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Surrogate Modelling of Liquid Nitrogen Pumping in Hydrocarbon and Allied Fluids Piping Systems

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Abstract
This paper presented surrogate modelling of liquid nitrogen pumping in hydrocarbon and allied fluids piping systems. It was aimed at the development of multi-objective frameworks for optimal liquid nitrogen pumping for leak test of hydrocarbon and allied fluids piping systems thereby reducing the uncertainties and variability in the pumping operation due to energy consumption, operational cost and other performance parameters. Surrogate modelling is a data-driven computational and stochastic process. The data set was collected from a real time hydrocarbon and allied fluids piping systems' nitrogen-helium leak test scenario. The pumping process was parameterized, and the performance parameters are volume of liquid nitrogen pumped, test duration, pumping duration and volume of liquid nitrogen used; while the control parameters are pressurized volume of the piping systems, test pressure of the piping systems, maximum discharge pressure of the liquid nitrogen pump and maximum flow rate of the liquid nitrogen pump. The result showed that cubic polynomials were adequate approximation of the relationship between the identified performance and control parameters of the liquid nitrogen pumping process. Also, the sensitivity analysis performed based on the process performance indices of the performance parameters and the set means and standard deviations of the control parameters showed that each of the control parameters affected each of the performance parameters at varying degrees. Thus, the developed models will provide good estimation of total volume of liquid nitrogen pumped and pumping duration.
KEYWORDS: *Liquid nitrogen pumping, Hydrocarbon fluids, Piping systems, Models, Energy consumption.*

I. Introduction

Background

Liquid nitrogen pumping in hydrocarbon and allied fluids piping systems is characterized by lots of uncertainties and variability bothering on flow loss, energy consumption, operational cost, leakages, performance parameters of the pump and geometrical configuration of the piping systems. Managing these uncertainties and variability to achieve efficient pumping operation and minimized cost simultaneously involves complex and expensive computational tasks with multi-parametric considerations; hence surrogate modelling, which is a data-driven, global approximation method [1] provides the gateway to best solution set.

Thus, surrogate modelling provides means to reduce the complexity of intrinsically computationally expensive analysis by developing cost-effective models [2, 3]. It has been applied in solving different problems associated with fluid pumping in different fluids piping systems considering uncertainties and variability, hence it is stochastic. To solve the complex optimization problems associated with the pumping of liquefied light hydrocarbon [4] developed a multi-scenario and multi-objective optimization model for the pipeline system thereby minimizing the operational cost of the pump and number of switching operation. This considered several parameters to include mode of transportation, inventory limits of injection stations, flow rate and safe operating pressures of the pipeline segments; and the results indicated that the model improved the profit margin, safety and robustness of the process [5] modelled steady flow of gas mixtures in pipeline networks to determine the pressures and flows of the system at fixed boundary conditions and compressor settings. Khlebnikova *et al.* [6] formulated optimization models for economic operation of liquid petroleum products pipeline system with a view to maximizing economic value and minimizing operating costs of the systems. Thus, the use of operating settings of the pump and flow allocations were considered. Duan *et al.* [7] developed practically implementable model for fluid and heat flow in pipeline that provided tool for estimation of some technical processes. Torregrossa and Capitanescu [8] also developed optimization models for operation of water pumping systems in order to save energy and increase the operational life of the system. More so, [9] developed optimization model to minimize cost considering some technical parameters like pipe flow velocity and water pressure. A stochastic approach was adopted for a fluid pumping to optimize the pressure of the system, energy produced and plant cost simultaneously [10]. Thus, each of these studies illustrated the application of surrogate modelling in improving the performance of fluid pumping process and systems in piping systems.

According to [1], surrogate models are response surface models used to predict the system performance and develop a relationship between inputs and outputs; perform complex optimization and sensitivity analysis. Response surface models are constructed using response surface methodology (RSM). In their use of RSM for process optimization, [11] noted that it is an experimental design tool widely used in industrial processes. Thus, experimental design is an advanced statistical approach [12] for empirical model development. Hence development of surrogate models is parametric and data-driven; and to avoid unjustified biases in data collection, a suitable experimental design framework is necessary [13]. Antony *et al.* [14] noted that design of experiment is a powerful technique for process optimization. It is a powerful means of improving process efficiency [15]. Anderson and Whitcomb [16] outlined the characteristics of a good experimental design. Therefore, the essence of planning statistical design of experiment is to ensure appropriate data collection and statistical analysis [17].

Randomized Experimental Design Frameworks

Surrogate models of liquid nitrogen pumping for pneumatic test of hydrocarbon and allied fluids piping systems involves some parameters that must be measured randomly, and development of such models involves taking decisions under uncertainty. In the development of stochastic models, [18] presented experimental design frameworks that guided the way to a successful generation of equation. According to [19], an experimental design is the laying out of a detailed experimental plan in advance of doing the experiment. Anderson and Whitcomb [16] outlined the characteristics of a good experimental design. Antony *et al.* [14] noted that design of experiment is a powerful technique for process optimization.

Design of experiment has a wide range of applications in diverse fields, including non-manufacturing environments to identify the key variables that influence the process performance, process parameters, etc. [14]. Anderson and Kraber [15] noted that design of experiment is a powerful means of improving process efficiency. Thus, the efficiency of liquid nitrogen pumping for pneumatic test of hydrocarbon fluids piping systems could be seriously improved using design of experiment. Then [15] outlined the keys to a successful design of experiment to include setting of objectives, quantitatively measuring the responses, reducing uncontrollable variations by replication, randomizing the experimental run order, ensuring that unknown sources of variations are blocked, aliasing possible effects, doing a sequential series of experiments and always confirming series of critical findings. According to [20], some of the basic principles of experimental design used to remove or reduce bias in practical applications are randomization, replication and blocking. It was noted that experimental design methodology is basically divided into the planning phase, designing phase, conducting phase and analysis phase. It should also be noted that the essence of planning statistical design of experiment is to ensure appropriate data collection and statistical analysis [17]. The data so collected and analysed leads the formulation of empirical models which may be linear or nonlinear, depending on the set objectives. These models show the functional relationship between the response (output) variable of interest and the input (control) variable. The relationship is referred to as the response surface model, and its objectives include hitting a target, maximizing or minimizing a response, reducing variation, making a process more robust and seeking multiple goals [19].

Response Surface Designs for Empirical Modelling

An experimental design method popularly used for empirical model formulation is the RSM, which consists of a group of mathematical and statistical techniques used in the development of an adequate functional relationship between a response of interest and a number of associated control (input) variables [21]. They can be two-level, three-level or more, and factorial or fractional factorial depending on the expected behaviour of the response as functions of the factor settings [19]. Two-level factorial (fractional factorial), Plackett-Burman and simplex designs are the most common designs to fit first-order regression models; while the most common designs to fit second and higher order regression models are the third-level factorial (fractional factorial), central composite and Box-Behnken designs [21]. The designs for first, second and higher order models normally give rise to the formulation of first-order (linear), second-order (quadratic) or higher order models respectively, hence third-order (cubic) models are possible with response surface designs [22]. Results of the statistical analyses determine whether cubic models are significant or not.

Different response surface designs have been applied in different multidisciplinary domains including of a system or process analysis, particularly to check the effect of one parameter on the other. Adeyi *et al.* [23] used Box-Behnken experimental design in their preliminary assessment and uncertainty analysis of a process to check the effects of interdependent parameters. Faria Filho *et al.* [24] used response surface models to predict the performance and conduct economic analysis of a system. They concluded that response surface models are effective in predicting performance of the system and elaboration of the implied analysis to optimize profit. In their use of RSM for process optimization, Pais-Chanfrau *et al.* [11] noted that it is an experimental design tool that is widely used in industrial processes. Ascough II *et al.* [25] applied RSM to assess the economic and environmental trade-offs at the farm level and thus concluded that it provides a useful mechanism to quantitatively evaluate the process under study. The effects of the parameters of a combined system were studied using RSM (central composite design) as an experimental design approach in optimization project [26]. They affirmed that it is a computer-based modelling tool that has proven itself in multidisciplinary and energy systems applications. Also, in the stochastic techno-economic optimization of a power system, Bendato *et al.* [27] utilized it to formulate meta-models that measured the impact of technical parameters on the economic performance of the system. Bellotti, *et al.* [28] used RSM for stochastic sensitivity analysis of the economic sustainability of a system to ascertain the best economic condition of the system.

II. Materials and Methods

Experimental Design Methods

Liquid nitrogen pumping for pressure test of hydrocarbon and allied fluids piping systems is a multi-parametric, stochastic and probabilistic process; the development of its optimization models requires strategies that will enhance empirical data collection. Thus, a first order, randomized factorized experimental design approach was adopted in this study. This was to demystify the complexity associated with the process and obtain data that would be analysed to yield valid and objective conclusion. Since the process was parameterized and characterized, so an orthogonal experimental design [29, 16, 19, 30, 31, 17, 32, 33] was set up. The idea was to establish practically implementable and defensible relationships between the input parameters and their corresponding output parameters. The experimental design is economical and allows for the estimation of the significance of the characteristics of the parameters. It has been applied successfully in many areas of industrial processes, engineering and research [34, 35, 36, 37, 38, 39].

The experimental design method allowed the control (input) parameters to be set at the maximum and minimum points for a two-level factorial design. Also, an intermediate (centre) point that allows for statistical analysis of lack-of-fit was introduce to the design points, hence a three-level factorial design was generated for each of the control parameters. Actual values of the parameters at these points were utilized accordingly for each of the control parameters, and they set the points for empirical data collection. Table (1) showed the three-level full factorial randomized experimental design matrix. Thus X_1, X_2, X_3 and X_4 are the coded values of the control parameters – pressurized volume of the piping systems (V_p), test pressure of the piping systems (P_t), maximum discharge pressure of the nitrogen pump (P_d) and maximum flow rate of the nitrogen pump (Q_N). The coded values were determined from the actual maximum and minimum values ($C_{(max)}$ and $C_{(min)}$) of the control parameters as follows [38]:

$$X_r = \frac{C_{(max,min)} - \mu}{\sigma} \tag{1}$$

$$\mu = \frac{C_{(max)} + C_{(min)}}{2} \tag{2}$$

$$\sigma = \frac{C_{(max)} - C_{(min)}}{2} \tag{3}$$

where $r = 1, 2, \dots, k$; k is the number of factors or control parameters (i.e. 4), C represents actual values of the control parameters. Number of experimental points (n) is given by;

$$n = 2^k + n_c \tag{4}$$

where n_c is number of centre points taken as 4 for each control parameter.

Table 1: First-order three level full factorial experimental design matrix with centre points

Std	Run	X_1	X_2	X_3	X_4
15	1	-1.00	1.00	1.00	1.00
19	2	0.00	0.00	0.00	0.00
6	3	1.00	-1.00	1.00	-1.00
8	4	1.00	1.00	1.00	-1.00
2	5	1.00	-1.00	-1.00	-1.00
12	6	1.00	1.00	-1.00	1.00
1	7	-1.00	-1.00	-1.00	-1.00
4	8	1.00	1.00	-1.00	-1.00
7	9	-1.00	1.00	1.00	-1.00
22	10	0.00	0.00	0.00	0.00
5	11	-1.00	-1.00	1.00	-1.00
13	12	-1.00	-1.00	1.00	1.00
3	13	-1.00	1.00	-1.00	-1.00
16	14	1.00	1.00	1.00	1.00
20	15	0.00	0.00	0.00	0.00
18	16	0.00	0.00	0.00	0.00
10	17	1.00	-1.00	-1.00	1.00
17	18	0.00	0.00	0.00	0.00

9	19	-1.00	-1.00	-1.00	1.00
11	20	-1.00	1.00	-1.00	1.00
14	21	1.00	-1.00	1.00	1.00
21	22	0.00	0.00	0.00	0.00

If the statistical analysis result of the factorial design indicated that the predicted equations, curvature and lack-of-fit are statistically significant for each of the performance parameters – total volume of liquid nitrogen pumped (V_1 in m^3), test duration (D_t in hr), pumping duration (D_p in hr) and total volume of liquid nitrogen used (V_2 in m^3), the three level factorial designs would be augmented to five level designs. These are central composite circumscribed designs (CCD) obtained by the addition of two axial points to the three level designs. Different experimental designs of this category have been utilized in different applications, particularly, based on the objective of the implied study and availability of resources. This type of experimental design approach [17, 32, 31, 19] was utilized for its convenience, flexibility and robustness [40, 41]. It is a form of experimental design that allows for the exploration of wider design spaces due to the new extremes inherently established, thereby making the design robust [19, 42, 43, 44]. The design of experiment was conducted using DESIGN EXPERT version 13.0.5.0 (Stat-Ease on 64-bit Microsoft windows platform), and the design points so established and validated finally set off efficient guide for data collection. Thus, the design matrix is given in table (2). The number of experimental points is given by;

$$n = 2^k + n_c + n_\alpha \quad (5)$$

where n_α is number of axial points determined from equation (6);

$$n_\alpha = \sqrt{k} \quad (6)$$

Table 2: First-order three level full factorial experimental design matrix with centre points

Std	Run	X_1	X_2	X_3	X_4
6	1	1.00	-1.00	1.00	-1.00
27	2	0.00	0.00	0.00	0.00
11	3	-1.00	1.00	-1.00	1.00
28	4	0.00	0.00	0.00	0.00
15	5	-1.00	1.00	1.00	1.00
30	6	0.00	0.00	0.00	0.00
19	7	0.00	-2.00	0.00	0.00
8	8	1.00	1.00	1.00	-1.00
18	9	2.00	0.00	0.00	0.00
10	10	1.00	-1.00	-1.00	1.00
20	11	0.00	2.00	0.00	0.00
3	12	-1.00	1.00	-1.00	-1.00
14	13	1.00	-1.00	1.00	1.00
5	14	-1.00	-1.00	1.00	-1.00
12	15	1.00	1.00	-1.00	1.00
4	16	1.00	1.00	-1.00	-1.00
7	17	-1.00	1.00	1.00	-1.00
16	18	1.00	1.00	1.00	1.00
24	19	0.00	0.00	0.00	2.00
2	20	1.00	-1.00	-1.00	-1.00
21	21	0.00	0.00	-2.00	0.00
26	22	0.00	0.00	0.00	0.00
29	23	0.00	0.00	0.00	0.00
17	24	-2.00	0.00	0.00	0.00
9	25	-1.00	-1.00	-1.00	1.00
25	26	0.00	0.00	0.00	0.00
13	27	-1.00	-1.00	1.00	1.00
1	28	-1.00	-1.00	-1.00	-1.00
23	29	0.00	0.00	0.00	-2.00
22	30	0.00	0.00	2.00	0.00

Data Collection Methods

This study utilized primary and quantitative data collected with the consideration of its nature, scope and objectives [45, 46]. The three main methods employed in the data collection are direct observation, practical experience and field experimentation [45, 46]. The values of the selected parameters were based on the set points of the experimental design previously conducted. The data was collected from real-time process and pipeline pre-commissioning and commissioning [47] liquid nitrogen pumping for pressure test on surface piping systems of hydrocarbon and allied fluids facility, which is Egina FPSO. The pressure values of the piping systems under test were recorded from the indication on the installed pressure gauge. In addition, the operating parameters of the pump were taken from the indicated values on the control panel. The collected data values for each of the randomly selected test packs [48] were tabulated and analysed. Therefore, only the set experimental design points were considered in the experimental data analysis.

Data Analysis and Model Development Methods

The data collected was analysed according to the factorial experimental design setup to predict regression models which were statistically analysed to check for lack-of-fit test and existence of errors in the data set. Also, further analysis was performed to check for the need for curvature and interactions, check the standard error in the regression and significance test. These were performed with the aid of DESIGN EXPERT, version 13.0.5.0 64-bit software [49]. According to [46] statistical data analyses offer more reliable quantitative data evaluation results. Also, the uncertainties associated with the data set and their inherent stochastic nature brought about parametric considerations in the data analyses processes [50].

The design was statistically analysed to generate first-order (linear), second-order (quadratic) and third-order (cubic) models for each of the performance parameters. This model development approach is stochastic [51, 52], and the models are nonlinear, continuous and multidimensional [53, 54, 55, 56, 57]. They were developed, evaluated and validated using DESIGN EXPERT 13.0.5.0 64-bit software [58, 59, 60] and thus, were presented according to [55 61, 59, 63]. Then the quadratic and cubic models were compared on the basis of their respective lack-of-fit, R^2, $Adj - R^2$, $Pred - R^2$ and Adequate precision. Therefore the model with significant lack-of-fit was rejected, the model with increasing R^2, $Adj - R^2$ and $Pred - R^2$, whose $Adj - R^2$ and $Pred - R^2$ are in reasonable agreement such that their difference is less than 0.2, and adequate precision is greater than 4 was considered. According to StatEase [49], the linear, quadratic and cubic models are respectively represented in equations (7), (8) and (9) respectively;

$$Y = \beta_0 + \sum_{i=1}^{q} \beta_i X_i + \epsilon \tag{7}$$

$$Y = \beta_0 + \sum_{i=1}^{q} \beta_i X_i + \sum_{i<j}^{q-1} \sum_{j}^{q} \beta_{ij} X_i X_j + \epsilon \tag{8}$$

$$Y = \beta_0 + \sum_{i=1}^{q} \beta_i X_i + \sum_{i<j}^{q-1} \sum_{j}^{q} \beta_{ij} X_i X_j + \sum_{i<j}^{q-1} \sum_{j}^{q} \beta_{ij} X_i X_j (X_i - X_j) + \sum_{i<j}^{q-2} \sum_{j<k}^{q-1} \sum_{k}^{q} \beta_{ijk} X_i X_j X_k + \epsilon \tag{9}$$

Where Y represents the performance parameters, β represents coefficients of the control parameters, ϵ represents prediction error.

Model Verification Methods

To ascertain the validity of the predicted equations to fit the data set well, the Spearman's rank correlation coefficient denoted by R, the coefficients of determination denoted by R^2, $Adj - R^2$, error standard deviation denoted by S, the $F - statistic$ or $F - value$, and the predicted probability value denoted by $p - val$ [63, 64, 65, 66] were computed from DESIGN EXPERT 13.0.5.0 64-bit software. Thus R ($0 \le R \le 1$) was used to show the correlation between the measured data values and the data values estimated from the fitted models such that the more the value of R (for each of the performance parameters) approaches 1, the more the estimated data values correlate with the measured data values and vice versa. The coefficient of determination (R^2) was to indicate how the fitted data values represent the measured data values; its value lies between zero and one ($0 \le R^2 \le 1$) and the closer it is to one (1), the better the model represents the measured data, and vice versa. Adjusted coefficient of determination ($Adj - R^2$) was to indicate the need for model adjustment; and the more the value of $Adj - R^2$ approaches the value of R^2, the less the need for model adjustment. The error standard deviation (S) was used to show how well the predicted value of the performance parameter model approximates the measured data value in that the closer the value of S approaches zero, the better the model approximates the measured value. $F - statistic$ was calculated to show how well the model fits the measured data; the larger the value of $F - statistic$, the better the model fits the data and vice versa. The calculated value of $F - statistic$ (F_{cal}) was compared with the critical or tabulated value (F_{tab}) at 95% confidence interval (i.e. $\alpha = 0.05$); so if $F_{cal} > F_{tab}$, the model fits the data adequately and vice versa. The probability value ($p - val$) was estimated at $\alpha = 0.05$ such that a $p - val < 0.05$ indicates statistically significant result, while a $p - val > 0.05$ indicates statistical insignificance. Then the validity analyses were performed as follows [49];

$$R^2 = 1 - \left[\frac{SS_R}{SS_R + SS_M}\right] = 1 - \left[\frac{SS_R}{SS_T - SS_C - SS_B}\right] \tag{10}$$

$$Adj - R^2 = 1 - \left[\frac{SS_R}{df_R}\left(\frac{df_R + df_M}{SS_R + SS_M}\right)\right] = 1 - \left[\frac{SS_R}{df_R}\left(\frac{df_T - df_C + df_B}{SS_T - SS_C + SS_B}\right)\right] \tag{11}$$

$$Pred - R^2 = 1 - \left[\frac{PRESS}{SS_R + SS_M}\right] = 1 - \left[\frac{PRESS}{SS_T - SS_C - SS_B}\right] \tag{13}$$

$$PRESS = \sum_{i=1}^{n} (\epsilon_{-i})^2 \tag{14}$$

$$\epsilon_{-i} = Y_i - \hat{Y}_{-i} = \frac{\epsilon_i}{1 - H_{ii}} \tag{15}$$

$$Adequate\ Precision = \frac{max(\hat{Y}) - min(\hat{Y})}{\sqrt{\bar{V}_{\hat{Y}}}} > 4 \tag{16}$$

$$\bar{V}_{\hat{Y}} = \frac{p\hat{\sigma}^2}{n} \tag{17}$$

Where SS_R is residual sum of square, SS_M is model sum of squares, SS_T is total sum of squares, SS_C is curvature sum of squares, SS_B is block sum of squares, df_R is residual degree of freedom, df_T is total degree of freedom, df_C is curvature degree of freedom, df_B is block degree of freedom, ϵ_{-i} *is deletion residual computed by fitting a model without the ith run then trying to predict the* ith *observation with the resulting model*, ϵ_i is the residual for each observation left over from the model fit to all the data, H_{ii} is the leverage of the run in the design, p is number of parameters including intercept (β_0) and any block coefficients, n is number of runs in the experiment, σ^2 is residual mean square from the ANOVA table.

III. Result and Discussions

Factorial Experimental Data Set

The three level randomized full factorial experimental design data set generated with design expert 13 is presented in table (3). It showed the actual values of the control parameters at the factorial (maximum and minimum) and centre points of the design with the corresponding values of the performance parameters. The data set was analysed for first order models of the performance parameters.

Table 3 Factorial Design Layout

Std	Run	$V_p(m^3)$	$P_t(bar)$	$P_d(bar)$	$Q_N(m^3/hr)$	$V_1(m^3)$	$D_t(hr)$	$D_p(hr)$	$V_2(m^3)$
15	1	10.80	330.30	350.00	1699.00	3.94	7.50	4.19	4.07
19	2	1326.60	168.20	215.00	976.58	1.72	2.42	1.51	1.72
6	3	2642.40	6.10	350.00	254.15	3.40	6.42	3.62	3.42
8	4	2642.40	330.30	350.00	254.15	4.57	7.00	3.93	4.57
2	5	2642.40	6.10	80.00	254.15	0.59	4.00	2.35	0.61
12	6	2642.40	330.30	80.00	1699.00	5.27	9.25	5.11	5.68
1	7	10.80	6.10	80.00	254.15	0.54	1.92	1.25	0.55
4	8	2642.40	330.30	80.00	254.15	1.62	5.33	3.05	1.63
7	9	10.80	330.30	350.00	254.15	2.62	6.33	3.57	2.71
22	10	1326.60	168.20	215.00	976.58	0.80	2.67	1.65	0.80
5	11	10.80	6.10	350.00	254.15	0.69	4.42	2.57	0.69
13	12	10.80	6.10	350.00	1699.00	1.32	5.83	3.31	1.42
3	13	10.80	330.30	80.00	254.15	0.75	3.58	2.12	0.75
16	14	2642.40	330.30	350.00	1699.00	6.25	12.83	7.00	6.25
20	15	1326.60	168.20	215.00	976.58	1.26	2.50	1.56	1.42
18	16	1326.60	168.20	215.00	976.58	1.20	2.17	1.38	1.22
10	17	2642.40	6.10	80.00	1699.00	1.36	6.42	3.62	1.36
17	18	1326.60	168.20	215.00	976.58	1.42	2.50	1.56	1.51
9	19	10.80	6.10	80.00	1699.00	1.11	5.65	3.22	1.13
11	20	10.80	330.30	80.00	1699.00	3.20	6.08	3.44	4.01
14	21	2642.40	6.10	350.00	1699.00	3.15	9.75	5.38	3.26
21	22	1326.60	168.20	215.00	976.58	1.14	2.25	1.42	1.15

Data Analyses Results for Factorial Designs and First Order Models

The analyses of the data set yielded first order models that were fitted with the factorial experimental design previously conducted. The analysis results include the normal plots of residuals, curvature terms, ANOVA and fit statistics for total volume of liquid nitrogen gas pumped, test duration, pumping duration and total volume of liquid nitrogen used. The normal plots of residuals for the main effects are shown in figs. (1), (2), (3) and (4). The colour band from blue to red indicates the skewness of the normal probability of residuals from the minimum to the maximum values of the performance parameters.

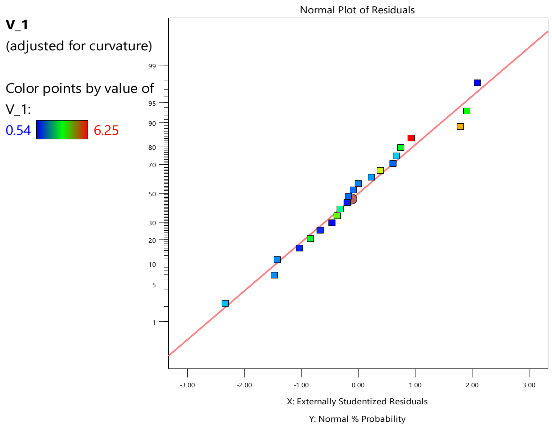

Fig. 1: Normal Probability Plot of Residuals for Main Effects of V_1

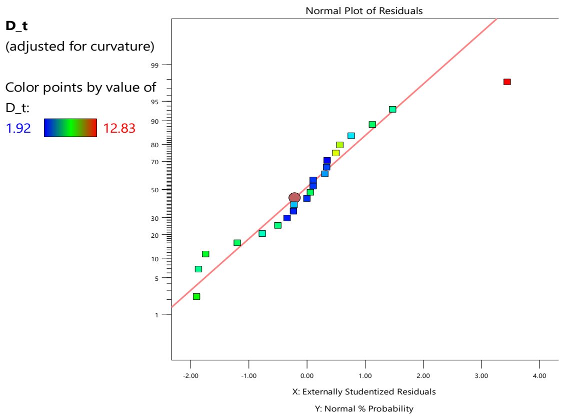

Fig. 2: Normal Probability Plot of Residuals for Main effects of D_t

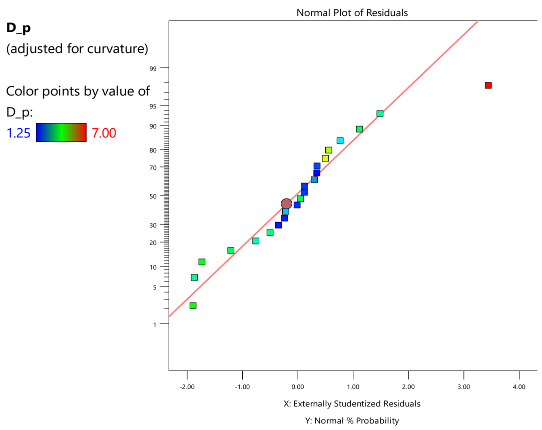

Fig. 3: Normal probability plot of residuals for main effects of D_p

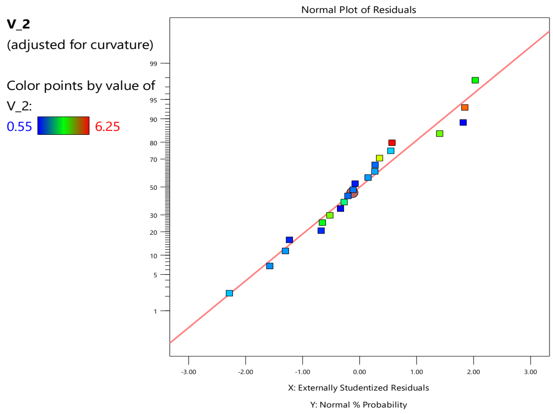

Fig. 4: Normal probability plot of residuals for main effects of V_2

The distribution of points along the regression line indicates that there are many outliers, possible lack of fit and presence of possible curvatures in the models of total volume of liquid nitrogen pumped, test duration, pumping duration and total volume of liquid nitrogen used respectively.

Then the analyses of curvature terms are shown in tables (4) for performance parameters. Each of the tables for curvature terms indicates that the overall model of each of the performance parameters is significant, their respective curvature is significant and their respective lack of fit is also significant. This is justified by the model p-value of less than 0.05 (p-value < 0.05) for each of the model terms of the respective performance parameters. Significant models indicate the models are true representation of the respective measured data set, and so should not be discarded. Significant curvatures indicate the presence of possible curvature (which can be quadratic, cubic, quartic, etc.) in the model of each the performance parameters. Then significant lack of fit indicates that none of the models fit the measured data well. Thus, since the models should not be discarded, have significant need for curvature and have significant lack of fit; the factorial design should be augmented to the response surface design. This checks what causes the significant curvature terms in the models.

Table 4: Curvature Term for the Performance Parameters

Term	Adjusted F-value				Model p-value				
	V_1	D_t	D_p	V_2	V_1	D_t	D_p	V_2	
Model	18.63	37.68	37.63	16.51	7.22908E-06	5.92469E-08	5.97869E-08	1.55865E-05	significant
Curvature	12.81	112.46	112.24	11.76	0.0025	1.20657E-08	1.22278E-08	0.0034	significant
Lack of Fit	8.07	26.31	24.59	8.82	0.0160	0.0010	0.0012	0.0131	significant

The analysis of variance (ANOVA) results for the selected factorial design of total volume of liquid nitrogen pumped, test duration, pumping duration and total volume of liquid nitrogen used are respectively shown in tables (5), (6), (7) and (8). According to the tables, all the control parameters are significant to each of the performance parameters since their respective p-values are less than 0.05 (p-value < 0.05). Thus, the ANOVA tables show that each of the control parameters affects each of the performance parameters. Hence none of the control parameters should discarded from the respective models of the performance parameters. The direction of their effects is indicated by their respective coefficients.

Table 5: ANOVA Table for Selected Factorial Model of V_1

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	40.76	4	10.19	18.63	7.22908E-06	significant
V_p	9.06	1	9.06	16.56	0.0009	
P_t	16.12	1	16.12	29.47	5.57417E-05	
P_d	8.27	1	8.27	15.11	0.0013	
Q_N	7.32	1	7.32	13.38	0.0021	
Curvature	7.01	1	7.01	12.81	0.0025	significant
Residual	8.75	16	0.5470			
Lack of Fit	8.29	11	0.7532	8.07	0.0160	significant

Pure Error	0.4667	5	0.0933			
Cor Total	56.52	21				

Table 6: ANOVA Table for Selected Factorial Design of D_t

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	92.45	4	23.11	37.68	5.92469E-08	significant
V_p	24.23	1	24.23	39.50	1.08696E-05	
P_t	11.37	1	11.37	18.54	0.0005	
P_d	19.91	1	19.91	32.46	3.30034E-05	
Q_N	36.94	1	36.94	60.21	8.21959E-07	
Curvature	68.98	1	68.98	112.46	1.20657E-08	
Residual	9.82	16	0.6134			
Lack of Fit	9.65	11	0.8771	26.31	0.0010	significant
Pure Error	0.1667	5	0.0333			
Cor Total	171.25	21				

Table 7: ANOVA Table for Selected Factorial Model of D_p

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	25.68	4	6.42	37.63	5.97869E-08	significant
V_p	6.75	1	6.75	39.55	1.07914E-05	
P_t	3.14	1	3.14	18.42	0.0006	
P_d	5.53	1	5.53	32.44	3.31282E-05	
Q_N	10.26	1	10.26	60.12	8.30161E-07	
Curvature	19.15	1	19.15	112.24	1.22278E-08	
Residual	2.73	16	0.1706			
Lack of Fit	2.68	11	0.2436	24.59	0.0012	significant
Pure Error	0.0495	5	0.0099			
Cor Total	47.56	21				

Table 8: ANOVA Table for Selected Factorial Model of V_2

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	43.24	4	10.81	16.51	1.55865E-05	significant
V_p	8.19	1	8.19	12.51	0.0027	
P_t	18.55	1	18.55	28.33	6.86712E-05	
P_d	7.12	1	7.12	10.87	0.0046	
Q_N	9.38	1	9.38	14.32	0.0016	
Curvature	7.70	1	7.70	11.76	0.0034	
Residual	10.48	16	0.6549			
Lack of Fit	9.96	11	0.9059	8.82	0.0131	significant
Pure Error	0.5137	5	0.1027			
Cor Total	61.42	21				

The fit statistics presented, among other statistical parameters, the values of R-squared, adjusted R-squared, predicted R-squared and adequate precision for each of the performance parameters. It showed that the values of R-square (R^2), adjusted R-square, ($adj - R^2$), predicted R-square ($pred - R^2$) and adequate precision are good to navigate the design space for each of the respective performance parameters. The predicted R-square and the adjusted R-square are in reasonable agreement since the difference is less than 0.2, and adequate precision, which measures the signal to noise ratio, is greater than 4 in each instance of the performance parameters. Thus, the models are not discarded.

Augmented Experimental Design Data Set

Since the linear models of the total volume of liquid nitrogen pumped, test duration, pumping duration and total volume of liquid nitrogen used show evidence of lack of fit and significant curvature terms, the factorial designs were augmented to polynomial designs by the introduction of star points as presented in table (9).

Table 9: Data Set for the Polynomial Designs

S/N	$V_p\ (m^3)$	$P_t(bar)$	$P_d\ (bar)$	$Q_N\ (m^3/hr)$	$V_1\ (m^3)$	$D_t\ (hr)$	$D_p\ (hr)$	$V_2\ (m^3)$
1	792.00	99.00	200.00	1.22	1.36	4.00	1.50	1.36
2	1577.92	190.80	300.00	2.44	3.94	6.33	4.58	4.07
3	55.44	99.00	80.00	0.37	0.54	2.50	1.58	0.55
4	1601.10	45.00	260.00	2.44	4.57	6.42	3.83	4.57
5	293.60	8.60	180.00	1.22	1.32	5.83	1.50	1.42
6	439.20	219.60	290.00	0.49	3.40	9.25	7.00	3.42
7	192.60	192.60	200.00	0.98	1.62	2.67	2.25	1.63
8	1387.58	6.30	180.00	1.10	1.72	7.50	3.83	1.72
9	22.77	99.00	100.00	0.37	0.69	2.50	1.92	0.69
10	495.00	9.90	150.00	0.98	1.11	2.42	2.55	1.13
11	281.34	54.00	135.00	0.85	1.20	2.17	1.45	1.22
12	22.77	99.00	100.00	0.37	0.69	2.50	1.92	0.69
13	219.36	6.10	140.00	0.49	0.80	5.33	3.08	0.80
14	69.30	31.50	100.00	0.49	1.14	6.42	4.00	1.15
15	1577.92	190.80	300.00	2.44	3.94	6.33	4.58	4.07
16	312.01	192.60	200.00	0.98	0.75	1.92	1.25	0.75
17	1785.60	45.00	276.00	2.44	5.27	7.00	4.13	5.68
18	315.00	31.50	180.00	1.22	2.62	5.65	2.75	2.71
19	495.00	9.90	150.00	0.98	1.11	2.42	2.55	1.13
20	1530.62	150.80	280.00	2.44	3.15	6.08	2.83	3.26
21	2642.40	330.30	350.00	1.95	6.25	12.83	4.67	6.25
22	10.80	108.00	100.00	0.37	0.59	2.25	1.58	0.61
23	10.80	108.00	100.00	0.37	0.59	2.25	1.58	0.61
24	437.85	31.50	160.00	1.22	1.26	3.58	1.67	1.42
25	1785.60	45.00	276.00	2.44	5.27	7.00	4.13	5.68
26	293.60	8.60	180.00	1.22	1.32	5.58	1.50	1.42
27	464.54	40.50	160.00	1.22	1.42	4.42	1.67	1.51
28	1620.00	54.00	207.00	1.46	3.20	9.75	4.58	4.01
29	439.20	219.60	290.00	0.49	3.40	9.25	7.00	3.42
30	1601.10	45.00	260.00	2.44	4.57	6.42	3.83	4.57

Data Analyses Results for Quadratic Models

The augmented designs were analysed to generate quadratic polynomials. Results of the analyses presented the fit summaries (table 10). It showed that the total volume of liquid nitrogen pumped has no significant lack of fit for both the quadratic and cubic terms, test duration has significant lack of fit for quadratic terms but no significant lack of fit for cubic terms, pumping duration has significant lack of fit for quadratic terms and no significant lack of fit for cubic terms, and total volume of liquid nitrogen used has no significant lack of fit for both the quadratic terms and cubic terms. So quadratic model is suggested and the cubic model is aliased for each of the performance parameters. The adjusted R^2 and predicted R^2 are seen to increase from the quadratic toward the cubic models of the respective parameters. This indicated that there is improvement in the performance of the models from the quadratic to the cubic. The fit summaries for quadratic models of total volume of liquid nitrogen pumped (V_1), test duration (D_t), pumping duration (D_d) and total volume of liquid nitrogen used were shown in tables (10).

Table 10: Fit summary for Quadratic Model of the Performance Parameters

Parameters	Source	Sequential p-value	Lack of Fit p-value	Adjusted R²	Predicted R²	
V_1	Linear	1.20246E-07	0.0090	0.7316	0.6797	
	2FI	0.1536	0.0121	0.7746	0.6837	
	Quadratic	**0.0003**	**0.1282**	**0.9264**	**0.8052**	**Suggested**
	Cubic	0.0311	0.7966	0.9744	0.9144	**Aliased**
D_t	Linear	0.0007	1.49369E-05	0.4467	0.4037	
	2FI	0.9829	7.4813E-06	0.3082	0.0602	
	Quadratic	**5.61273E-06**	**0.0004**	**0.8660**	**0.6045**	**Suggested**
	Cubic	2.62093E-05	0.3144	0.9944	0.9269	**Aliased**
D_p	Linear	0.0007	1.76738E-05	0.4464	0.4034	
	2FI	0.9830	8.85053E-06	0.3077	0.0595	
	Quadratic	**5.75061E-06**	**0.0004**	**0.8655**	**0.6032**	**Suggested**
	Cubic	2.91959E-05	0.3413	0.9942	0.9285	**Aliased**
V_2	Linear	3.32578E-07	0.0077	0.7083	0.6483	
	2FI	0.1424	0.0105	0.7578	0.6471	
	Quadratic	**0.0007**	**0.0844**	**0.9090**	**0.7536**	**Suggested**
	Cubic	0.0163	0.8084	0.9742	0.9186	**Aliased**

Also, sequential model sum of squares (table (11)) showed that quadratic versus two-factor-interaction terms were significant and suggested, and the cubic versus quadratic terms are significant and aliased for all the performance parameters. But [49] recommends that the highest order polynomial where the additional terms are significant and the model is not aliased should be selected. The sequential model sum of squares for quadratic polynomials of total volume of liquid nitrogen pumped (V_1), test duration (D_t), pumping duration (D_d) and total volume of liquid nitrogen used are shown in tables (11).

Table 11: Sequential Model Sum of Squares for Quadratic Model of the Performance Parameters

Parameters	Source	Sum of Squares	df	Mean Square	F-value	p-value	
V_1	Mean vs Total	157.83	1	157.83			
	Linear vs Mean	63.50	4	15.87	20.76	1.20246E-07	
	2FI vs Linear	6.92	6	1.15	1.80	0.1536	
	Quadratic vs 2FI	**9.05**	**4**	**2.26**	**10.80**	**0.0003**	**Suggested**
	Cubic vs Quadratic	2.63	8	0.3291	4.51	0.0311	Aliased
	Residual	0.5112	7	0.0730			
	Total	240.44	30	8.01			
D_t	Mean vs Total	928.41	1	928.41			
	Linear vs Mean	102.85	4	25.71	6.85	0.0007	
	2FI vs Linear	4.67	6	0.7778	0.1658	0.9829	
	Quadratic vs 2FI	**75.50**	**4**	**18.87**	**20.78**	**5.61273E-06**	**Suggested**
	Cubic vs Quadratic	13.36	8	1.67	44.15	2.62093E-05	Aliased
	Residual	0.2648	7	0.0378			
	Total	1125.04	30	37.50			
D_p	Mean vs Total	301.53	1	301.53			
	Linear vs Mean	28.54	4	7.13	6.85	0.0007	
	2FI vs Linear	1.29	6	0.2153	0.1653	0.9830	
	Quadratic vs 2FI	**20.96**	**4**	**5.24**	**20.70**	**5.75061E-06**	**Suggested**
	Cubic vs Quadratic	3.72	8	0.4652	42.76	2.91959E-05	Aliased
	Residual	0.0761	7	0.0109			
	Total	356.12	30	11.87			

V_2	Mean vs Total	170.50	1	170.50			
	Linear vs Mean	67.21	4	16.80	18.60	3.32578E-07	
	2FI vs Linear	8.33	6	1.39	1.85	0.1424	
	Quadratic vs 2FI	**10.02**	**4**	**2.51**	**8.89**	**0.0007**	**Suggested**
	Cubic vs Quadratic	3.67	8	0.4585	5.74	0.0163	Aliased
	Residual	0.5593	7	0.0799			
	Total	260.29	30	8.68			

The model summary statistics showed that the standard deviation of errors and prediction residual sum of squares decrease from the quadratic to the cubic terms, the quadratic terms were suggested while the cubic terms are aliased for all the performance parameters. It also showed that the R^2, $adjusted\ R^2$ and $predicted\ R^2$ increase from the quadratic to the cubic functions; while the predicted residual sum of squares (PRESS) decreases as the model order increases for each of the parameters. Then [49] recommends that the model maximizing the $adjusted\ R^2$ and $predicted\ R^2$ should be focused on. The model summary statistics for quadratic polynomials of total volume of liquid nitrogen pumped (V_1), test duration (D_t), pumping duration (D_d) and total volume of liquid nitrogen used are shown in table (12).

Table 12: Model Summary Statistics for Quadratic Models

Parameters	Source	Std. Dev.	R²	Adjusted R²	Predicted R²	PRESS	
V_1	Linear	0.8744	0.7686	0.7316	0.6797	26.46	
	2FI	0.8012	0.8524	0.7746	0.6837	26.13	
	Quadratic	**0.4578**	**0.9619**	**0.9264**	**0.8052**	**16.09**	**Suggested**
	Cubic	0.2702	0.9938	0.9744	0.9144	7.07	Aliased
D_t	Linear	1.94	0.5230	0.4467	0.4037	117.25	
	2FI	2.17	0.5468	0.3082	0.0602	184.80	
	Quadratic	**0.9531**	**0.9307**	**0.8660**	**0.6045**	**77.76**	**Suggested**
	Cubic	0.1945	0.9987	0.9944	0.9269	14.37	Aliased
D_p	Linear	1.02	0.5228	0.4464	0.4034	32.57	
	2FI	1.14	0.5465	0.3077	0.0595	51.34	
	Quadratic	**0.5032**	**0.9304**	**0.8655**	**0.6032**	**21.66**	**Suggested**
	Cubic	0.1043	0.9986	0.9942	0.9285	3.90	Aliased
V_2	Linear	0.9504	0.7485	0.7083	0.6483	31.58	
	2FI	0.8660	0.8413	0.7578	0.6471	31.69	
	Quadratic	**0.5308**	**0.9529**	**0.9090**	**0.7536**	**22.13**	**Suggested**
	Cubic	0.2827	0.9938	0.9742	0.9186	7.31	Aliased

More so, lack-of-fit test results show suggested quadratic and aliased cubic functions for each of the performance parameters; though insignificant quadratic functions for total volume of liquid nitrogen pumped and total volume of liquid nitrogen used, significant lack of fit for quadratic function of the test duration and pumping durations, and insignificant lack fit for the cubic functions of all the performance parameters. Stat-Ease [49] recommends that the model with insignificant lack of fit should be selected. Hence, though the quadratic models for the total volume of liquid nitrogen pumped and the total volume of liquid nitrogen used have insignificant lack of fit, the insignificance of the lack of fit of their cubic models is higher. It can also be seen that the quadratic models of the test duration and the pumping duration have significant lack of fit, while their respective cubic models have insignificant lack of fit. The lack of fit for quadratic polynomials of total volume of liquid nitrogen pumped (V_1), test duration (D_t), pumping duration (D_d) and total volume of liquid nitrogen used are shown in table (13).

Table 13: Lack of Fit Tests for the Quadratic Models

Parameters	Source	Sum of Squares	df	Mean Square	F-value	p-value	
V_1	Linear	18.65	20	0.9325	9.99	0.0090	
	2FI	11.73	14	0.8379	8.98	0.0121	
	Quadratic	**2.68**	**10**	**0.2677**	**2.87**	**0.1282**	**Suggested**
	Cubic	0.0444	2	0.0222	0.2380	0.7966	Aliased
	Pure Error	0.4667	5	0.0933			
D_t	Linear	93.62	20	4.68	140.42	1.49369E-05	
	2FI	88.96	14	6.35	190.60	7.4813E-06	

	Quadratic	**13.46**	**10**	**1.35**	**40.37**	**0.0004**	**Suggested**
	Cubic	0.0981	2	0.0491	1.47	0.3144	Aliased
	Pure Error	0.1667	5	0.0333			
D_p	Linear	26.00	20	1.30	131.22	1.76738E-05	
	2FI	24.71	14	1.76	178.15	8.85053E-06	
	Quadratic	**3.75**	**10**	**0.3748**	**37.83**	**0.0004**	**Suggested**
	Cubic	0.0266	2	0.0133	1.34	0.3413	Aliased
	Pure Error	0.0495	5	0.0099			
V_2	Linear	22.07	20	1.10	10.74	0.0077	
	2FI	13.74	14	0.9812	9.55	0.0105	
	Quadratic	**3.71**	**10**	**0.3713**	**3.61**	**0.0844**	**Suggested**
	Cubic	0.0456	2	0.0228	0.2219	0.8084	Aliased
	Pure Error	0.5137	5	0.1027			

The analyses of variance (ANOVA) results of the quadratic functions show that there is no significant lack of fit for total volume of liquid nitrogen pumped and total volume of liquid nitrogen used but there is significant lack of fit for test duration and pumping duration. For V_1, V_p and P_t are the significant main effects; the interactions of V_p and P_d, P_t and Q_N, and P_d and Q_N are significant; while all the quadratic terms are also significant. For D_t and D_d, only Q_N is the insignificant main effect, no two-factor interaction term is significant and all the quadratic terms are significant. For V_2, no main effect is significant, the two-factor interactions of V_p and P_d, P_t and Q_N, and P_d and Q_N; and all the quadratic terms are significant. The ANOVA for quadratic polynomials of V_1, D_t, D_d and V_2 are shown in tables (14), (15), (16) and (17) respectively.

Table 14: ANOVA for Quadratic Model of V_1

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	79.47	14	5.68	27.08	4.19829E-08	significant
V_p	1.02	1	1.02	4.89	0.0430	
P_t	1.31	1	1.31	6.23	0.0247	
P_d	0.0538	1	0.0538	0.2568	0.6197	
Q_N	0.0030	1	0.0030	0.0145	0.9058	
V_pP_t	0.3481	1	0.3481	1.66	0.2170	
V_pP_d	1.93	1	1.93	9.22	0.0083	
V_pQ_N	0.0484	1	0.0484	0.2309	0.6378	
P_tP_d	0.1560	1	0.1560	0.7445	0.4018	
P_tQ_N	3.40	1	3.40	16.24	0.0011	
P_dQ_N	1.03	1	1.03	4.92	0.0425	
V_p^2	4.75	1	4.75	22.68	0.0003	
P_t^2	4.55	1	4.55	21.73	0.0003	
P_d^2	1.51	1	1.51	7.23	0.0168	
Q_N^2	1.55	1	1.55	7.38	0.0159	
Residual	3.14	15	0.2096			
Lack of Fit	2.68	10	0.2677	2.87	0.1282	not significant
Pure Error	0.4667	5	0.0933			
Cor Total	82.61	29				

Table 15: ANOVA for Quadratic Model of D_t

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	183.01	14	13.07	14.39	3.16287E-06	significant
V_p	8.86	1	8.86	9.75	0.0070	
P_t	6.81	1	6.81	7.50	0.0153	
P_d	8.77	1	8.77	9.65	0.0072	
Q_N	3.83	1	3.83	4.22	0.0579	
V_pP_t	0.2889	1	0.2889	0.3181	0.5811	
V_pP_d	1.08	1	1.08	1.19	0.2935	
V_pQ_N	2.80	1	2.80	3.08	0.0997	
P_tP_d	0.0613	1	0.0613	0.0674	0.7986	
P_tQ_N	0.4001	1	0.4001	0.4404	0.5170	
P_dQ_N	0.0431	1	0.0431	0.0474	0.8306	
V_p^2	34.82	1	34.82	38.34	1.72508E-05	
P_t^2	28.64	1	28.64	31.52	4.9288E-05	
P_d^2	23.56	1	23.56	25.94	0.0001	
Q_N^2	20.08	1	20.08	22.10	0.0003	
Residual	13.63	15	0.9084			
Lack of Fit	13.46	10	1.35	40.37	0.0004	significant
Pure Error	0.1667	5	0.0333			
Cor Total	196.64	29				

Table 16: ANOVA for Quadratic Model of D_p

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	50.79	14	3.63	14.33	3.25375E-06	significant
V_p	2.46	1	2.46	9.71	0.0071	
P_t	1.90	1	1.90	7.50	0.0152	
P_d	2.44	1	2.44	9.64	0.0072	
Q_N	1.06	1	1.06	4.20	0.0582	
V_pP_t	0.0827	1	0.0827	0.3265	0.5762	
V_pP_d	0.2998	1	0.2998	1.18	0.2937	
V_pQ_N	0.7700	1	0.7700	3.04	0.1016	
P_tP_d	0.0176	1	0.0176	0.0693	0.7959	
P_tQ_N	0.1106	1	0.1106	0.4367	0.5187	
P_dQ_N	0.0116	1	0.0116	0.0456	0.8337	
V_p^2	9.66	1	9.66	38.15	1.77091E-05	
P_t^2	7.95	1	7.95	31.41	5.02338E-05	
P_d^2	6.54	1	6.54	25.85	0.0001	
Q_N^2	5.58	1	5.58	22.03	0.0003	
Residual	3.80	15	0.2532			
Lack of Fit	3.75	10	0.3748	37.83	0.0004	significant
Pure Error	0.0495	5	0.0099			
Cor Total	54.59	29				

Table 17: ANOVA for Quadratic Model of V_2

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	85.56	14	6.11	21.69	1.97086E-07	significant
V_p	1.07	1	1.07	3.79	0.0705	
P_t	1.01	1	1.01	3.60	0.0773	
P_d	0.0067	1	0.0067	0.0238	0.8795	
Q_N	0.0052	1	0.0052	0.0184	0.8938	
V_pP_t	0.1871	1	0.1871	0.6638	0.4280	
V_pP_d	2.08	1	2.08	7.38	0.0159	
V_pQ_N	0.0095	1	0.0095	0.0337	0.8567	
P_tP_d	0.0095	1	0.0095	0.0337	0.8567	
P_tQ_N	4.46	1	4.46	15.84	0.0012	
P_dQ_N	1.58	1	1.58	5.61	0.0317	
V_p^2	5.79	1	5.79	20.53	0.0004	
P_t^2	4.65	1	4.65	16.50	0.0010	
P_d^2	1.47	1	1.47	5.23	0.0372	
Q_N^2	1.69	1	1.69	5.99	0.0272	
Residual	4.23	15	0.2818			
Lack of Fit	3.71	10	0.3713	3.61	0.0844	not significant
Pure Error	0.5137	5	0.1027			
Cor Total	89.79	29				

Then the fit statistics showed that the predicted R^2 is in reasonable agreement with the adjusted R^2 since the difference between them is less than 0.2 for total volume of liquid nitrogen pumped and total volume of liquid nitrogen used, but they are not as close as one might normally expect since the difference between them is more than 0.2 for test duration and pumping duration. Also, the adequate precision which measures the signal to noise ratio is greater than 4 in all the performance parameters. The cases of the test duration and pumping duration, according to [49], may indicate a large block effect or a possible problem with the models and/or data set and so model reduction, response transformation, outliers, etc. may be considered; however, each of the models is good to navigate their respective design spaces. Thus, the analyses are advanced to the cubic polynomials to check possible improvement in the models' fit.

Data Analyses for Cubic Models

Considering the results of the quadratic polynomials which indicated possible improvement of the models' fit in the cubic function, the polynomial designs were analysed to fit cubic polynomials. Therefore, the analysis results of the cubic models show that the cubic models represented the data more precisely than the quadratic models as their respective lacks of fit were not significant, R^2 is closer to 1 than that of the quadratic models, adjusted R^2 and predicted R^2 are in reasonable agreement and adequate precision increased. The cubic models are selected due to their adequacy level in prediction of the data set and significance of some of the three factor interactions; though some of cubic terms are aliased (all the aliased terms are removed from the models). The results also presented the fit summary, sequential model sum of squares, model summary statistics, lack of fit tests, analysis of variance (ANOVA) and fit statistics for each of the total volume of liquid nitrogen pumped, the test duration, the pumping duration and the total volume of liquid nitrogen used.

The fit summaries show that the V_1, D_t, D_d and V_2 have no significant lack of fit for the cubic terms. There is a large increase in the fit probability of the cubic models, though some of the cubic terms were aliased and the quadratic models were suggested as shown. Since the aliased terms were removed from the models and the difference between $adjusted\ R^2$ and $predicted\ R^2$ reduced tremendously in the cubic models of all the performance parameters, the cubic models produced better fits for the respective data sets. Also, the $adjusted\ R^2$ and $predicted\ R^2$ were seen to increase from the quadratic toward the cubic functions of the respective parameters. This indicates that there is improvement in the performance of the models from the quadratic to the cubic functions. The fit summaries of the cubic polynomials are presented in table (18).

Table 18: Fit Summary for Cubic Models

Parameters	Source	Sequential p-value	Lack of Fit p-value	Adjusted R²	Predicted R²	
V_1	Linear	1.20246E-07	0.0090	0.7316	0.6797	
	2FI	0.1536	0.0121	0.7746	0.6837	
	Quadratic	**0.0003**	**0.1282**	**0.9264**	**0.8052**	**Suggested**
	Cubic	0.0311	0.7966	0.9744	0.9144	**Aliased**
D_t	Linear	0.0007	1.49369E-05	0.4467	0.4037	
	2FI	0.9829	7.4813E-06	0.3082	0.0602	
	Quadratic	**5.61273E-06**	**0.0004**	**0.8660**	**0.6045**	**Suggested**
	Cubic	2.62093E-05	0.3144	0.9944	0.9269	**Aliased**
D_p	Linear	0.0007	1.76738E-05	0.4464	0.4034	
	2FI	0.9830	8.85053E-06	0.3077	0.0595	
	Quadratic	**5.75061E-06**	**0.0004**	**0.8655**	**0.6032**	**Suggested**
	Cubic	2.91959E-05	0.3413	0.9942	0.9285	**Aliased**
V_2	Linear	3.32578E-07	0.0077	0.7083	0.6483	
	2FI	0.1424	0.0105	0.7578	0.6471	
	Quadratic	**0.0007**	**0.0844**	**0.9090**	**0.7536**	**Suggested**
	Cubic	0.0163	0.8084	0.9742	0.9186	**Aliased**

Also, sequential model sum of squares for the cubic models of the respective performance parameters considered can be seen in table (19). Since [49] recommended that the highest order polynomial where the additional terms are significant and the model is not aliased should be selected. Then the cubic models (with the aliased terms removed) were selected as the best fit for the data set. The summary statistics for the cubic polynomials of the performance parameters – V_1, D_t, D_d and V_2 – are shown in table (22). It showed that the standard deviation of errors and prediction residual sum of squares decrease from the quadratic to the cubic terms, the quadratic terms were suggested while the cubic terms were aliased for all the performance parameters. It can also be seen that the R^2, $adjusted\ R^2$ and $predicted\ R^2$ increase from the quadratic to the cubic functions; while the predicted residual sum of squares (PRESS) decreases as the model order increases for each of the parameters. Then [49] recommended that the model maximizing the $adjusted\ R^2$ and $predicted\ R^2$ should be focused on.

Table 19: Sequential Model Sum of Squares [Type I] Cubic Models

Parameters	Source	Sum of Squares	df	Mean Square	F-value	p-value	
V_1	Mean vs Total	157.83	1	157.83			
	Linear vs Mean	63.50	4	15.87	20.76	1.20246E-07	
	2FI vs Linear	6.92	6	1.15	1.80	0.1536	
	Quadratic vs 2FI	**9.05**	**4**	**2.26**	**10.80**	**0.0003**	**Suggested**
	Cubic vs Quadratic	2.63	8	0.3291	4.51	0.0311	Aliased
	Residual	0.5112	7	0.0730			
	Total	240.44	30	8.01			
D_t	Mean vs Total	928.41	1	928.41			
	Linear vs Mean	102.85	4	25.71	6.85	0.0007	
	2FI vs Linear	4.67	6	0.7778	0.1658	0.9829	
	Quadratic vs 2FI	**75.50**	**4**	**18.87**	**20.78**	**5.61273E-06**	**Suggested**
	Cubic vs Quadratic	13.36	8	1.67	44.15	2.62093E-05	Aliased
	Residual	0.2648	7	0.0378			
	Total	1125.04	30	37.50			
D_p	Mean vs Total	301.53	1	301.53			
	Linear vs Mean	28.54	4	7.13	6.85	0.0007	
	2FI vs Linear	1.29	6	0.2153	0.1653	0.9830	
	Quadratic vs 2FI	**20.96**	**4**	**5.24**	**20.70**	**5.75061E-06**	**Suggested**

	Cubic vs Quadratic	3.72	8	0.4652	42.76	2.91959E-05	Aliased
	Residual	0.0761	7	0.0109			
	Total	356.12	30	11.87			
V_2	Mean vs Total	170.50	1	170.50			
	Linear vs Mean	67.21	4	16.80	18.60	3.32578E-07	
	2FI vs Linear	8.33	6	1.39	1.85	0.1424	
	Quadratic vs 2FI	**10.02**	**4**	**2.51**	**8.89**	**0.0007**	**Suggested**
	Cubic vs Quadratic	3.67	8	0.4585	5.74	0.0163	Aliased
	Residual	0.5593	7	0.0799			
	Total	260.29	30	8.68			

Table 20: Summary Statistics for the Cubic Models

Parameters	Source	Std. Dev.	R^2	Adjusted R^2	Predicted R^2	PRESS	
V_1	Linear	0.8744	0.7686	0.7316	0.6797	26.46	
	2FI	0.8012	0.8524	0.7746	0.6837	26.13	
	Quadratic	**0.4578**	**0.9619**	**0.9264**	**0.8052**	**16.09**	**Suggested**
	Cubic	0.2702	0.9938	0.9744	0.9144	7.07	Aliased
D_t	Linear	1.94	0.5230	0.4467	0.4037	117.25	
	2FI	2.17	0.5468	0.3082	0.0602	184.80	
	Quadratic	**0.9531**	**0.9307**	**0.8660**	**0.6045**	**77.76**	**Suggested**
	Cubic	0.1945	0.9987	0.9944	0.9269	14.37	Aliased
D_p	Linear	1.02	0.5228	0.4464	0.4034	32.57	
	2FI	1.14	0.5465	0.3077	0.0595	51.34	
	Quadratic	**0.5032**	**0.9304**	**0.8655**	**0.6032**	**21.66**	**Suggested**
	Cubic	0.1043	0.9986	0.9942	0.9285	3.90	Aliased
V_2	Linear	0.9504	0.7485	0.7083	0.6483	31.58	
	2FI	0.8660	0.8413	0.7578	0.6471	31.69	
	Quadratic	**0.5308**	**0.9529**	**0.9090**	**0.7536**	**22.13**	**Suggested**
	Cubic	0.2827	0.9938	0.9742	0.9186	7.31	Aliased

The lack of fit test results of the cubic models of the performance parameters were shown in table (21). Though [49] suggested the quadratic models while the cubic models are aliased, it noted that the selected models should have insignificant lack of fit for each of the performance parameters. A closer study of the tables reveals that the insignificance of lack of fit increased from 0.1282 to 0.7966, 0.0004 to 0.3144, 0.0004 to 0.3413 and 0.0844 to 0.8084 from the quadratic to the cubic models of the V_1, D_t, D_d and V_2. Results of the analysis of variance (ANOVA) for the cubic models show that there is no significant lack of fit for the V_1, D_t, D_d and V_2. The significant model terms for the respective performance parameters are shown in tables (22), (23), (24) and (25) respectively.

Table 21: Lack of Fit Tests for the Cubic Models

Parameters	Source	Sum of Squares	df	Mean Square	F-value	p-value	
V_1	Linear	18.65	20	0.9325	9.99	0.0090	
	2FI	11.73	14	0.8379	8.98	0.0121	
	Quadratic	**2.68**	**10**	**0.2677**	**2.87**	**0.1282**	**Suggested**
	Cubic	0.0444	2	0.0222	0.2380	0.7966	Aliased
	Pure Error	0.4667	5	0.0933			
D_t	Linear	93.62	20	4.68	140.42	1.49369E-05	
	2FI	88.96	14	6.35	190.60	7.4813E-06	
	Quadratic	**13.46**	**10**	**1.35**	**40.37**	**0.0004**	**Suggested**
	Cubic	0.0981	2	0.0491	1.47	0.3144	Aliased
	Pure Error	0.1667	5	0.0333			
D_p	Linear	26.00	20	1.30	131.22	1.76738E-05	

	2FI	24.71	14	1.76	178.15	8.85053E-06	
	Quadratic	**3.75**	**10**	**0.3748**	**37.83**	**0.0004**	**Suggested**
	Cubic	0.0266	2	0.0133	1.34	0.3413	Aliased
	Pure Error	0.0495	5	0.0099			
$\boldsymbol{V_2}$	Linear	22.07	20	1.10	10.74	0.0077	
	2FI	13.74	14	0.9812	9.55	0.0105	
	Quadratic	**3.71**	**10**	**0.3713**	**3.61**	**0.0844**	**Suggested**
	Cubic	0.0456	2	0.0228	0.2219	0.8084	Aliased
	Pure Error	0.5137	5	0.1027			

Table 22: ANOVA for Cubic Model (Aliased) of $\boldsymbol{V_1}$

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	82.10	22	3.73	51.00	9.7839E-06	significant
V_p	0.1293	1	0.1293	1.77	0.2255	
P_t	2.02	1	2.02	27.56	0.0012	
P_d	0.4652	1	0.4652	6.36	0.0397	
Q_N	0.1286	1	0.1286	1.76	0.2265	
V_pP_t	0.2441	1	0.2441	3.34	0.1105	
V_pP_d	0.1607	1	0.1607	2.20	0.1819	
V_pQ_N	0.1702	1	0.1702	2.33	0.1711	
P_tP_d	0.8938	1	0.8938	12.21	0.0101	
P_tQ_N	0.9988	1	0.9988	13.65	0.0077	
P_dQ_N	0.0002	1	0.0002	0.0026	0.9611	
V_p^2	0.0382	1	0.0382	0.5215	0.4936	
P_t^2	2.84	1	2.84	38.84	0.0004	
P_d^2	1.51	1	1.51	20.67	0.0026	
Q_N^2	1.53	1	1.53	20.91	0.0026	
$V_pP_tP_d$	0.5329	1	0.5329	7.28	0.0307	
$V_pP_tQ_N$	0.3136	1	0.3136	4.29	0.0772	
$V_pP_dQ_N$	0.2304	1	0.2304	3.15	0.1193	
$P_tP_dQ_N$	0.2862	1	0.2862	3.91	0.0885	
$V_p^2P_t$	0.1906	1	0.1906	2.60	0.1506	
$V_p^2P_d$	0.1160	1	0.1160	1.59	0.2484	
$V_p^2Q_N$	0.1360	1	0.1360	1.86	0.2150	
$V_pP_t^2$	0.8227	1	0.8227	11.24	0.0122	
Residual	0.5122	7	0.0732			
Lack of Fit	0.0455	2	0.0227	0.2436	0.7926	not significant
Pure Error	0.4667	5	0.0933			
Cor Total	82.61	29				

Table 23: ANOVA for Cubic Model (Aliased) of $\boldsymbol{D_t}$

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	196.37	22	8.93	238.77	4.65592E-08	significant
V_p	4.48	1	4.48	119.86	1.17396E-05	
P_t	0.6605	1	0.6605	17.67	0.0040	
P_d	2.20	1	2.20	58.91	0.0001	
Q_N	0.0045	1	0.0045	0.1200	0.7392	
V_pP_t	0.8652	1	0.8652	23.14	0.0019	
V_pP_d	4.06	1	4.06	108.61	1.62843E-05	

V_pQ_N	3.39	1	3.39	90.66	2.95312E-05	
P_tP_d	0.0014	1	0.0014	0.0377	0.8516	
P_tQ_N	0.5199	1	0.5199	13.91	0.0074	
P_dQ_N	1.79	1	1.79	47.94	0.0002	
V_p^2	3.03	1	3.03	81.12	4.24492E-05	
P_t^2	4.41	1	4.41	117.83	1.24255E-05	
P_d^2	23.53	1	23.53	629.51	4.07686E-08	
Q_N^2	19.97	1	19.97	534.21	7.19782E-08	
$V_pP_tP_d$	0.2475	1	0.2475	6.62	0.0368	
$V_pP_tQ_N$	1.87	1	1.87	50.02	0.0002	
$V_pP_dQ_N$	2.62	1	2.62	69.98	6.84466E-05	
$P_tP_dQ_N$	0.2475	1	0.2475	6.62	0.0368	
$V_p^2P_t$	1.61	1	1.61	43.13	0.0003	
$V_p^2P_d$	4.55	1	4.55	121.64	1.11793E-05	
$V_p^2Q_N$	2.24	1	2.24	59.88	0.0001	
$V_pP_t^2$	0.0024	1	0.0024	0.0635	0.8082	
Residual	0.2617	7	0.0374			
Lack of Fit	0.0950	2	0.0475	1.42	0.3238	not significant
Pure Error	0.1667	5	0.0333			
Cor Total	196.64	29				

Table 24: ANOVA for Cubic Model (Aliased) of D_p

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	54.51	22	2.48	230.36	5.27488E-08	significant
V_p	1.25	1	1.25	116.33	1.29662E-05	
P_t	0.1830	1	0.1830	17.01	0.0044	
P_d	0.6108	1	0.6108	56.79	0.0001	
Q_N	0.0011	1	0.0011	0.1062	0.7540	
V_pP_t	0.2390	1	0.2390	22.22	0.0022	
V_pP_d	1.14	1	1.14	105.78	1.77705E-05	
V_pQ_N	0.9494	1	0.9494	88.27	3.22325E-05	
P_tP_d	0.0004	1	0.0004	0.0401	0.8469	
P_tQ_N	0.1437	1	0.1437	13.36	0.0081	
P_dQ_N	0.5006	1	0.5006	46.55	0.0002	
V_p^2	0.8463	1	0.8463	78.68	4.68777E-05	
P_t^2	1.21	1	1.21	112.78	1.43733E-05	
P_d^2	6.54	1	6.54	607.75	4.60538E-08	
Q_N^2	5.55	1	5.55	515.86	8.12255E-08	
$V_pP_tP_d$	0.0689	1	0.0689	6.41	0.0392	
$V_pP_tQ_N$	0.5148	1	0.5148	47.86	0.0002	
$V_pP_dQ_N$	0.7353	1	0.7353	68.36	7.37968E-05	
$P_tP_dQ_N$	0.0689	1	0.0689	6.41	0.0392	
$V_p^2P_t$	0.4415	1	0.4415	41.05	0.0004	
$V_p^2P_d$	1.27	1	1.27	118.21	1.22952E-05	
$V_p^2Q_N$	0.6255	1	0.6255	58.15	0.0001	

$V_pP_t^2$	0.0009	1	0.0009	0.0879	0.7755	
Residual	0.0753	7	0.0108			
Lack of Fit	0.0258	2	0.0129	1.30	0.3511	not significant
Pure Error	0.0495	5	0.0099			
Cor Total	54.59	29				

Table 25: ANOVA for Cubic Model (Aliased) of V_2

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	89.23	22	4.06	50.66	1.00097E-05	significant
V_p	0.0827	1	0.0827	1.03	0.3432	
P_t	2.83	1	2.83	35.31	0.0006	
P_d	0.4313	1	0.4313	5.39	0.0533	
Q_N	0.1350	1	0.1350	1.69	0.2353	
V_pP_t	0.3000	1	0.3000	3.75	0.0941	
V_pP_d	0.2268	1	0.2268	2.83	0.1362	
V_pQ_N	0.0618	1	0.0618	0.7718	0.4088	
P_tP_d	1.03	1	1.03	12.84	0.0089	
P_tQ_N	2.04	1	2.04	25.50	0.0015	
P_dQ_N	0.0005	1	0.0005	0.0060	0.9402	
V_p^2	0.0187	1	0.0187	0.2336	0.6436	
P_t^2	3.96	1	3.96	49.43	0.0002	
P_d^2	1.47	1	1.47	18.36	0.0036	
Q_N^2	1.67	1	1.67	20.83	0.0026	
$V_pP_tP_d$	0.4865	1	0.4865	6.08	0.0431	
$V_pP_tQ_N$	0.2093	1	0.2093	2.61	0.1499	
$V_pP_dQ_N$	0.1463	1	0.1463	1.83	0.2185	
$P_tP_dQ_N$	0.7700	1	0.7700	9.62	0.0173	
$V_p^2P_t$	0.4388	1	0.4388	5.48	0.0518	
$V_p^2P_d$	0.0490	1	0.0490	0.6126	0.4595	
$V_p^2Q_N$	0.0508	1	0.0508	0.6348	0.4518	
$V_pP_t^2$	1.51	1	1.51	18.85	0.0034	
Residual	0.5604	7	0.0801			
Lack of Fit	0.0467	2	0.0233	0.2271	0.8046	not significant
Pure Error	0.5137	5	0.1027			
Cor Total	89.79	29				

More so, the fit statistics for all the parameters showed there is reasonable improvement in the goodness of fit of the models from the quadratic to the cubic. Therefore, the cubic models were better representation of the measured data set. Comparatively, the fit statistics for all the model types is presented in table (26). Thus, the cubic functions of the performance parameters offered the best reliability-based optimization models of the liquid nitrogen pumping process.

Table 26: Comparative Fit Statistics for all the Model Types

Param	Std. Dev.			Mean			C.V.%			R^2			$Adj-R^2$			$Pred-R$			$Adeq\ Pre.$		
Models	**Linear**	**Quadratic**	**Cubic**	**Linear**	**Quadratic**	**Cubic**	**Linear**	**Quadratic**	**Cubic**	**Linear**	**Quadratic**	**Cubic**	**Linear**	**Quadratic**	**Cubic**	**Linear**	**Quadratic**	**Cubic**	**Linear**	**Quadratic**	**Cubic**

V_1	D_t	D_p	V_2
24.2275	64.0646	62.9655	23.0294
19.9146	13.2148	13.1903	17.6228
16.3172	23.0243	23.0065	15.2618
0.9144	0.9269	0.9285	0.9186
0.8052	0.6045	0.6032	0.7536
0.6324	0.7981	0.7979	0.5938
0.9744	0.9944	0.9942	0.9742
0.9264	0.8660	0.8655	0.9090
0.7790	0.8800	0.8799	0.7562
0.9938	0.9987	0.9986	0.9938
0.9619	0.9307	0.9304	0.9529
0.8232	0.9040	0.9039	0.8049
11.78	3.50	3.29	11.86
19.96	17.13	15.87	22.27
33.96	14.75	13.60	35.66
2.29	5.56	3.17	2.38
2.29	5.56	3.17	2.38
2.18	5.31	3.04	2.27
0.2702	0.1945	0.1043	0.2827
0.4578	0.9531	0.5032	0.5308
0.7396	0.7832	0.4130	0.8093

Summary of the Surrogate Models

The selected models of the performance parameters in terms of actual value of the control parameters are given in equations (4.1), (4.2), (4.3) and (4.4);

$$V_1 = 1.0333 - 0.01223P_t - 0.005027P_d + 0.000021P_tP_d + 0.003178P_tQ_N + 0.000031P_t^2 + 0.000013P_d^2 + 0.2196Q_N^2 - 0.000000006338V_pP_tP_d - 0.00000001136V_pP_t^2 \quad (18)$$

$$D_t = 2.4849 + 0.003702V_p - 0.007001P_t - 0.01094P_d - 0.000006021V_pP_t - 0.000011V_pP_d - 0.001384V_pQ_N - 0.002293P_tQ_N - 0.004213P_dQ_N - 0.000001034V_p^2 + 0.000038P_t^2 + 0.0000515P_d^2 + 0.7934Q_N^2 - 0.000000004319V_pP_tP_d + 0.000001549V_pP_tQ_N + 0.000002199V_pP_dQ_N + 0.000005491P_tP_dQ_N + 0.000000001959V_p^2P_t + 0.000000003951V_p^2P_d + 0.0000003610V_p^2Q_N \quad (19)$$

$$D_p = 1.5482 + 0.001956V_p - 0.003685P_t - 0.005760P_d - 0.000003164V_pP_t - 0.000006027V_pP_d - 0.000733V_pQ_N - 0.001205P_tQ_N - 0.002227P_dQ_N - 0.0000005460V_p^2 + 0.00002P_t^2 + 0.000027P_d^2 + 0.4182Q_N^2 - 0.000000002279V_pP_tP_d + 0.00000008125V_pP_tQ_N + 0.000001166V_pP_dQ_N + 0.000002897P_tP_dQ_N + 0.000000001025V_p^2P_t + 0.000000002089V_p^2P_d + 0.0000001908V_p^2Q_N \quad (20)$$

$$V_2 = 1.0110 - 0.01457P_t + 0.000023P_tP_d + 0.004543P_tQ_N + 0.000036P_t^2 + 0.000013P_d^2 + 0.2293Q_N^2 - 0.000000006056V_pP_tP_d - 0.000009686P_tP_dQ_N - 0.00000001539V_pP_t^2 \quad (21)$$

Model Validation

The models were practically validated using the data of leak test at AKPO FPSO during the last full field shutdown. Hence, the results were presented in table (28). The table showed the results of selected ten test packs. Therefore, the optimization models are practically implementable.

Table 28: Results of Model Varification

S/N	$V_p(m^3)$	$P_t(bar)$	$P_d(bar)$	$Q_N(m^3/hr)$	$V_1(m^3)$	$D_t(hr)$	$D_p(hr)$	$V_2(m^3)$
1	0.03	0.02	127.61	0.97	0.81	2.15	1.37	1.44
2	0.05	0.05	236.88	0.83	0.72	2.50	1.55	1.90
3	0.42	0.80	194.44	0.74	0.66	2.13	1.35	1.62
4	14.54	4.50	229.04	0.70	0.65	2.37	1.48	1.77
5	3389.18	194.00	233.69	0.75	0.69	2.45	1.52	1.85
6	42.88	64.00	159.76	0.59	0.31	1.63	1.09	0.98
7	2374.32	8.00	175.27	0.67	0.58	4.29	2.24	1.42
8	369.30	9.30	181.85	0.70	0.60	2.37	1.45	1.47
9	1665.76	58.00	127.64	1.32	0.64	3.18	1.85	1.16
10	133.49	10.30	175.39	0.49	0.53	2.09	1.32	1.37

IV. Conclusion

In this paper, we demonstrated and discussed the application of a surrogate modelling technique in liquid nitrogen pumping for leak testing of hydrocarbon and allied fluids piping systems. The main objective of the paper is to develop multi-objective mathematical models of the key performance parameters of the pumping process that will act as frameworks for energy efficient pumping process and cost optimization. The available literatures revealed that surrogate modelling has been applied in different flow and fluid pumping scenarios but there is no literature pertaining to liquid nitrogen pumping in hydrocarbon and allied fluids piping systems. Thus, this paper filled the knowledge gap in the applications of surrogate modelling in fluid pumping process The key performance parameters were volume of liquid nitrogen and pumping duration. Other performance parameters considered were the test duration and the total volume of liquid nitrogen used. Analysis of the collected data set showed that cubic polynomials were adequate representation of the relationship between the performance parameters (total volume of liquid nitrogen pumped, test duration, pumping duration and total volume of liquid nitrogen used) and the control parameters (pressurized volume of the piping systems, test of the piping systems, maximum discharge pressure of the liquid nitrogen pump and maximum flow rate of the liquid nitrogen pump). Then the sensitivity analysis result indicated that each of the control parameters affected the performance parameters in varying degrees. So, the models were recommended for use in the cost minimization and optimization of liquid nitrogen pumping in hydrocarbon and allied fluids piping systems.

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