(3; r_2 + 2^3)$, $(3; s_2 + 2^3)$, $(3; p + 2^3)$, $(3; q + 2^3)$ of L_3 in the right copy of BF(4) \ F. Now the vertices $(3; r_2 + 2^2)$, $(3; s_2 + 2^2)$, $(3; p + 2^2)$, $(3; q + 2^2)$ of L_3 in the left copy of BF(4) \ F are undominated.
For the case r = 0, s = 1, r_2 = 0, s_2 = 1, p = 2, q = 3, we get
$D_1 = \{(1; 2), (1; 3), (1; 4), (1; 5), (2; 0), (2; 1), (2; 2), (2; 3)\}$.
The following figure illustrates the selection of vertices into D_1.

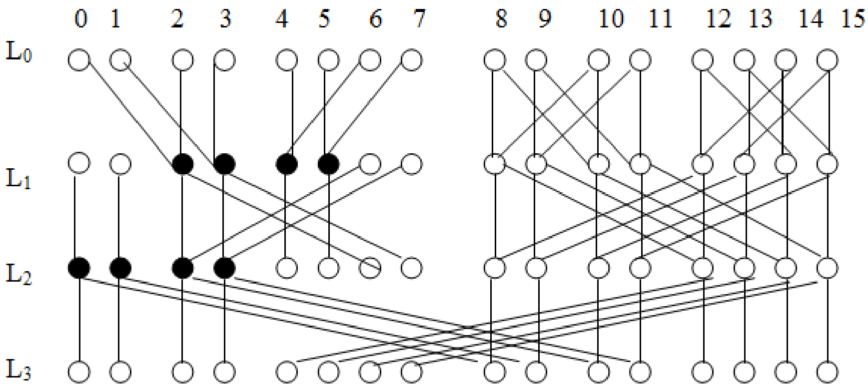

Fig. 6

From the figure, observe that the vertices in D_1, dominate all the vertices except $(3; r_2 + 2^2) = (3; 4)$, $(3; s_2 + 2^2) = (3; 5)$, $(3; p + 2^2) = (3; 6)$, $(3; q + 2^2) = (3;7)$ in the left copy of BF(4) \ F .
The mirror image of D_1 in the right copy, denoted by D_2 contains the vertices $\{(1; r^1), (1; s^1), (1; r_1^1), (1; s_1^1), (2; r_2^1), (2; s_2^1), (2; p^1), (2; q^1)\}$ where $r + 2 + r^1 = 15$, $s + 2 + s^1 = 15$, $r_1 + r_1^1 = 15$, $s_1 + s_1^1 = 15$, $r_2 + r_2^1 = 15$, $s_2 + s_2^1 = 15$, $p + p^1 = 15$, $q + q^1 = 15$. Then as per the above values of r , s , r_2, s_2 , p , q , D_2 becomes {(1; 13), (1; 12), (1; 11), (1; 10), (2; 15), (2; 14), (2; 13), (2; 12) }.
As above it can be shown that the vertices in D_2 dominate all the vertices in the right copy of BF(4) \ F except the vertices $(3; r_2^1 - 7)$, $(3; s_2^1 - 5)$, $(3; p^1 - 3)$, $(3; q^1 - 1)$ of L_3 in the right copy of BF(4) \ F.
The following figure illustrates the selection of vertices into D_2 .

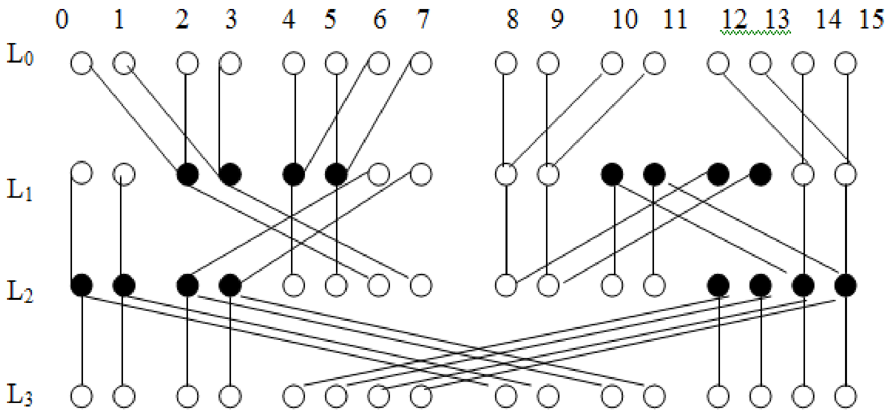

Fig. 7

Again it can be verified from the figure that D_2 dominates all the vertices in the right copy of BF(4) \ F except the vertices $(3; r_2^1 -7) = (3; 8)$, $(3; s_2^1 - 5) = (3; 9)$, $(3; p^1 - 3) = (3; 10)$, $(3; q^1 -1) = (3; 11)$ in the right copy of BF(4) \ F .Now the undominated vertices (3; 4), (3; 5), (3; 6), (3; 7) in the left copy are dominated by the selected vertices (2; 12), (2; 13), (2; 14), (2; 15) and the undominated vertices (3; 8), (3; 9) , (3; 10), (3; 11) in the right copy is dominated by the selected vertices (2; 0), (2; 1) , (2; 2), (2; 3) .Let $D = D_1 \cup D_2$. Thus all vertices of BF(4) \ F are dominated by the 16 selected vertices in D. Therefore γ (BF(4)) = γ (BF(3) \ F) .
For other choices of edges in these two $K_{2,2}$ s, it can be shown that the domination number for BF(4) and BF(4) \ F is unaltered. Similar is the case if $K_{2,2}$ s are taken between L_0 , L_1 and L_1 , L_2 in the right copy .
Case 2 : Suppose three edges are selected into F , between two levels L_2 and L_3 .Let $F = \{\{(2; r), (3; r)\}, \{(2; r+ 2), (3 ; r+ 2)\},\{(2; r), (3; r + 2^3)\}$, r = 0, 1, 2, …7.
As the selection of vertices into a dominating set D of BF(4) is from any two consecutive levels , select the vertices of L_2 and L_3 into D.
Consider the six $K_{2,2}$ s in the left copy of BF(4) between L_2 and L_3 given by
$K_{2,2} = \{(2; r), (3; r), (2; r + 2^3), (3; r + 2^3)\}$, r = 0, 2, 4, 5, 6 , 7 .
$K_{2,2} = \{(2; s), (3; s), (2; s+ 2^3), (3; s + 2^3)\}$, s = r + 2
$K_{2,2} = \{(2; p), (3; p), (2; p + 2^3), (3; p + 2^3)\}$, p = s + 2
$K_{2,2} = \{(2; q), (3; q), (2; q + 2^3), (3; q + 2^3)\}$, q = p + 1
$K_{2,2} = \{(2; t), (3; t), (2; t + 2^3), (3; t + 2^3)\}$, t = q + 1
$K_{2,2} = \{(2; w), (3; w), (2; w + 2^3), (3; w + 2^3)\}$,w = t + 1
Let D_1 denote a dominating set of BF(4) \ F in the left copy . Taking into consideration the deletion of edges in F , select the following vertices from these $K_{2,2}$ s into D_1 .
Let D_1 = {(3; r), (3; s) , (3; p) , (2; p), (2; q), (3; t), (2; t), (2; w) }.
Here observe that the vertices in D_1 dominate all vertices in the left copy of BF(4) \ F , except the vertices (2; r +1), (2; s +1) of L_2 and (3; r +1), (3; s +1) of L_3 For the values of r = 0 , s = 2 , p = 4, q = 5 , t = 6, w = 7 , D_1 becomes {(3; 0), (3; 2) , (3; 4) , (2; 4), (2; 5), (3; 6), (2; 6), (2; 7) } and the following figure illustrates the selection of vertices into D_1 .

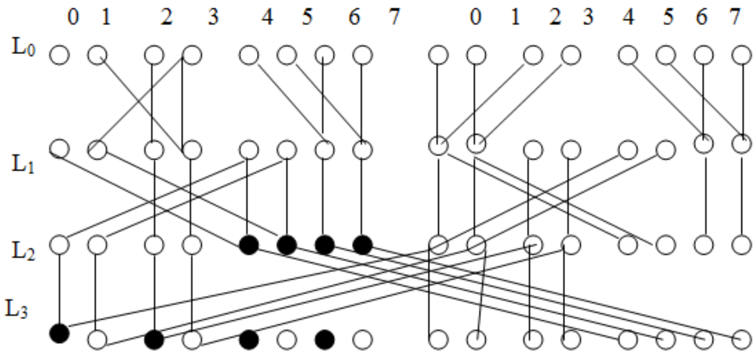

Fig. 8

Here observe that the vertices (2; r +1) = (2; 1) , (2; s +1) = (2; 3), (3; r +1) = (3; 1) , (3; s +1) = (3; 3) are not dominated by the vertices of D_1 .

Now the vertices in the mirror image of D_1 denoted by D_2 are given by D_2 = {(3; r^1), (3; s^1) , (3; p^1) , (2; p^1), (2; q^1), (3; t^1), (2; t^1), (2; w^1) } where r + r^1 = 15, s + s^1 = 15, p + p^1 = 15, q + q^1 = 15 , t + t^1 = 15 , w + w^1 = 15.

Again it can be shown that the vertices in D_2 dominate all vertices in the right copy of BF(4) \ F except the vertices (2; r^1 -1) , (2; s^1 -1) of L_2 and (3; r^1 -1) (3; s^1 -1) of L_3 .

For above said values of r, s, p, q, t, w, the set D_2 becomes D_2 = {(3; 15), (3; 13) , (3; 11) , (2; 11), (2; 10), (3; 9), (2; 9), (2; 8) }.Here observe that the vertices (2; r^1 -1) = (2; 14), (2; s^1 - 1) =(2; 12), (3; r^1 - 1) = (3; 14), (3; s^1 - 1) = (3; 12) are not dominated by the vertices of D_2 .

The following figure illustrates the selection of vertices into D_2 .

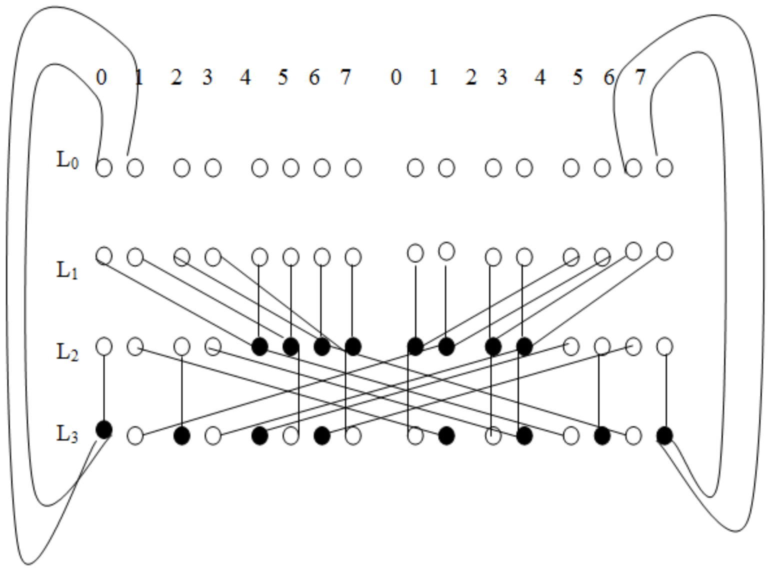

Fig. 9

Now the undominated vertices (2; r +1) = (2; 1) , (2; s +1) = (2; 3) , (3; r +1) = (3; 1) , (3; s +1) = (3; 3) in the left copy are dominated by the selected vertices (3;9),(3;11), (2; 9), (2; 11) respectively. Also the undominated vertices (2; r^1 - 1) = (2; 14), (2; s^1 - 1) = (2; 12), (3; r^1 - 1) = (3; 14), (3; s^1 - 1) = (3; 12) in the right copy are dominated by the selected vertices (3; 6), (3; 4), (2; 6) , (2; 4) respectively .Let D = $D_1 \cup D_2$. Then all vertices of BF(4) \ F are dominated by the 16 selected vertices in D. Therefore γ (BF(4)) = γ (BF(4) \ F) .

For other choices of edges in these two $K_{2,2}$ s, it can be shown that the domination number for BF(4) and BF(4) \ F is unaltered.

Similarly for any choice of three edges in BF(4) between L_1 , L_2 and L_0 , L_3 , it can be shown that the domination number of BF(4) and BF(4) \ F is unaltered. Thus b(BF(4)) = 4. □

Lemma 4 : The bondage number of BF(n) is

b(BF(n)) = b(BF(4)) = 4, for n > 4.

Proof : From Recursive Construction 1, we know that for n > 4, every BF(n) has a copy of BF(4) between the first 4 levels L_0, L_1, L_2 and L_3. Removal of 4 edges incident on a single vertex in one of these BF(4) copies increases the domination number of this copy of BF(4). Since all these copies are disjoint, this results in the increase of the domination number of BF(n).

Thus b(BF(n)) = b(BF(4)) = 4. □

This is illustrated for the graph BF(5) in the figure given below.

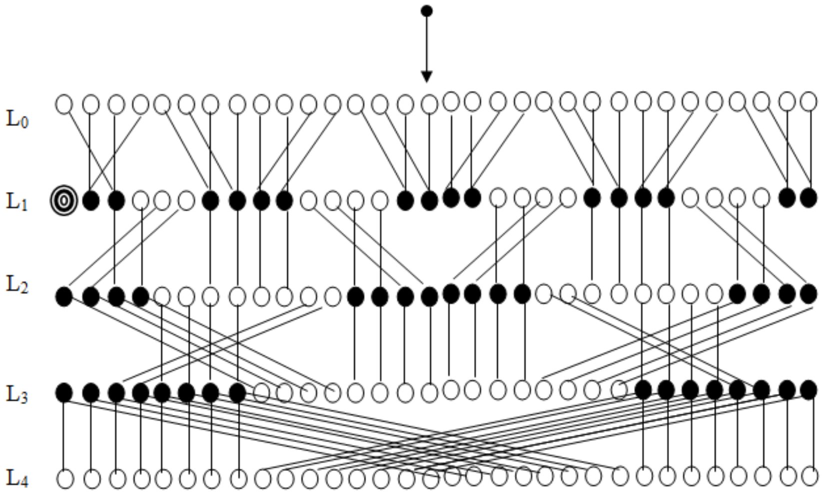

Fig.10 BF(5)

The above results from Lemma 1 through Lemma 4 can be compiled as follows :

Theorem 5 : The bondage number of BF(n) is

b(BF(n)) = 2 for n = 2.

= 4 for n ≥ 3.□

REFERENCES

[1]. Barth, D., Raspaud, A., Two edge-disjoint Hamiltonian cycles in the Butterfly Graph, Infrom. Process.

[2]. Lett. 51,(1994), 175-179

[3]. Berge, C., Theory of graphs and its Aplications ,Methuen, London(1962)

[4]. Bermond, J.C. Darrot, E. Delmos, Perennes, S., Hamilton Cycle

[5]. Decomposition of the butterfly Network, Parallel rocessing Letters,

[6]. Vol. 8,(1998),371-385

[7]. Chang, G.J., Algorithmic aspects of domination in graphs, in :D.-Z.Du, P.M. Pardalos (Eds.), Hand book of combinational Otimization, Vol. 3, Kluwer, Boston, MA,(1998),pp.339-405.

[8]. Fink, J.F., Jacobson, M.S., Kinch L.F., Roberts, J., The bondage number of graph,Descrete Math. Vol.86(1990) ,47-57.

[9]. Haynes, T.W., Hedetniemi, S.T., Slater, P.J., Fundamentals of Domination in Graphs: Advanced topics, Marcel Dekker, New York,(1998)

[10]. Kelkar, I.P., Some Studies On Domination Paramaters of Butterfly Graphs, Ph.D Thesis, Sri Padmavati Mahila Visvavidyalayam Tirupati India,(2006)

[11]. Leighton F.T., Introduction to Parallel Algorithms and Architecture Arrays, Trees, Hypercudes,Morgan Kaufman,San Mateo,CA (1992).

[12]. Ore, O., Theory of Graphs, Amer, Maths. Soc. Colloq. Pub. 38, Providence (1962)