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Closed Solution of EM Wave Scattering Problems and Creating Materials with Specific Performances

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Abstract: **-** The new results on EM wave scattering by small particles embedded in homogeneous medium are discussed. Such particles can be as the thin perfectly conducting wires or small impedance spheres. The proposed approach is applied to creating the media with a given refraction coefficient or a piecewise constant magnetic permeability. The explicit analytical formulas for the refraction coefficient and magnetic permeability of new inhomogeneous media are main engineering results.

Keywords: *- Asymptotic Approach, EM Wave Scattering, Inhomogeneous Media, Magnetic Permeability, Refraction Coefficient.*

I. INTRODUCTION

The problem of EM wave scattering by the small bodies is interesting because of its many applications in engineering. This topic was investigated firstly by Rayleigh [1] and Mie [2], and it has found wide applications in optics [3]-[6], electromagnetics [7]-[11], and other engineering topics [12], [13] during last decades. The areas where it is actively applied are the light scattering by atmospheric and cosmic dust, colloidal particles in water, finding the location and size of the small distinct inhomogeneities using the observation of the far field, scattered by these inhomogeneities. In this paper, we study the EM wave scattering by many small bodies and on this base creating materials with a desired refraction coefficient [14], [15] or magnetic permeability [16]-[19]. The particles are ideally conducting or having prescribed boundary impedances, and they are distributed in the given material with a prescribed density. A variety of the desired refraction coefficients (in the case of the ideally conducting or impedance thin wires) or given magnetic permeability (in the case of impedance spheres) can be obtained by embedding many thin wires or many small spheres into a given material. We assume that in an arbitrary fixed bounded domain D, belonging to R^3, the refraction coefficient $n_0^2(x)$ of the material is known, and outside $n_0^2(x) = \mathrm{const}$. We try to create in D a desired refraction coefficient $n^2(x)$ that is different from $n_0^2(x)$. This limiting medium is created in a way that size a of the small embedded particles tends to zero while the total number $M = M(a)$ of the particles tends to infinity. As a result of such procedure, the refraction coefficient $n^2(x)$ in the limiting medium is spatially inhomogeneous. An approach to create a desired refraction coefficient in such resulting media was proposed in [20]. It allows to substantiate what distribution density (number of particles per unit volume) of embedded small particles in D should be, and what boundary impedances these small particles should have in order to get the resulting medium having the refraction coefficient that differs from the desired refraction coefficient $n^2(x)$ as much as possible. Following to the case of an acoustic scattering [23], it was proved that one can create materials with negative refraction [15]. This can be used in construction of meta-materials [24], [25]. If to implement small impedance particles in a given material then the resulting medium has new physical properties. Although the initial medium has a constant permeability μ_0, the limiting medium, obtained by embedding many small particles with prescribed boundary impedances, has a non-homogeneous permeability $\mu(x)$ that is expressed analytically in terms of the distribution density of the small particles and their boundary impedances. Therefore, a new engineering result was predicted theoretically, namely, appearance of a spatially inhomogeneous permeability as a result of embedding of many small particles, whose physical properties are described by their boundary impedances [26].

II. EM WAVE SCATTERING BY SET OF SMALL PARTICLES

2.1. Scattering by Thin Wires

Let us assume that the wires are perfect conductors and their radius $a = 0.5\,\mathrm{diam}\,D_m$, where D_m is the cross-section of wire. Our "smallness" (thinness) assumption is $ka << 1$, where k is the wave number in the region exterior to the union of the wires. We assume that the thin wires C_m are distributed in D according to the following law:

$$\mathrm{N}(\Delta) = \ln \frac{1}{a} \int_{\Delta} N(x)dx[1 + o(1)], \ \ a \to 0 , \tag{1}$$

where $\mathrm{N}(\Delta) = \sum_{\tilde{x} \in \Delta} 1$ is the number of the wires in an arbitrary open sub-domain Δ of D, $N(x) \geq 0$ is a continuous function, which can be chosen for our requirements.

The EM wave scattering problem consists of finding the solution to Maxwell's equations (see [27]):

$$\nabla \times E = i\omega\mu H , \tag{2}$$

$$\nabla \times H = -i\omega\varepsilon E \tag{3}$$

in C' that is the complement of C, such that

$$E_t = 0 \text{ on } \partial C , \tag{4}$$

where ∂C is the union of the surfaces C_m, E_t is the tangential component of E, μ and ε are constants in C', ω is the frequency, $k^2 = \omega^2 \varepsilon_0 \mu_0$. Denote by $n_0^2 = \varepsilon_0 \mu_0$, so $k^2 = \omega^2 n_0^2$.

We look for the solution to problem (2)-(4) of the form

$$E(x) = E_0(x) + v(x), \ x = (x_1, x_2, x_3) = (x, y, z) = (\tilde{x}, z) , \tag{5}$$

where $E_0(x)$ is the incident field, $v(x)$ is the scattered field satisfying the radiation condition

$$\sqrt{r}\left(\frac{\partial v}{\partial r} - ikv\right) = o(1) , \ r = (x_1^2 + x_2^2)^{1/2} . \tag{6}$$

We assume also that

$$E_0(x) = \exp(i\kappa y + ik_3 z)e_1 , \ \kappa^2 + k_3^2 = k^2 , \tag{7}$$

where $\{e_j\}, j = 1,2,3,$ are the unit basis vectors along the Cartesian coordinate axes x, y, z. We consider EM waves with $H_3 := H_z = 0$, i.e., E-waves, or TH-waves,

$$E = \sum_{j=1}^{3} E_j e_j , \ H = H_1 e_1 + H_2 e_2 = \frac{\nabla \times E}{i\omega\mu} . \tag{8}$$

It was proved in [14] that the components of E can be expressed by the formulas:

$$E_j = \frac{ik_3}{\kappa^2} u_{x_j} e^{ik_3 z} , \ j = 1,2 , \ E_3 = u e^{ik_3 z} , \tag{9a}$$

and components of H by

$$H_j = \frac{i\omega\varepsilon}{\kappa^2} u_{x_j} e^{ik_3 z} , \ j = 1,2 , \ H_3 = 0 , \tag{9b}$$

where $u_{x_j} := \frac{\partial u}{\partial x_j}$, $u = u(x, y)$ solves the problem

$$(\Delta^2 + \kappa^2)u = 0 \text{ in } C' , \tag{10}$$

$$u\big|_{\partial C} = 0 , \ u = e^{i\kappa y} + w , \tag{11}$$

and w satisfies the radiation condition (6)

The unique solution to (10)-(11) was given by the formulas [14]:

$$E_1 = \frac{ik_3}{\kappa^2} u_x e^{ik_3 z} , \ E_2 = \frac{ik_3}{\kappa^2} u_y e^{ik_3 z} , \ E_3 = u e^{ik_3 z} , \tag{12}$$

$$H_1 = \frac{i\omega\mu}{\kappa^2} u_y e^{ik_3 z} , \ H_2 = \frac{i\omega\mu}{\kappa^2} u_x e^{ik_3 z} , \ H_3 = 0 , \tag{13}$$

where $u_x := \frac{\partial u}{\partial x}$, u_y is defined similarly, and $u = u(\tilde{x})$, $(\tilde{x} = (x, y))$ solves scalar two-dimensional problem (10)-(11). It was proven in [28] that such a problem has a unique solution.

2.2. Scattering by Impedance Spheres

We assume that small spheres $D_m, 1 \le m \le M$, are embedded in a homogeneous medium with constant parameters ε_0, μ_0. EM wave scattering problem consists of finding vectors E and H satisfying the Maxwell equations (2)-(3) in $D := R^3 \setminus \bigcup_{m=1}^{M} D_m$, the impedance boundary conditions:

$$[N,[E,N]] = \zeta_m [H,N] \tag{14}$$

on $S_m, 1 \le m \le M$, and the radiation conditions:

$$E = E_0 + v_E\,,\ H = H_0 + v_H\,. \tag{15}$$

In formula (14), ζ_m is the impedance, N is the unit normal to S_m pointing out of D_m, E_0, H_0 are the incident fields satisfying equations (2)-(3) in all R^3.

Let us the incident wave is a plane wave, i.e., $E_0 = \beta e^{ik\alpha\cdot x}$, β is a constant vector, $\alpha \in S^2$ is a unit vector, S^2 is the unit sphere in R^3, $\alpha \cdot \beta = 0$, v_E and v_H satisfies the radiation condition:

$$r\left(\frac{\partial v}{\partial r} - ikv\right) = o(1)\,. \tag{16}$$

We restrict ourselves to the constant impedance ζ_m, and $\mathrm{Re}\,\zeta_m \ge 0$. In general, ζ_m can be a $2\text{x}2$ matrix function acting on the tangential to S_m vector fields, such that

$$\mathrm{Re}(\zeta_m E^t, E^t) \ge 0\,\forall E^t \in T_m\,, \tag{17}$$

where T_m is the set of all tangential to S_m continuous vector fields. In this case, we should superimpose additional restriction $\mathrm{Div} E^t = 0$, where Div is the surface divergence, and E^t is the tangential component of E. Using this assumption, we write down the impedance boundary conditions on S_m as:

$$[N,[E,N]] = \frac{\zeta_m}{i\omega\mu_0}[\nabla \times E, N],\ , 1 \le m \le M\ . \tag{18}$$

In this way, we have reduced problem (2), (3), (16), (17) to finding one vector $E(x)$ satisfying the impedance boundary condition (18). If $E(x)$ is found, then $H = \frac{\nabla E}{i\omega\mu_0}$.

III. CLOSED FORM OF SOLUTION

3.1. Solution for Thin Wires

We look for the solution to problem (10)-(11) of the form

$$u(\tilde{x}) = u_0(\tilde{x}) + \sum_{m=1}^{M} \int_{S_m} g(\tilde{x},t)\sigma_m(t)dt\,, \tag{19}$$

where S_m is the boundary of D_m, and dt is the element of the arc-length of S_m. The distribution of the points $\tilde{x}_m = (x_m, y_m)$ in a bounded domain Δ on the plane $P = xOy$ is given by formula (1). The field $u_0(\tilde{x}) := e^{ikx_2}$, and

$$g(\tilde{x},t) := \frac{i}{4} H_0^{(1)}(\kappa\,|\tilde{x} - t|)\,. \tag{20}$$

The effective field acting on the D_j defined as

$$u_e(\tilde{x}) = u_0(\tilde{x}) + \sum_{m=1, m\ne j}^{M} \int_{S_m} g(\tilde{x},t)\sigma_m(t)dt\,. \tag{21}$$

It is assumed that the distance $d = d(a)$ between neighboring cylinders is much greater than a:

$$d >> a,\ \lim_{a\to 0}\frac{a}{d(a)} = 0\,. \tag{22}$$

Let us rewrite (21) as

$$u = u_0 + \sum_{m=1}^{M} g(\tilde{x},\tilde{x}_m)Q_m + \sum_{m=1, m\ne j}^{M} \int_{S_m} [g(\tilde{x},t) - g(\tilde{x},\tilde{x}_m)]\sigma_m(t)dt, \tag{23}$$

where $Q_m := \int_{S_m} \sigma_m(t)dt$.

It was proven in [14] that the second sum in (23) is negligible compared with the first one as $a \to 0$. The asymptotic formula for functions Q_m is:

$$Q_m = \frac{-2\pi u_e(x_m)}{\ln \frac{1}{a}}[1 + o(1)], \; a \to 0 . \tag{24}$$

The main feature of our method consists of finding numbers Q_m instead of determination of the unknown boundary functions $\sigma_m(t)$. This leads to a significant increase of the numerical efficiency of developed method, and does not lead to the loss of its accuracy because a is small. From formulas (21) and (24), we obtain the solution to problem (10)-(11) of the form, which is asymptotically exact as $a \to 0$:

$$u_e(\tilde{x}_j) = u_0(\tilde{x}_j) - \frac{2\pi}{\ln \frac{1}{a}} \sum_{m=1, m\neq j}^{M} g(\tilde{x}_j, \tilde{x}_m) u_e(\tilde{x}_m), \; 1 \leq j \leq M . \tag{25}$$

The numbers $u_e(\tilde{x}_m), 1 \leq m \leq M$, in (23) are not known. The LAS (linear algebraic systems) for their finding is

$$u_e(\tilde{x}_j) = u_0(\tilde{x}_j) - \frac{2\pi}{\ln \frac{1}{a}} \sum_{m=1, m\neq j}^{M} g(\tilde{x}_j, \tilde{x}_m) u_e(\tilde{x}_m), \; 1 \leq j \leq M . \tag{26}$$

This system can be solved numerically if the number M is not very large, say $M < O(10^3)$.

3.2. Solution for Impedance Spheres

We look for E in the form

$$E(x) = E_0(x) + \nabla \times \int_{S_m} g(x,t)\sigma_m(t)dt, \; g(x,y) = \frac{e^{ik(x-y)}}{4\pi|x-y|}, \tag{27}$$

where $t \in S_m$ and dt is an element of the area of S_m.

Similarly to (21), define the effective field $E_e(x) = E_e^m(x) = E_e^m(x,a)$, acting on the m-th body D_m, by the formula:

$$E_e(x) = E(x) - \nabla \times \int_{S_m} g(x,t)\sigma_m(t)dt := E_e^m(x) . \tag{28}$$

Using the assumptions analogous to the case of thin cylinders, we write down the effective field as

$$E(x) = E_0(x) + \sum_{m=1}^{M} [\nabla_x g(x, x_m), Q_m], \; \min_m |x - x_m| > a , \tag{29}$$

with an error that tends to zero under our assumptions as $a \to 0$. If $|x - x_j| << a$ then the term with $m = j$ in the sum (29) should be dropped according to the definition of the effective field.

This representation in the limit $a \to 0$ is equivalent to the following integral equation:

$$E(x) = E_0(x) + \nabla \times \int_D g(x,y)N(y)Q(y)dy , \tag{30}$$

where $Q(y)$ is the function uniquely defined in the points x_m: $Q_m = Q(x_m)a^{2-\kappa}$.

The function $Q(y)$ can be expressed in terms of E:

$$Q(y) = -\frac{8\pi i}{3\omega\mu_0} h(y)(\nabla \times E)(y) . \tag{31}$$

The factor $\frac{8\pi}{3}$ appears if D_m are spheres. Alternatively, a tensorial factor c_m, depending on the shape of S_m, should be used in place of $\frac{8\pi}{3}$.

From equations (30) and (31), we obtain

$$E(x) = E_0(x) - \frac{8\pi i}{3\omega\mu_0} \nabla \times \int_D g(x,y)h(y)N(y)\nabla \times E(y)dy . \tag{32}$$

This equation will be used to derive the explicit formula for magnetic permeability $\mu(x)$ of resulting medium.

IV. EXPLICIT FORMULAS FOR REFRACTION COEFFICIENT AND PERMEABILITY OF RESULTING MEDIUM

If to act by the operator $\Delta_2 + \kappa^2$ on equation for effective field $u(\tilde{x})$, we receive:

$$\Delta_2 u(\tilde{x}) + \kappa^2 u(\tilde{x}) - 2\pi N(\tilde{x}) u(\tilde{x}) = 0 \,. \tag{33}$$

This is a Shrödinger equation, and $u(\tilde{x})$ is the scattering solution corresponding to the incident wave $u_0 = e^{iky}$.

If to put $N(\tilde{x}) = N$ is a constant, then it follows from (33) that the resulting medium, obtained by embedding many perfectly conducting circular cylinders, has new parameter $\kappa_N^2 = \kappa^2 - 2\pi N$. This means that $k^2 = \kappa^2 + k_3^2$ is replaced by $\overline{k}^2 = k^2 - 2\pi N$. The quantity k_3^2 is not changed. In such a way $\overline{k}^2 = \omega^2 n^2$, $k^2 = \omega^2 n_0^2$. Therefore $n^2 / n_0^2 = (k^2 - 2\pi n)/k^2$. It means that the new refraction coefficient n^2 is

$$n^2 = n_0^2 (1 - 2\pi N k^{-2}) \,. \tag{34}$$

Since the number $N > 0$ is at our disposal, equation (34) shows that choosing suitable N, we can create a medium with a smaller than n_0^2 refraction coefficient including its negative values. This fact was substantiated firstly by computational modelling in [29]. In practice one does not go to the limit $a \to 0$, but chooses a sufficiently small a. As a result, we obtain a medium with refraction coefficient n_a^2, which can differ from (34) insignificantly, and $\lim_{a \to 0} n_a^2 = n^2$.

If to act by the operator $\Delta_2 + \kappa^2$ to equation for effective field created by embedding into homogeneous medium small impedances particles, we receive the equation

$$\nabla \times \nabla \times E = \omega^2 \varepsilon(x) \mu(x) E + [\frac{\nabla \mu(x)}{\mu(x)}, \nabla \times E] \,. \tag{35}$$

It follows from (35) that the electromagnetic property of the resulting medium is described by the variable permeability:

$$\mu(x) = \frac{\mu_0(x)}{\Psi(x)} = \frac{\mu_0}{1 + \frac{8\pi i}{3\omega\mu_0} h(x) N(x)} \,, \tag{36}$$

and this medium is described by the new refraction coefficient $K^2 = \omega^2 \varepsilon_0 \mu(x)$. The numerical algorithms providing the possibility to create the inhomogeneous media with piecewise constant distribution of permeability $\mu(x)$ were developed in [30].

V. COMPUTATIONAL MODELING

The algorithms for numerical investigation of the problem of EM wave scattering by many small particles were developed in [15], [19]; they applied here for the modelling the media with negative refraction by embedding in the initial medium many thin cylinders and media with piecewise-constant distribution of magnetic permeability by embedding in such medium many impedance spheres. One can conclude from formula (34) that the value of refraction coefficient n^2 depend on the wave number k, geometry of the domain D, namely on the parameter N, which contains the value a of the cylinder's radius, number M of cylinders and distance d between them. In Fig. 1 and Fig. 2, the dependence of n^2 on the wave number k for the various a and d is shown. In Fig. 1, the results are presented for the number M of cylinders is equal to 900, they are placed equidistantly in a square 30 cylinders on side. The value of k has dimension of m^{-1}, values of a and d have dimension of m, and the values n^2 of refraction coefficient are normalized to the constant $11.1254 \cdot 10^{-18} \frac{\mathrm{sek}^2}{\mathrm{m}^2}$. At such $d = 0.1386\mathrm{m}$ the values of n^2 much differ from the refraction coefficient $n_0^2 = 1$ of initial media.

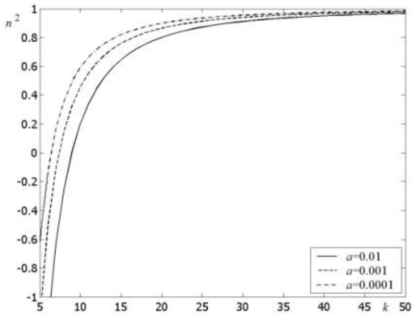

Fig. 1. Values of n^2 versus wave number k, $d = 0.1386\text{m}$.

The numerical results presented in Fig. 2 testify the possibility to create the medium with various n^2 depending on the distance d between cylinders at fixed a and k. The results are shown for $a = 0.001\text{m}$ and $k = 20.0$. At small M the values of n^2 are changed notably and at the increasing M they tend to $n^2 = -0.45$, $n^2 = 0.06$, $n^2 = 0.35$, and $n^2 = 0.64$ for $d = 20a$, $d = 25a$, $d = 30a$, and $d = 40a$, respectively. One should note that at the considered parameters of the domain D the relative error of solution to LAS (26) does not exceed 2.34%, 1.69%, 1.18%, and 0.93% for $d = 20a$, $d = 25a$, $d = 30a$, and $d = 40a$ at $\sqrt{M} = 10$, and relative error diminishes if M grows.

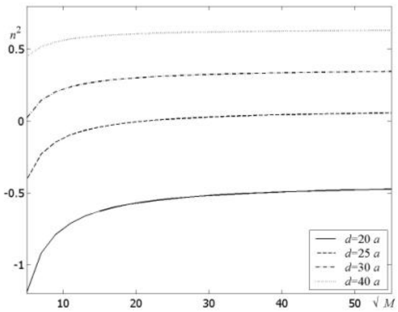

Fig. 2. Values of n^2 versus the number M of cylinders.

In such a way, the forming the desired refraction coefficient n^2 differing of refraction coefficient n_0^2 of the initial medium can be carried out by the changing the values of wave number k at the fixed geometries of D as well as by the changing the parameters a, M, and d that determine the geometry of the domain D.

In Fig. 3, the respective values of the calculated by formula (36) permeability μ depending on the radius a of particle are shown. Ones can see that value of μ for greater d are more close to the initial permeability $\mu_0 = 1$ of media without embedded particles. It is physically based forecast, because at the increasing d, the properties of D stand similar to properties of such medium.

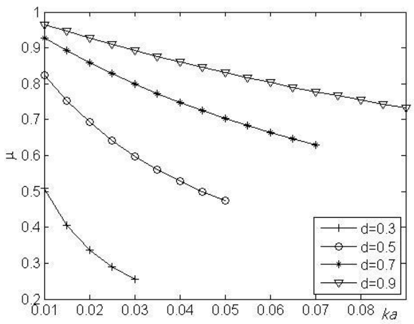

Fig. 3. Amplitude of permeability μ versus the radius of particle a.

The values of $\mu(x)$ depend on the values of function $h(x)$ too. The results, shown in Fig. 4, demonstrate that the obtained μ at small $|\text{Im } h(x)|$ are more close to $\mu_0 = 1$, and difference between μ and μ_0 grows if $|\text{Im } h(x)|$ increases.

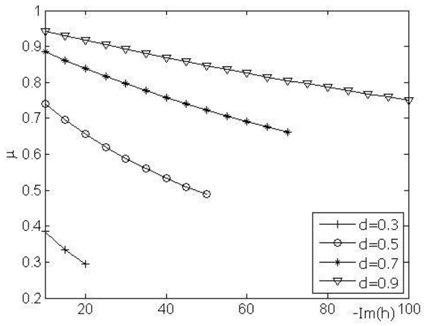

Fig. 4. Amplitude of permeability μ versus the value of $|\operatorname{Im} h(x)|$.

The proposed approach allows to create the media with piecewise-constant distribution of μ. Such distribution can be realized either by embedding the various number M_m of particles into subdomains Δ_m or by variation of function $h(x_m)$ in these subdomains. The both above approaches have the own advantages depending on the physical and geometrical parameter of D. At the engineering practice, it is necessary to have the constant distribution of μ along certain direction (for example, along $z-$ and y -axes), and piecewise-constant μ in the direction of $x-$ axis. In Fig. 5, such kind of μ is formed by embedding various number of particles M_m in three subdomains Δ_m of D with number of particles $M_{1,2,3} = 4\times 7\times 7, 7\times 7\times 7, 4\times 7\times 7$ respectively. Such distribution of particles allows to reach a difference in the μ values in range of 2.5%. In order to increase the difference μ_m for various D_m it is necessary to increase the difference between M_m.

Combining the ratio of values M_m, one can create the distribution of μ corresponding to various requirements. For example, distribution of μ which is shown in Fig. 6, is obtained at $M_1 = 9\times 9\times 9, M_2 = 7\times 9\times 9, M_3 = 5\times 9\times 9$. The amplitudes E_x and E_y for this case have more complicate structure because of larger variation of μ.

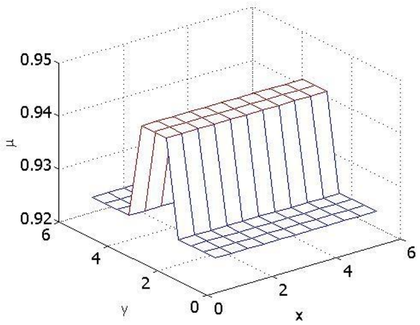

Fig. 5. The piecewise-constant distribution of μ corresponding to two various M_m.

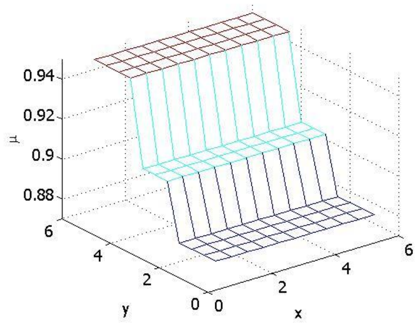

Fig. 6. The piecewise-constant distribution of μ corresponding to three various M_m.

VI. CONCLUSION

Asymptotic solution was given for the problem of EM wave scattering by many perfectly conducting parallel cylinders and impedance spheres of small radii a, $ka << 1$. An equation for the effective (self-consistent) field in the limiting medium is obtained when $a \to 0$ and the distribution of the embedded particles is given by formula (1). The theory yields the explicit formulas (34) and (36) for the refraction coefficient and magnetic permeability of the resulting medium obtained by embedding of these particles into the initial homogeneous structure. The numerical results confirm the possibility to create materials with inhomogeneous refraction coefficient and magnetic permeability.

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