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A Study of the Relationship between Arithmetic and Algebra

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Abstract
The purpose of this study is to investigate the development of relational thinking skills, which are an important component of the transition from arithmetic to algebra, in 12th grade students. In the study, the qualitative research method of teaching experiments was used. The research data were collected from +2 students by means of clinical interviews and teaching episodes. To observe the development of relational thinking, pre- and post-clinical interviews were also conducted before and after the 5-session teaching experiment. Qualitative analysis of the research data revealed that the relational thinking skills of all the students developed. It was also found that there was an interaction between the development of fundamental arithmetic concepts and relational thinking; the students developed concepts related to arithmetical operations such as addend and sum; minuend, subtrahend, and difference; multiplicator and product; and dividend, divisor, and quotient. Moreover, students were able to use these concepts effectively, although they failed to provide formal explanations about the relationships between them. In addition, the student.
Keywords: Arithmetic, Algebra, Mathematics

I. Introduction

Mathematics is among the most important tools for the development of thinking skills that individuals need to solve their daily life problems. On the other hand, mathematics is considered to be one of the most difficult subject areas. One basic reason why students find mathematics difficult to learn is that it consists of a specific network of abstract relations, and algebra is the area that includes these abstract relations the most.

Despite both being fundamental parts of mathematics and appearing very much the same, the processes behind arithmetic and algebra are very different (Herscovics & Linchevski, 1994). Arithmetic thinking is concrete and focuses on using procedures and operations to gain an answer to a problem, while algebraic thinking utilizes generalizations and reasoning to search out relations among values and variables (Stacey & MacGregor, 2000). The transition from arithmetic to algebra is one of the biggest hurdles for many students, as it requires less of a reliance on specific operational problem solving and more of a relational understanding between problems and a shift from concrete operations to more abstract representations and strategies (Cai & Moyer, 2008; Herscovics & Linchevski, 1994).

The generalization of strategies is a key component of school algebra and algebraic reasoning (Kinach, 2014). Generalization requires a flexible knowledge of numbers, operations, and symbols of arithmetic (Becker & Rivera, 2005; Humbersome & Reeve, 2008). Developing these generalizations requires knowledge of number systems and basic arithmetic (Kaput, Carraher, & Blanton, 2008). An early form of this generalization occurs when a student is required to solve for missing values in simple equations (CCSS, 2010). Students who fail to develop this generalization ability are at increased risk of incorrectly applying different algebraic ideas to problems and struggling to learn new ideas (Christou & Vosiadou, 2012).

The transition from arithmetic to algebra is a complicated process with many moving parts, like understanding equivalence and unknown quantities. The majority of research in the area of this transition focuses on those individual components rather than the transition as a whole. As stated earlier, much of the research on equivalence and variables occurs with middle school students, but there is research occurring with elementary school students in hopes of facilitating the transition sooner. Instructional methods to approach this goal include: teaching students to think of numbers as numbers and not just objects; changing language so instead of finding “what a problem equals," find “what makes the statement true”; constructing open number sentences; introducing symbols as variables prior to letters, as students may be confused by letters being a range of values; and using tables and graphs to show the changing nature of different values (Earnest & Balti, 2008; Rivera, 2006).

In related literature, there is no consensus on what algebraic thinking at an early age is or on what components it may have (Cai & Moyer, 2008). Moreover, it could be stated that algebraic thinking is necessary for the analysis of deeper mathematical structures rather than arithmetic and procedural fluency. Boulton et al. (2000) put forward a three-phase model for the development of algebraic knowledge to examine the developmental steps of algebraic thinking. According to this model, whose phases are called "arithmetic," “early algebra,” and "algebra," it is claimed that algebraic thinking in students is developed consecutively and that a transition to a higher step will not be healthy if the current step has not been developed efficiently. In arithmetic, as the first step of this model, students are expected to know the fundamental properties of operations such as commutative, associative, and distributive, to do work backward, and to be aware of the equal sign. Therefore, it is obvious that arithmetic constitutes the basis of algebra teaching, despite the differences resulting from the natures of arithmetic and algebra (Herscovics & Linchevski, 1994; Kieran, 1981; Knuth, Alibali, McNeil, Weinmberg & Stephens, 2005; Knuth, Stephens, McNeil & Alibali, 2006). It is also seen that students make use of their experiences in arithmetic in their transition to algebra (Hersovics & Linchevski, 1994; Mcneil & Alibali, 2005). The deficiencies resulting from the way of learning arithmetic during the transition from arithmetic to algebra may also have an influence on the development of algebraic thinking. For instance, learning subtraction without focusing on the relations between minuend, subtrahend, and difference might lead to deficiencies resulting from the way of teaching arithmetic. This situation causes elementary school students to perceive arithmetic as a set of rules.

Instead of memorizing the rules directly, it is a necessity for students to see the relationships underlying the rules and to develop fundamental arithmetic skills (Knuth, Stephens, Blanton, & Gardiner, 2016). The development of fundamental arithmetic skills allows for writing down number sentences with mathematical symbols, understanding the fundamental features of operations, and conceptualizing a number in a wide variety of forms (5=7-2, 5=3+2, etc.). Students can solve number sentences by focusing on the relationship between numbers (Molina & Ambrose, 2006). This focus requires relational thinking, which has an important place in the development of algebraic thinking. Relational thinking mostly concerns examining the relations between the given quantities rather than finding the result of operations. To clarify, relational thinking involves the use of fundamental properties of numbers and operations for the transformation of mathematical sentences. Koehler (2004) points out that relational thinking provides a different perspective for arithmetic and plays a key role in teaching and learning it. This key role brings about two benefits, which allow students not only to restructure arithmetic operations to change the given calculation but also to transform the number sentences with the use of fundamental arithmetic properties (Koehler, 2004). In relational thinking, the mere purpose is to help students become aware of the fact that both sides of an equation represent the same numbers without doing any calculations. Therefore, for relational thinking, first, students should use the relational meaning of the equal sign (Boulton et al., 2000; Carpenter & Franke, 2001; Yaman, Toluk, & Olkun, 2003). On the other hand, a number of previous studies conducted after the 1980s

demonstrated that most students have serious misconceptions regarding the meaning of the equal sign (Behr, Erlwanger & Nichols, 1980; Falkner, Levi, & Carpenter, 1999; Saenz-Ludlow & Walgamuth, 1998), and recent studies (Li, Ding, Capraro, & Capraro, 2008; Matthews, Rittle-Johnson, McEldoon &Taylor, 2012; McNeil & Alibali, 2005) support those previous results as well. As the key to relational thinking, students are supposed to understand that the equal sign refers to the relation and balance between numbers (Carpenter, Franke, & Levi, 2003), not to a direction (Kieran, 1981) or the result of an operation. True/false and open number sentences can be used as an important tool that will allow students to start thinking about relations, learn how to represent these relations and express the meanings they form, or help develop relational thinking and learn the relational meaning of the equal sign (Carpenter et al., 2003). For this reason, by focusing only on the relations between quantities, the number to be written down in the blank can be found. The answer to this sample question can be found by establishing the relationship between numbers, while some number sentences can be dealt with based on the properties of operations. The most difficult and striking one for internalizing the properties of operations is the distributive property. Distributive property is essential for understanding multiplication and for developing multiplicative reasoning.

Focusing on the change in the students’ relational thinking skills, the present study differs from others that examined secondary school students’ misconceptions and the thinking strategies they applied to solve open number sentences (Hunter, 2007; Stephens & Ribeiro, 2012), as well as from other studies that investigated students’ effective thinking methods for dealing with number sentences, including addition and subtraction (Stephens, 2006). In literature, there is one study in which a teaching model was designed to determine and develop secondary school students’ relational thinking skills (Napaphun, 2012); on the other hand, the present study included a wider variety of number sentences and those combining the associative and distributive properties and focused on the students’ understanding of these number sentences in the teaching process and on their use of these operational properties while dealing with the operations. In line with this research question, first, the students' relational thinking skills were determined. Following this, the teaching process was designed to develop these skills. Lastly, the changes in the students were examined via the interviews held with them.

II. Material and Methods

The study was conducted between 2022 and 2024 at the Department of Mathematics at Malwanchal University, Indore, MP. In this study, which involved the planning, application, and evaluation of a teaching process for the development of students' relational thinking skills, the method of teaching experiment was applied.

Purpose and Research Questions

The need for algebra intervention research is paramount because achievement data indicate that our students continue to struggle with algebra, but there is not a well-developed problem-analysis model for intervening with struggling students. Direct instructional activities and guidelines exist for a number of the content areas within algebra (Foegen, 2008; National Mathematics Advisory Panel, 2008), and academic interventions are more successful if they directly address the student deficit (Burns, VanDerHeyden, & Zaslofsky, 2014). As of now, there are no clear guidelines about how to assess for specific skill and knowledge-based deficits and then select appropriate, evidence-based interventions.

Participants

The study included participants from two charter schools located inside an Indore city. The first is a middle school with 225 students, and the second is a high school with 225 students. All of the students voluntarily enrolled in the charter school and were then randomly selected for enrollment through a lottery system. Together, the two schools consist of students from 6th through 12th grade, with approximately 41% receiving free or reduced-priced lunch and 17% receiving special education services. The ages of the participants ranged from 12 to 19 years old. The student population of the middle school consists of the following ethnicities: 54.1% white, 16.3% black, 16.3% Hispanic, 12.8% Asian and Pacific Islander, and 0.5% Native American. In 2013, 62% of students at the middle school scored within the proficient range for mathematics on the statewide test. The student population of the high school is made up of the following ethnicities: 54.4% white, 19.4% black, 15% Hispanic, 10.7% Asian and Pacific Islander, and 0.5% Native American. The 2013 data indicated that 47.4% of the students at the high school scored in the proficient range on the mathematics state test, and the school reported a graduation rate of 92.6%.

All of the students within the charter school were eligible to participate in the study. The study was discussed with the students during their mathematics classes, and because all students were eligible, passive consent was used.

A total of 376 students participated in testing, and 327 students were included in the final study. Of those participants, 42% were male and 58% were female. Less than 1% identify as American Indian or Alaskan Native, 11% identify as Asian or Pacific Islander, 16% identify as Hispanic, 23% identify as Black, and 50% identify as White. Of the total number of participants, 10% were receiving special education services, and 6% were receiving educational support through a 504 plan. The average age of the participants was 14.1, and the average grade was 8.5. The distribution of students among grades is as follows: grade 6 has 62 students (19%), grade 7 has 59 students (18%), grade 8 has 50 students (15%), grade 9 has 51 students (16%), grade 10 has 40 students (12%), grade 11 has 39 students (12%), and grade 12 has 26 students (8%).

The study evaluated the validity of using a five-part problem-solving model to identify core deficits for students struggling with algebra. Within the model, the five core sections of the problem-solving model support and inform each other and are all required to establish a basis for algebra proficiency.

III. DISCUSSION

The purpose of the study was to examine evidence for the validity of a proposed problem-solving model for identifying skill deficits in students struggling with algebra. Three research questions guided the study. The correlations between the subsections within the Basic Skills, Algebraic Thinking, and Content Knowledge skills sections range from weak to moderate, with the Algebraic Thinking section having the strongest correlations between subsections. The negative correlation between the Positive and Negative subsections on the Authentic Application section was strong, and the correlations between the subsections on the Classroom Engagement Inventory (CEI) were weak to moderate. The correlations between skill sections are larger, with the algebraic thinking section sharing the largest correlation with the other two sections. The correlations indicate that basic skills have the weakest connection to content knowledge and the strongest connection to algebraic thinking.

The correlations indicate that the subskills are related to one another, and each skill and subskill provides a unique set of information that, based on the evaluation of the five-factor model, can be assumed to give important information about separate mathematics skills critical to algebra proficiency. There is a weak to moderate correlation between the subskill measures within each subskill section. The correlations among the overall skills are higher than the subskills. Among the sections, Basic Skills is strongly correlated with Algebraic Thinking, indicating that a student's ability to order, calculate, and solve word problems using integers and rational numbers is closely related to their ability to perform tasks that utilize algebraic thinking. These skills seem to have less impact on their ability to solve problems and recall facts using content knowledge. One possible reason for the weaker correlation is that students may not directly use their arithmetic calculation skills in algebra, but they do so when developing skills related to arithmetic thinking.

Limitations

There were limitations to the study. First, the population used was a convenience sample. While all students were tested within the two schools, those schools only represented a certain sampling of middle and high school students. To enhance the generalizability and validity of the model, it is critical that the model and procedures be replicated and applied across different populations.

IV. Conclusions

This research adds to the literature supporting the skills required for algebra. It also adds to the sparse literature on problem analysis for students struggling with algebra. It provides a systematic way of identifying skill deficits, which can be used to deliver targeted interventions.

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