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Difference equations with weighted differences

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***Abstract:* -** In this paper we study properties of difference equations with weighted differences. Especially we treat difference equations with a space variable x and a time variable t. In the case that average differences by x define the values at next times, effects of weights for average difference s are investigated.

Keywords: *- difference, difference equation, weighted difference, average difference, discrete variable*

I. INTRODUCTION

In this paper we study properties of difference equations with weighted differences. Especially we treat difference equations with a space variable x and a time variable t. In the case that average differences by x define the values at next times, we investigate effects of weights for average differences and weights for current values.

II. DIFFERENCE EQUATIONS

We define the next weighted difference equation.

$f(x, t+1) = \{(k-2)/k\} f(x, t) + (1/k) \{f(x-1, t) + f(x+1, t)\}, \; x = 1, 2, \dots, \; t = 0, 1, 2, \dots$ (1)
$f(0, t+1) = \{(k-1)/k\} f(0, t) + (1/k) f(1, t), \; t = 0, 1, 2, \dots$
$f(x, 0) = 0, \; x = 1, 2, \dots$
$f(0, 0) > 0,$

where a real number k >= 2.

We call the term f(x-1, t) + f(x+1, t) the average difference. Constant values (k-2)/k and 1/k show weights for the current value f(x, t) and for the average difference, respectively.

If k = 2, equation (1) is the following;

$f(x, t+1) = (1/2) \{f(x-1, t) + f(x+1, t)\}, \; x = 1, 2, \dots, \; t = 0, 1, 2, \dots$ (2)
$f(0, t+1) = (1/2) f(0, t) + (1/2) f(1, t), \; t = 0, 1, 2, \dots$
$f(x, 0) = 0, \; x = 1, 2, \dots$
$f(0, 0) > 0$

Equation (2) is equivalent to the next discretization of a heat equation.

$f(x, t+1) - f(x, t) = (1/2) \{f(x+1, t) - f(x, t)\} - (1/2) \{f(x, t) - f(x-1, t)\}$
$= (1/2) f(x-1, t) - f(x, t) + (1/2) f(x+1, t), \; x = 1, 2, \dots, \; t = 0, 1, 2,$ (3)

where f(x+1, t) - f(x, t) and f(x, t) - f(x-1, t) are called the forward difference and backward difference, respectively. [1]

III. PROPERTIES OF DIFFERENCE EQUATIONS

On equation (1) the next properties are holds.

Property 1. $f(0, 0) = f(0, t) + f(1, t) + f(2, t) + \dots$ for $t = 0, 1, 2, \dots$

Property 2. The solution f(0, t) is monotone decreasing for t.

Property 3. The solution f(x, t) is monotone decreasing for x.

Property 4. In the case of k >= 3, it exists some integer T > 0 such that for any t > T the solution f(x, t) have an inflection point for x. In fact, the value of {f(x+1, t) - f(x, t)} - {f(x, t) - f(x-1, t)} changes negative to positive at some x if x increases from zero, for t > T and some T.

IV. CONCLUSION

We showed properties of difference equations with weighted average differences. Especially the existence of a inflection point for the solution is a key of the application to heat engineering, information engineering, and so on. The reason is that the existence of a inflection point may mean delay of transmission of heat and information.

V. ACKNOWLEDGEMENTS

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[1] S. Sugiyama, *difference/differential equations* (Kyoritsu Shuppan, 1999), in Japanese.