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Spectral Dynamic Analysis of Torsional Vibrations of Cantilever Thin-Walled Open Section Beam Elastically Restrained at the Other End

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Abstract: The present paper deals with spectral dynamic analysis of free torsional vibration of doubly symmetric thin-walled beams of open section. Spectral frequency equation is derived in this paper for the case of cantilever doubly symmetric thin-walled beam elastically restrained at the other end.. The resulting transcendental frequency equation with appropriate boundary conditions is derived and is solved for varying values of warping parameter and the rotational and transverse restraint parameters. The influence of the rotational restraint parameters and the warping parameter on the free torsional vibration frequencies is investigated in detail. A MATLAB computer program is developed to solve the spectral frequency equation derived in this paper. Numerical results for natural frequencies for various values of rotational and transverse restraint parametersfor various values of warping parameter are obtained and presented in both tabular as well as graphical form showing the influence of these parameters on the first fundamental torsional frequency parameter clearly.
Keywords: Beam, open section, torsion, dynamic stiffness, Winkler-Pasternak foundation.

I. INTRODUCTION

It is very well known that in practical situations, the boundary conditions of structural members will be quite complex and can be simulated by using translational springs and rotational springs with appropriate combinations of the same. There exist a good number of research efforts in this direction and many researchers have addressed this problem of vibrations of generally restrained beams with various combinations of boundary conditions [1-25]. The combined effect of rotary inertia, shear deformation and root flexibility has been investigated experimentally by Beglinger et al. [12].A considerable amount of theoretical work has been done in the field of vibration dealing with the computation of natural frequencies and mode shapes of cantilever beams with flexible roots [6, 9, 17, 19, 20, 22]. KameswaraRao [23] presented a closed form equation for computing fundamental frequency of cantilever blade taking into account the resilience of the clamped end. Experimental verification of the results for this case was also carried out by Abbas and Irretier [24]. KameswaraRao and Mirza [25] derived the transcendental frequency equation and mode shape expressions for the case of generally restrained Euler-Bernoulli beams and presented extensive numerical results for various values of linear and rotational restraint parameters.

While there are a number of publications on flexural vibrations of elastically restrained cantilever beams, the literature on torsional vibrations of doubly symmetric thin-walled beams of open section is rather rare. Free torsional vibrations and stability of doubly-symmetric long thin-walled beams of open section were investigated by Gere [26] and Christiano and Salmela [27]. Numerical values of exact torsional natural frequencies of beams with circular cross-section, where nonuniform warping does not arise, were presented by Gorman [26] and Belvins [27] for different classical boundary conditions. Torsional vibration frequencies for beams of open thin-walled sections, subjected to several combinations ofclassical boundary conditions, taking into account warpingeffectswere first derived by Gere [28]. Including elastic torsional and warping restraints, Carr [29] and Christino and Salmela [30] presented numerical results using approximate methods for the calculation of natural frequencies.For the case of torsional frequencies of circular shafts and piping with elastically restrained edges, KameswaraRao [31] derived exact frequency equation and presented corresponding numerical results for a wide range of non-dimensional parameters.

It can be seen from the very recent review presented by Sapountzakis [32]that the problem of free torsional vibration analysis of doubly-symmetric thin-walled I-beams or Z-beams subjected to partial warping restraint is not being addressed till now in the available literature. In view of the same, an attempt has been made in this paper to present aspectral dynamic analysis of free torsional vibration of cantilever doubly-symmetric thin-walled beams of open section elastically restrained at the other end of the beam including the effects of warping parameter. Spectral frequency equation is derived for this case and the resulting transcendental frequency equation is solved for varying values of warping parameter and the rotational and transverse restraint parameters. The influence of rotational and transverserestraint parameters along with warping parameter on the free torsional vibration frequencies is investigated in detail by utilising a MATLAB computer program developed especially to solve the spectral frequency equation derived in this paper. Numerical results for natural frequencies for various values of rotational and transverse restraint parameters are obtained and presented in both tabular as well as graphical form for use in design, showing their parametric influence clearly.

II. FORMULATION AND ANALYSIS

Consider a long doubly-symmetric thin-walled beam of open cross section of length L and the beam as undergoing free torsional vibrations. The corresponding differential equation of motion can be written as:

$$EC_W\frac{\partial^4\varphi}{\partial z^4} - GC_S\frac{\partial^2\varphi}{\partial z^2} + \rho I_P\frac{\partial^2\varphi}{\partial t^2} = 0 \qquad (1)$$

where,

E= young's modulus, C_W=warping constant,G =shear modulus,C_S = torsion constant, ρ =mass density of the material of the beam, I_P =polar moment of inertia, φ = angle of twist, z= distance along the length of the beam. For free torsional vibrations, the angle of twist $\varphi(z,t)$ can be expressed in the form.

$$\varphi(z,t) = x(z)e^{i\omega t} \qquad (2a)$$

$$x(z) = \mathrm{C}e^{mz} \qquad (2b)$$

In which $x(z)$ is the modal shape function corresponding to each beam torsional natural frequency ω.

The expression for $x(z)$which satisfies Eqn. (1) can be written as:

$$x(z) = \mathrm{A}e^{+\alpha z} + \mathrm{B}e^{-\alpha z} + \mathrm{C}e^{+i\beta z} + \mathrm{D}e^{-i\beta z} \qquad (3)$$

in which,

$$\beta\mathrm{L}, \alpha\mathrm{L} = \sqrt{\frac{\mp K^2+\sqrt{K^4+4\lambda^2}}{2}} \qquad (4)$$

where,

$K^2 = \left(\frac{GC_SL^2}{EC_W}\right)$; Non- dimensional warping parameter

$\lambda^2 = \left(\frac{\rho I_P\omega^2L^4}{EC_W}\right)$; Non- dimensional frequency parameter

From Eqn. (4),we have the following relation between αL and βL

$$(\alpha\mathrm{L})^2 = (\beta\mathrm{L})^2 + K^2 \qquad (5)$$

Knowingα and β, the frequency parameter λ can be evaluated using the following equation:

$$\lambda^2 = (\alpha\mathrm{L})(\beta\mathrm{L}) \qquad (6)$$

The four arbitrary constants A, B, C and D in Eqn. (3) can be determined from the boundary conditions of the beam. For any single-span beam, there will be two boundary conditions at each end and these four conditions then determine the corresponding frequency and mode shape expressions.

III. DERIVATION OF SPECTRAL FREQUENCY EQUATION

Consider a thin-walled doubly symmetric I-beam with one end rotationally restrained and the other end transversely restrained as shown in figure 1, undergoing free torsional vibrations. In order to derive the spectral frequency equation for this case, let us first introduce the related nomenclature.

The variation of angle of twist φ with respect to z is denoted byθ*(z).* The flange bending moment and the total twisting moment are given by *M(z)*and *T(z).* Considering clockwise rotations and moments to be positive, we have

$$\theta(z) = \frac{d\varphi}{dZ};\ hM(z) = -EC_W\frac{d^2\varphi}{dz^2} \qquad (7)$$

$$T(z) = -EC_W\frac{d^3\varphi}{dz^3} + GC_S\frac{d\varphi}{dZ} \qquad (8)$$

where $EC_W = \frac{I_f h^2}{2}$. $I_f$being the flange moment of inertia and h is the distance between the center lines of the flanges of a thin-walled I-beam.

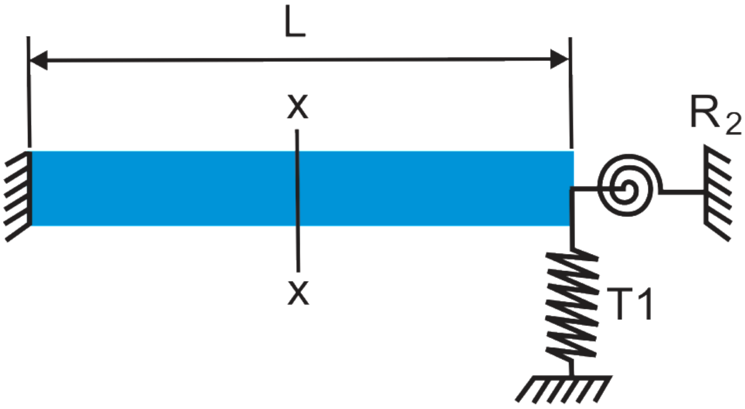


Fig. 1 (a) A cantilever thin-walled open section I-beam with clamped edge at one end and Elastically Restrained at the Other End

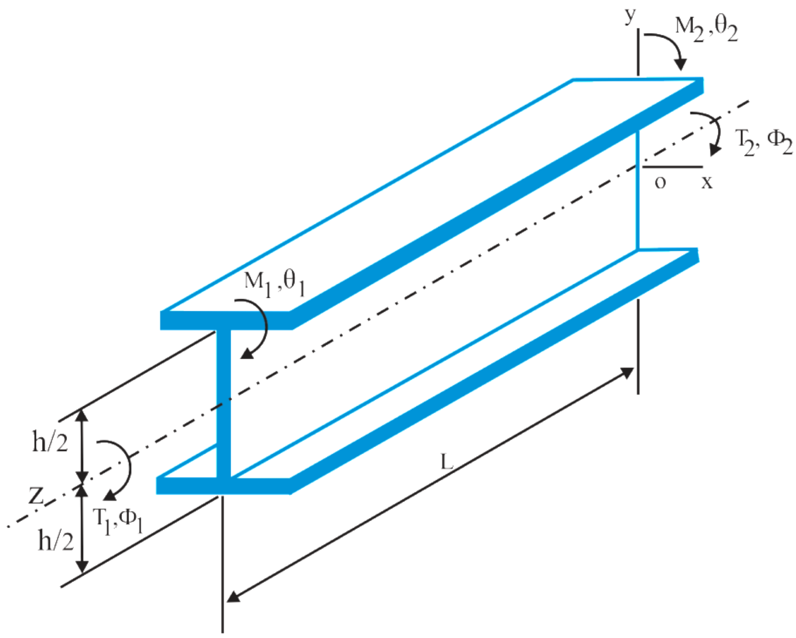


Fig. 1 (b) Cross-section of the beam at x-x

Taking S1 and S2 as the stiffnesses of the rotational springssituated at both ends and R_1 = (S_1L/EC_W) and R_2= (S_2L/EC_W)as the non-dimensional rotational spring stiffness parameters and Z=(z/L) as the non-dimensional length of the beam, the boundary conditions can be easily identified as follows:

At Z =0, φ =0,$\frac{d\varphi}{dZ}$= 0(9)

And at Z = L,$\frac{d^3\varphi}{dZ^3} - K^2\frac{d\varphi}{dZ} = T\varphi$, $\frac{d^2\varphi}{dZ^2}$ = -$R\frac{d\varphi}{dZ}$(10)

The spectral frequency equation obtained is as given below:

$S_1 + \frac{RT}{(\alpha^2\beta^2)}S_2 - RF_3S_3$ - T$F_4\,S_4$ = 0 (11)

where

$F_1 = \frac{(\alpha^2-\beta^2)}{(\alpha\beta)}$; $F_2 = \frac{(\alpha^2+\beta^2)}{(\alpha\beta)}$;$F_3 = \frac{(\alpha^2+\beta^2)}{(\alpha^2\beta^2)}$; $F_4 = \frac{(\alpha^2+\beta^2)}{(\alpha^3\beta^3)}$; F_5= $\frac{(\alpha^4+\beta^4)}{(\alpha^2\beta^2)}$ (12)

$Q_1 = \frac{1+e^{2L(\alpha+i\beta)}}{4e^{L(\alpha+i\beta)}}$; $Q_2 = \frac{1+e^{2L(\alpha-i\beta)}}{4e^{L(\alpha-i\beta)}}$; $Q_3 = \frac{1-e^{2L(\alpha+i\beta)}}{4ie^{L(\alpha+i\beta)}}$; $Q_4 = \frac{1-e^{2L(\alpha-i\beta)}}{4ie^{L(\alpha-i\beta)}}$(13)

$Q_{1p2} = (Q_1 + Q_2)$, $Q_{1m2} = (Q_1 - Q_2)$, $Q_{3p4} = (Q_3 + Q_4)$, $Q_{3m4} = (Q_3 - Q_4)$(14)

S1= $(2 + F_5Q_{1p2} + F_2Q_{1m2})$; S2= $[2(1 - Q_{1p2}) + F_1Q_{1m2}]$ (15)

S3=$(\alpha Q_{3p4} + \beta Q_{3m4})$;S4 =$(\alpha Q_{3m4} - \beta Q_{3p4})$(16)

Four degenerate cases spectral frequency equations can be easily obtained from Equation (11) as follows:

(1) For $R = 0$ and $T = 0$, we get the case of cantilever beam for which we obtain

$$(F_5Q_{1p2} + F_2Q_{1m2} + 2) = 0 \quad (17)$$

(2) For $R = 0$and $T = \infty$, we get the case ofa beamclamped at one end and simply-supported at the other end for which we obtain

$\left(\alpha Q_{3m4-}\beta Q_{3p4}\right) = 0$ (18)

(3) For $R = \infty$and $T = \infty$, we get the clamped at both ends of the beam case for which we obtain

$[2\left(1 - Q_{1p2}\right) + F_1 Q_{1m2}] = 0$ (19)

(4) For $R = \infty$and $T = 0$, we get the case of a beam clamped at one end and guided at the other end of the beam for which we obtain

$\left(\alpha Q_{3p4} + \beta Q_{3m4}\right) = 0$ (20)

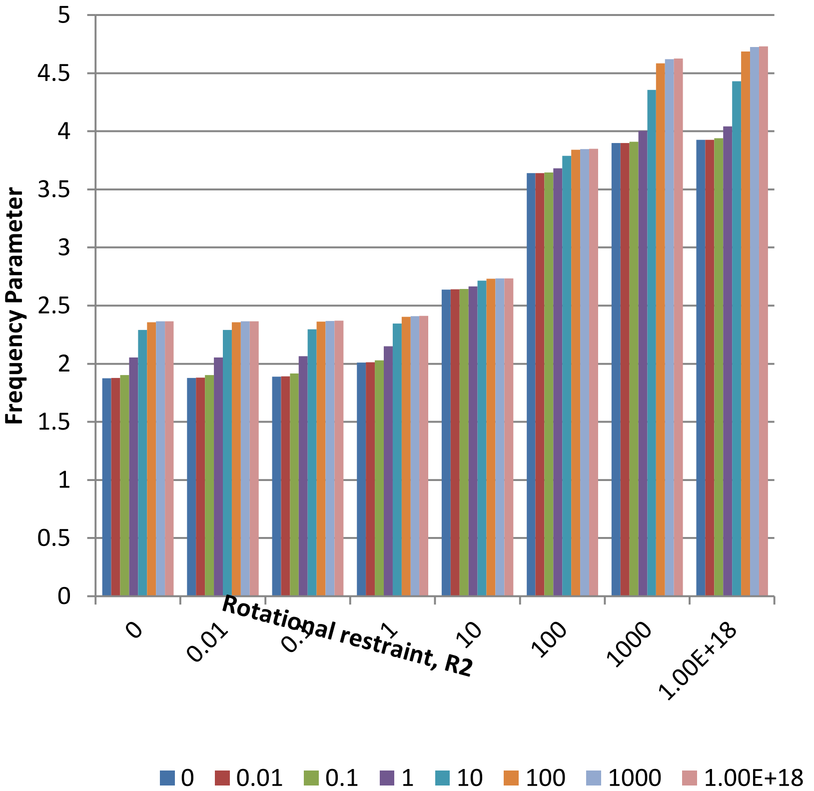
IV. RESULTS AND DISCUSSIONS

CLAMPED –ELASTICALLY RESTRAINED THIN-WALLED BEAM

R1=10^18 and T1=10^18

Table.1 First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 0.0$

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	1.8751	1.8766	1.8901	2.0100	2.6389	3.6405	3.8978	3.9266
0.01	1.8780	1.8795	1.8928	2.0121	2.6393	3.6411	3.8990	3.9279
0.1	1.9022	1.9036	1.9163	2.0304	2.6426	3.6456	3.9101	3.9398
1	2.0540	2.0550	2.0641	2.1491	2.6662	3.6818	4.0042	4.0418
10	2.2912	2.2917	2.2970	2.3470	2.7147	3.7888	4.3562	4.4303
100	2.3564	2.3569	2.3613	2.4037	2.7310	3.8403	4.5845	4.6853
1000	2.3641	2.3646	2.3689	2.4104	2.7330	3.8475	4.6205	4.7253
10^{18}	2.3650	2.3655	2.3698	2.4112	2.7332	3.8483	4.6247	4.7300



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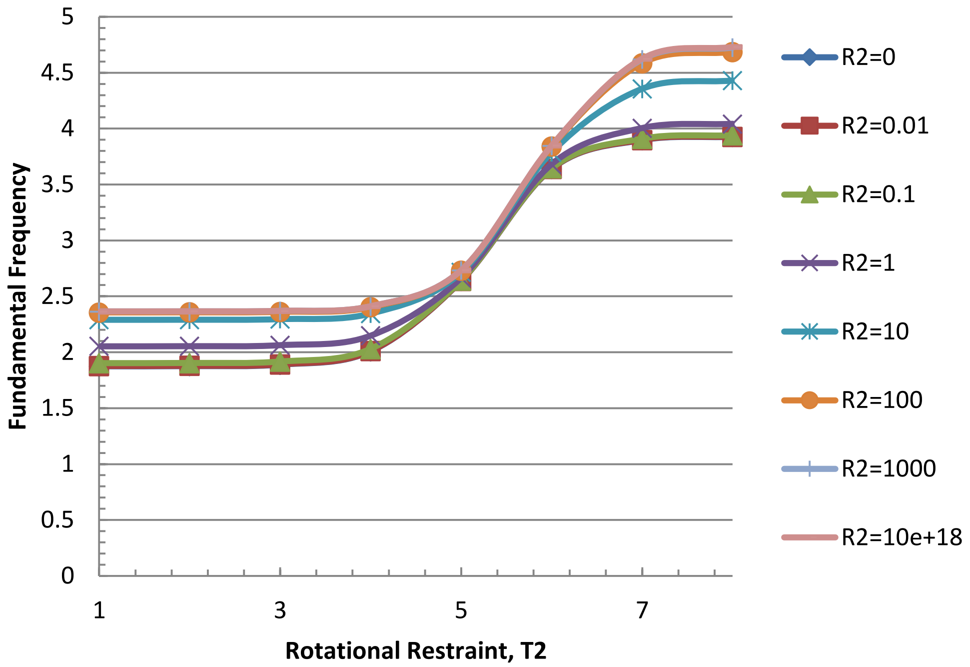

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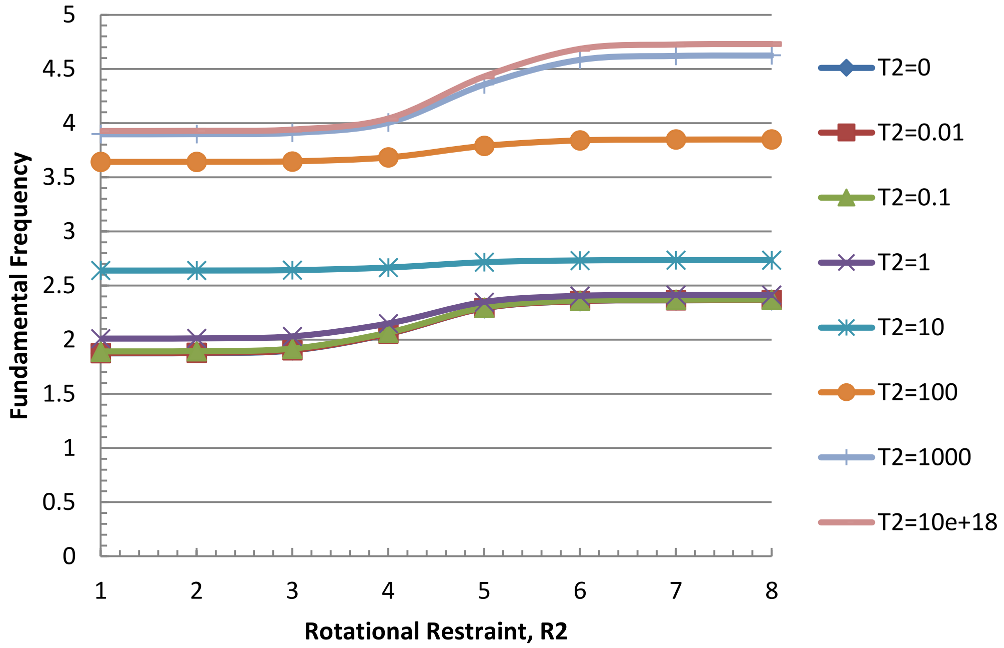
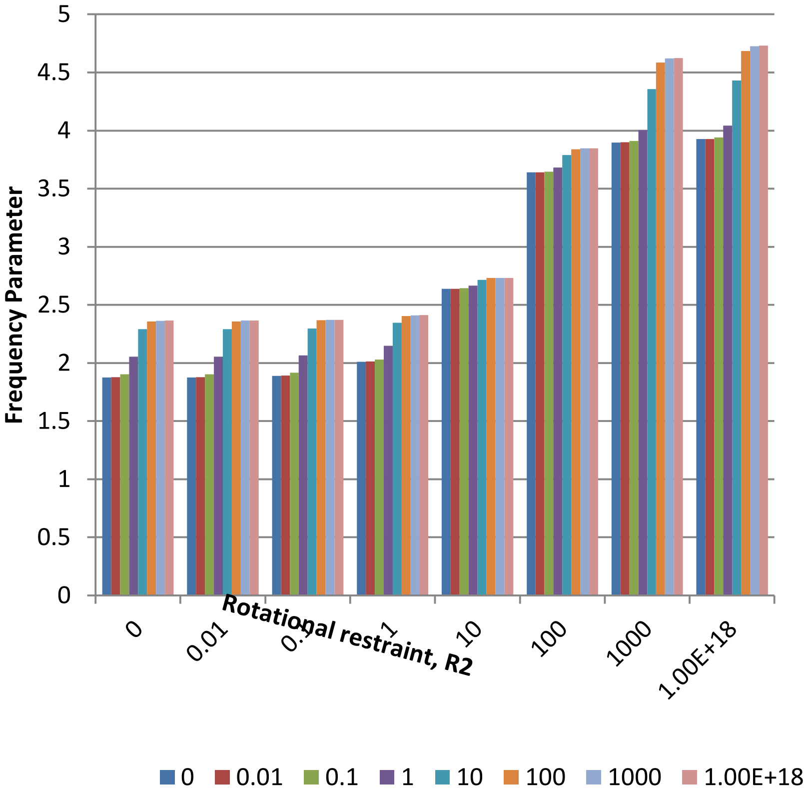


(c)

Fig. 2(a), (b) and (c). Variation of frequency parameter with elastic restraints (R2 &T2=0 to 10^{18}) for a given Warping parameter (K=0).

Table.2 First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 0.01$

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	1.8751	1.8766	1.8901	2.0100	2.6389	3.6405	3.8978	3.9266
0.01	1.8780	1.8795	1.8929	2.0122	2.6393	3.6411	3.8991	3.9280
0.1	1.9022	1.9036	1.9163	2.0304	2.6426	3.6456	3.9101	3.9398
1	2.0540	2.0550	2.0642	2.1491	2.6662	3.6818	4.0042	4.0418
10	2.2912	2.2918	2.2970	2.3470	2.7147	3.7888	4.3562	4.4303
100	2.3564	2.3569	2.3689	2.4037	2.7310	3.8403	4.5845	4.6853
1000	2.3642	2.3646	2.3698	2.4104	2.7330	3.8475	4.6205	4.7253
10^{18}	2.3650	2.3655	2.3698	2.4112	2.7332	3.8483	4.6247	4.7300



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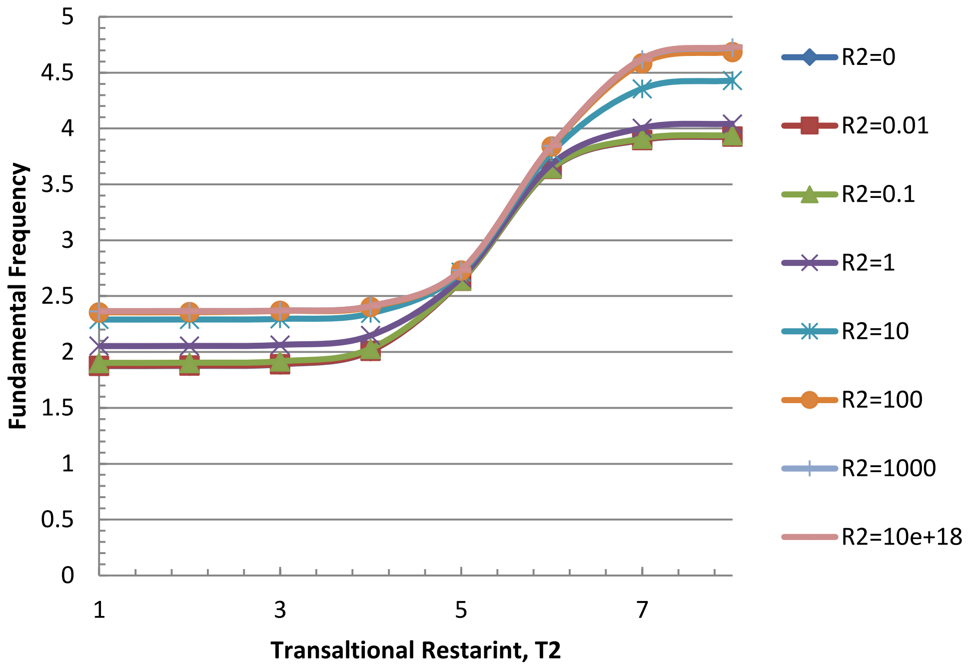

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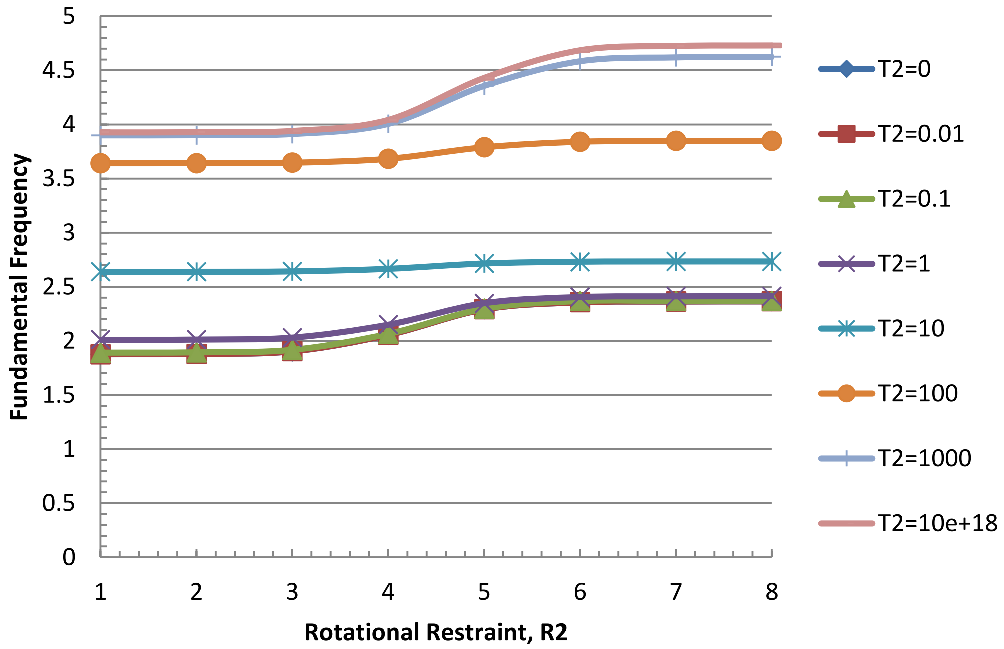
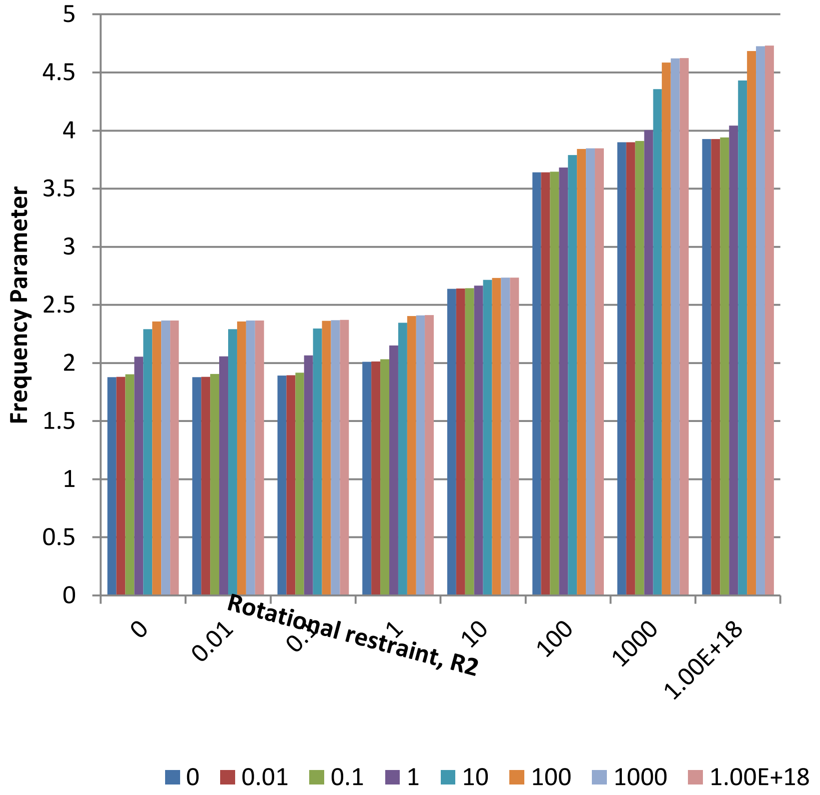


(c)

Fig. 3(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=0.01).

Table.3 First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 0.1$

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	1.8769	1.8784	1.8918	2.0114	2.6394	3.6408	3.8983	3.9271
0.01	1.8797	1.8812	1.8945	2.0135	2.6398	3.6413	3.8995	3.9284
0.1	1.9038	1.9052	1.9179	2.0317	2.6431	3.6458	3.9105	3.9403
1	2.0551	2.0561	2.0653	2.1501	2.6667	3.6821	4.0046	4.0423
10	2.2918	2.2924	2.2976	2.3477	2.7151	3.7890	4.3565	4.4306
100	2.3570	2.3575	2.3619	2.4042	2.7313	3.8405	4.5848	4.6856
1000	2.3647	2.3652	2.3695	2.4110	2.7333	3.8476	4.6207	4.7256
10^{18}	2.3656	2.3661	2.3703	2.4117	2.7336	3.8485	4.6250	4.7303



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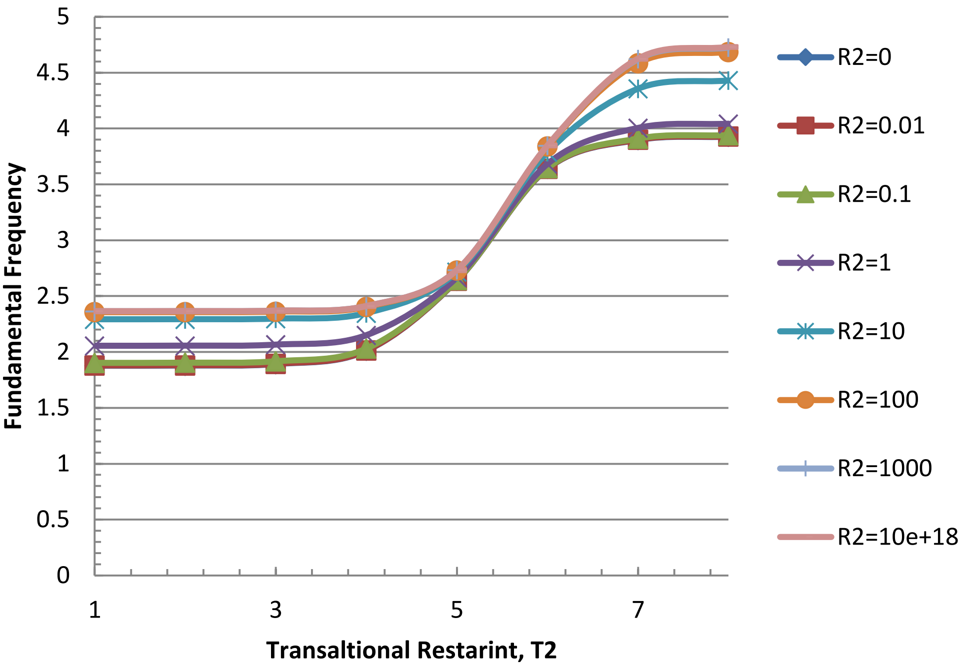

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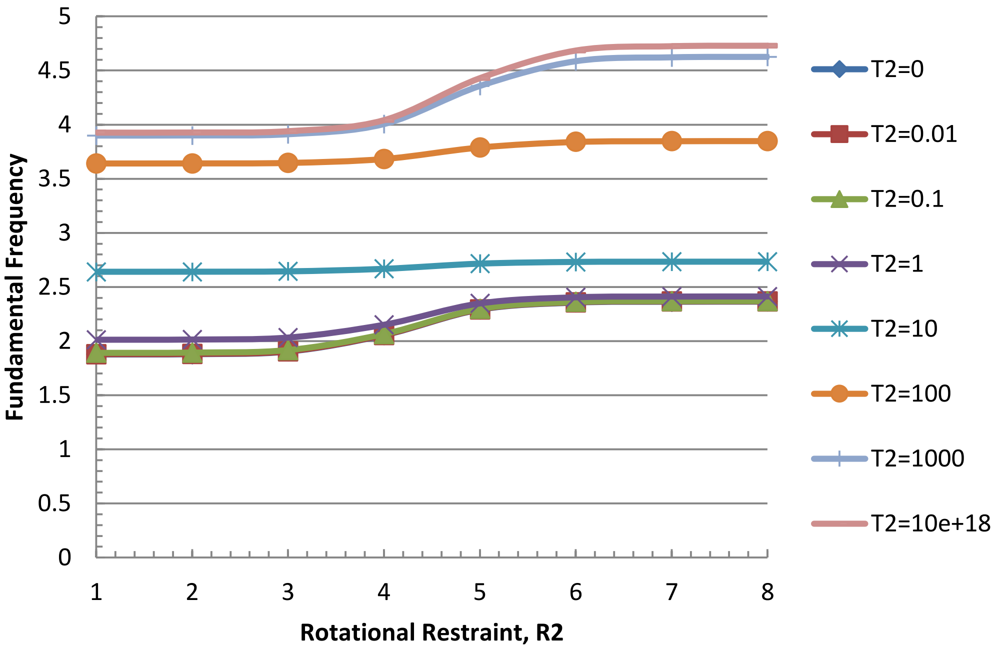
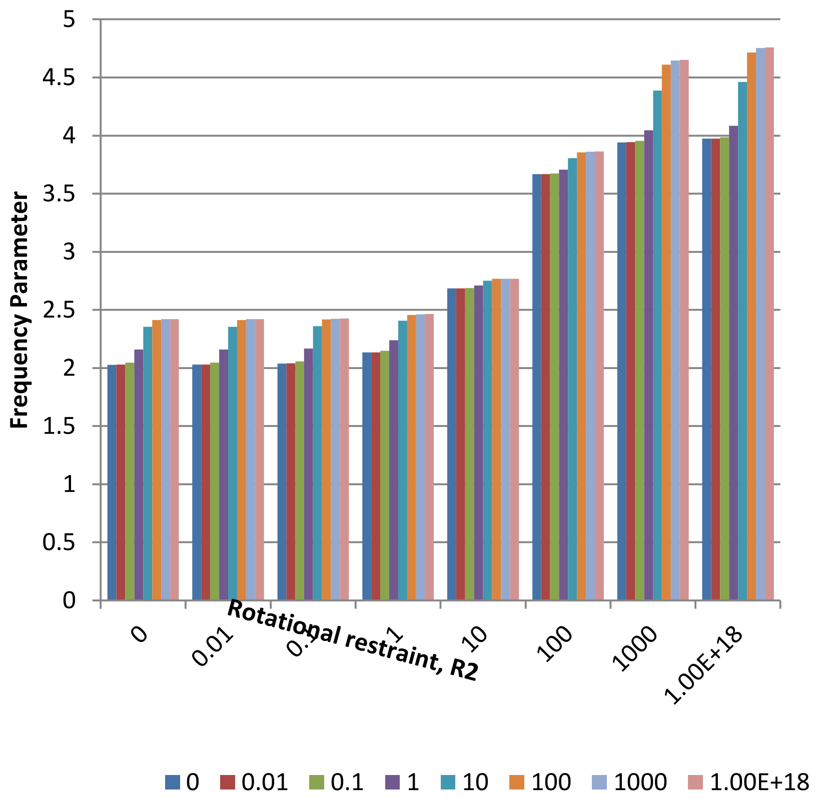


(c)

Fig. 4(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=0.1).

Table.4 First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 1.0$

$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	2.0274	2.0285	2.0389	2.1337	2.6854	3.6683	3.9420	3.9733
0.01	2.0294	2.0305	2.0408	2.1353	2.6858	3.6683	3.9432	3.9746
0.1	2.0463	2.0474	2.0574	2.1487	2.6885	3.6725	3.9537	3.9860
1	2.1590	2.1599	2.1676	2.2401	2.7086	3.7059	4.0441	4.0842
10	2.3551	2.3556	2.3604	2.4064	2.7517	3.8061	4.3857	4.4626
100	2.4132	2.4137	2.4177	2.4571	2.7668	3.8553	4.6104	4.7144
1000	2.4202	2.4206	2.4246	2.4633	2.7687	3.8622	4.6460	4.7541
10^{18}	2.4210	2.4214	2.4254	2.4640	2.7689	3.8630	4.6502	4.7588



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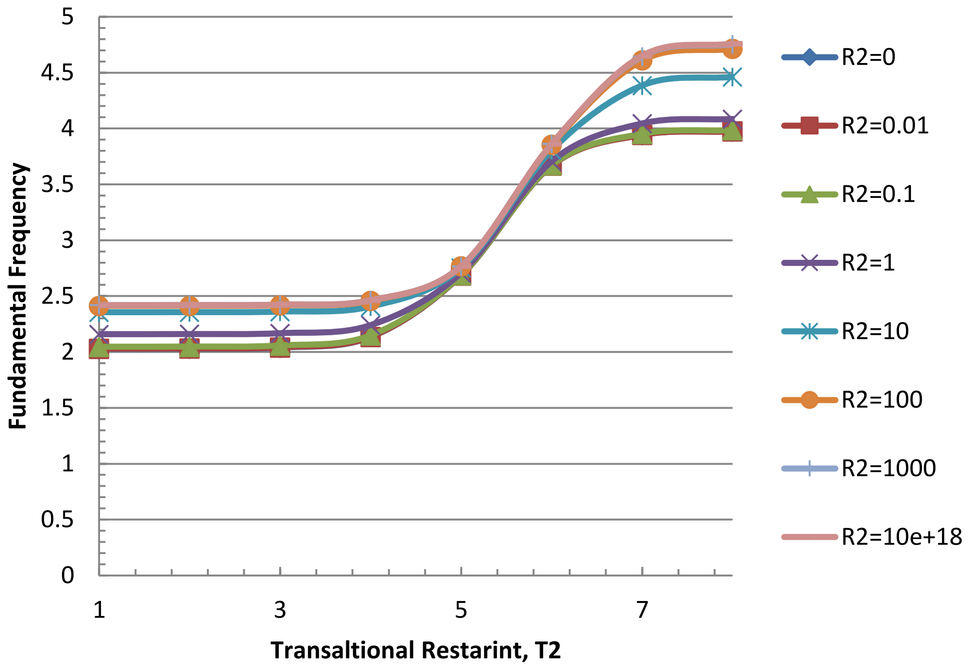

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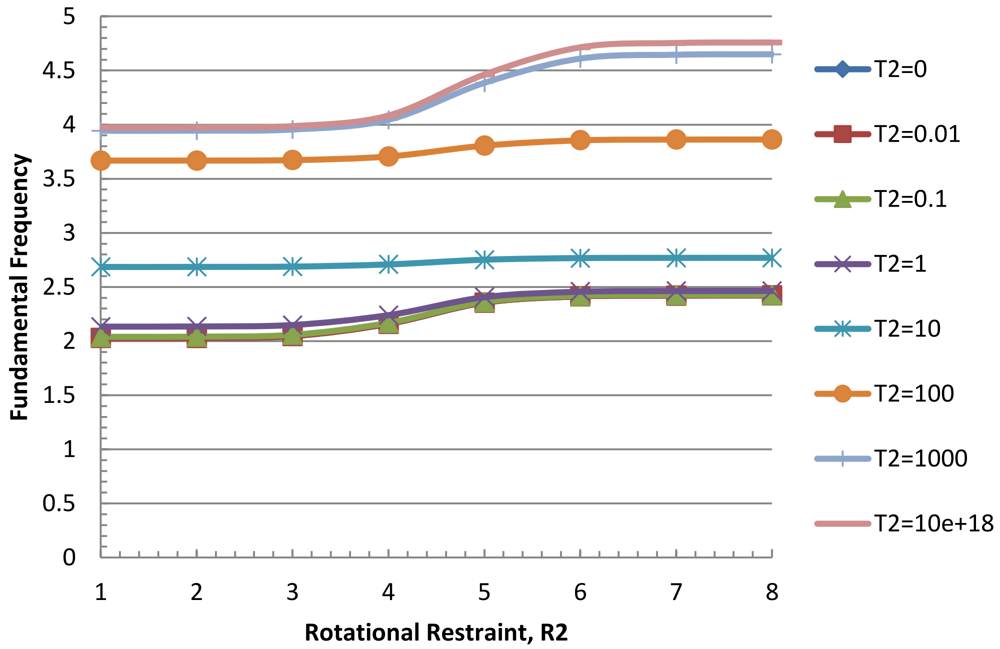
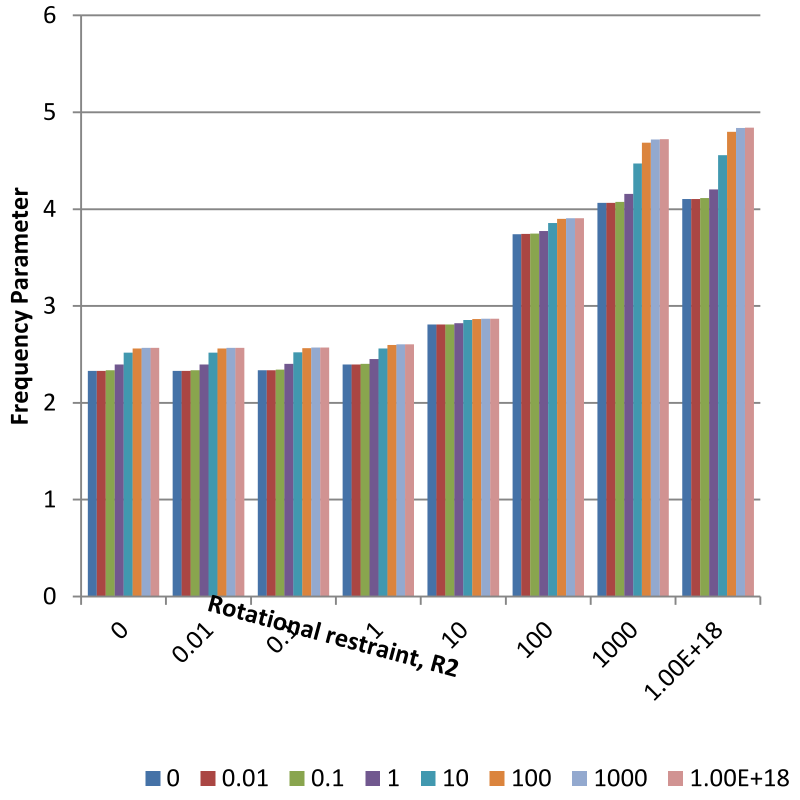


(c)

Fig. 5(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=1).

Table.5 First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 2.0

$R2$	$T2=0$	$T2=0.01$	$T2=0.1$	$T2=1$	$T2=10$	$T2=100$	$T2=1000$	$T2=10^{18}$
0	2.3283	2.3290	2.3353	2.3953	2.8073	3.7422	4.0649	4.1038
0.01	2.3292	2.3299	2.3362	2.3961	2.8075	3.7425	4.0660	4.1049
0.1	2.3372	2.3379	2.3441	2.4029	2.8092	3.7458	4.0753	4.1152
1	2.3952	2.3958	2.4012	2.4525	2.8224	3.7723	4.1558	4.2037
10	2.5187	2.5191	2.5230	2.5601	2.8539	3.8555	4.4700	4.5554
100	2.5618	2.5622	2.5655	2.5983	2.8662	3.8987	4.6848	4.7983
1000	2.5672	2.5676	2.5709	2.6031	2.8678	3.9049	4.7195	4.8374
10^{18}	2.5678	2.5682	2.5715	2.6036	2.8680	3.9056	4.7235	4.8420



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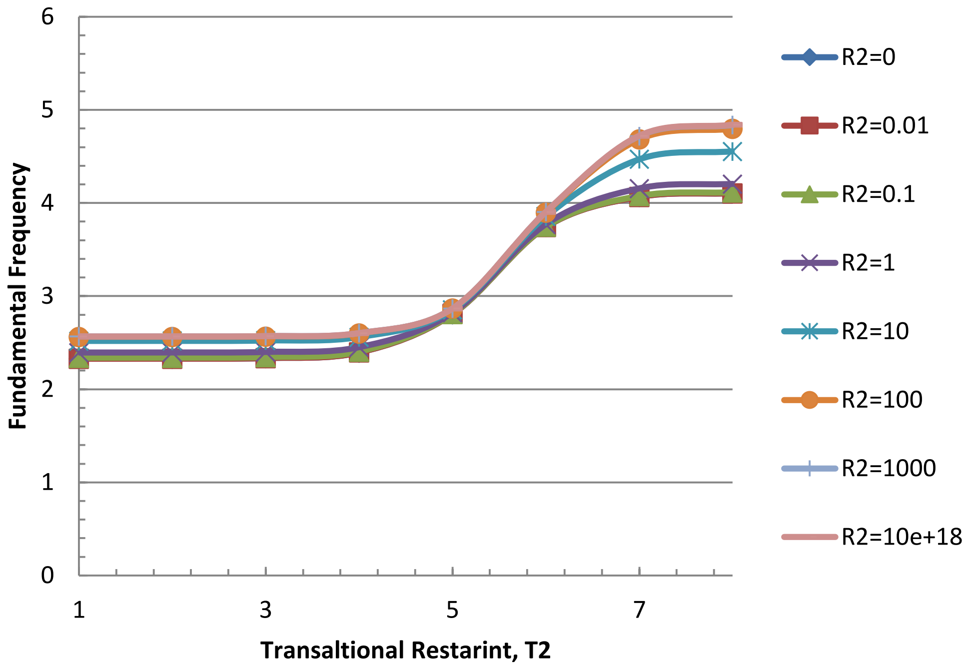

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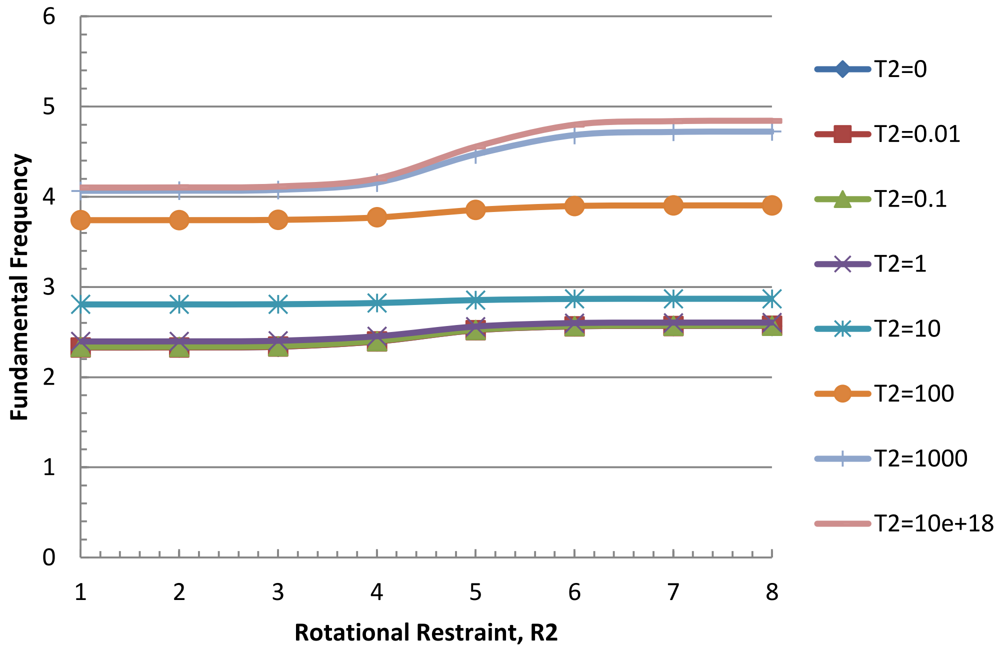
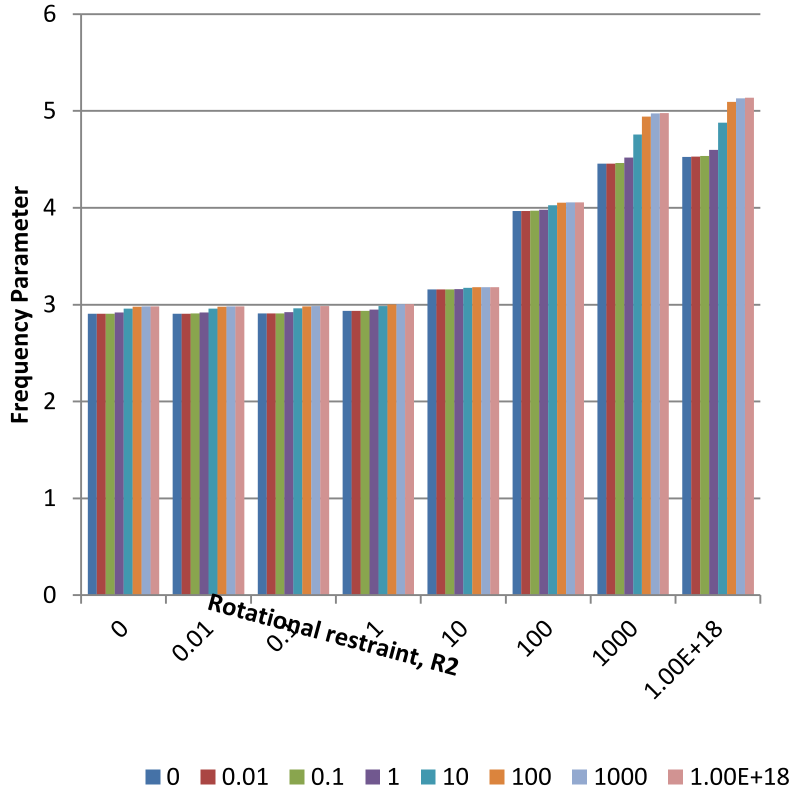


(c)

Fig. 6(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=2).

Table.6 First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 4.0

$R2$	$T2=0$	$T2=0.01$	$T2=0.1$	$T2=1$	$T2=10$	$T2=100$	$T2=1000$	$T2=10^{18}$
0	2.9057	2.9060	2.9087	2.9352	3.1564	3.9670	4.4550	4.5263
0.01	2.9059	2.9062	2.9089	2.9354	3.1564	3.9671	4.4557	4.5271
0.1	2.9075	2.9078	2.9105	2.9368	3.1569	3.9686	4.4620	4.5344
1	2.9204	2.9207	2.9232	2.9485	3.1611	3.9809	4.5178	4.5986
10	2.9593	2.9596	2.9618	2.9837	3.1740	4.0253	4.7567	4.8774
100	2.9786	2.9789	2.9809	3.0012	3.1807	4.0526	4.9414	5.0943
1000	2.9813	2.9816	2.9836	3.0037	3.1816	4.0568	4.9731	5.1312
10^{18}	2.9817	2.9819	2.9839	3.0040	3.1817	4.0573	4.9768	5.1356



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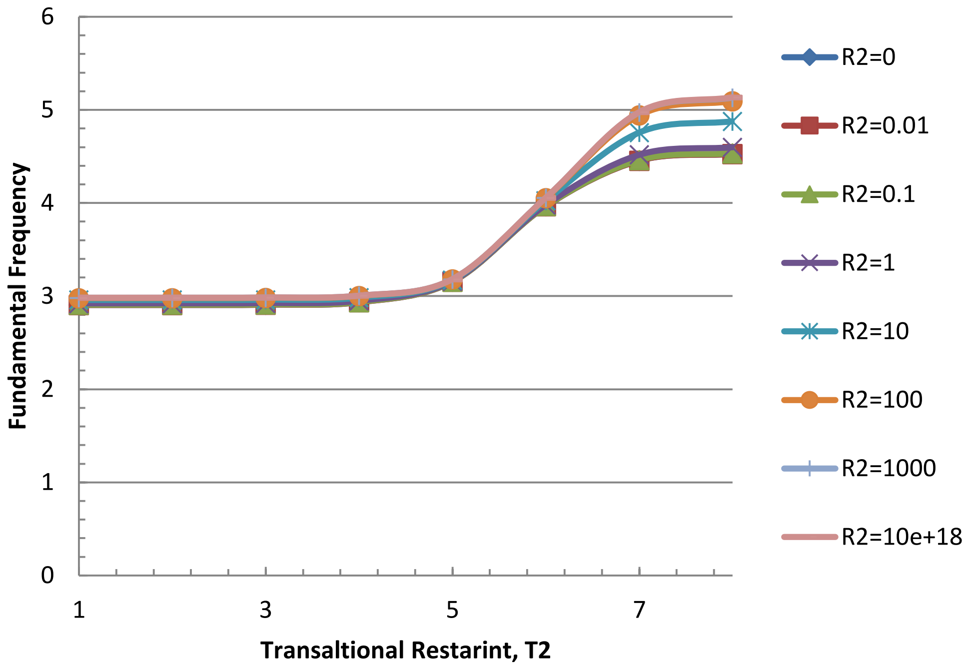
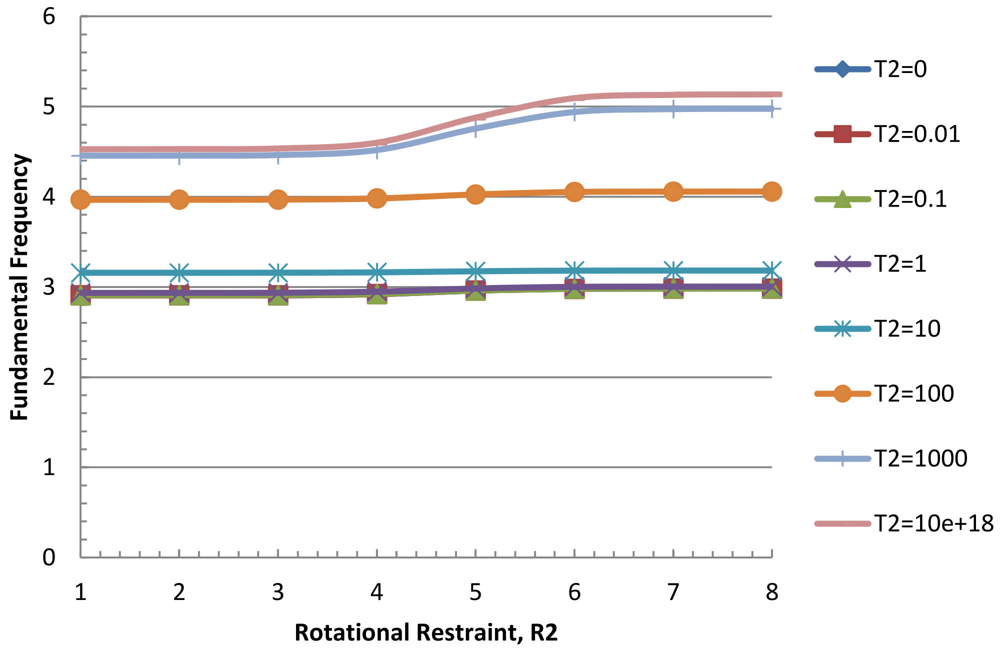


(b)

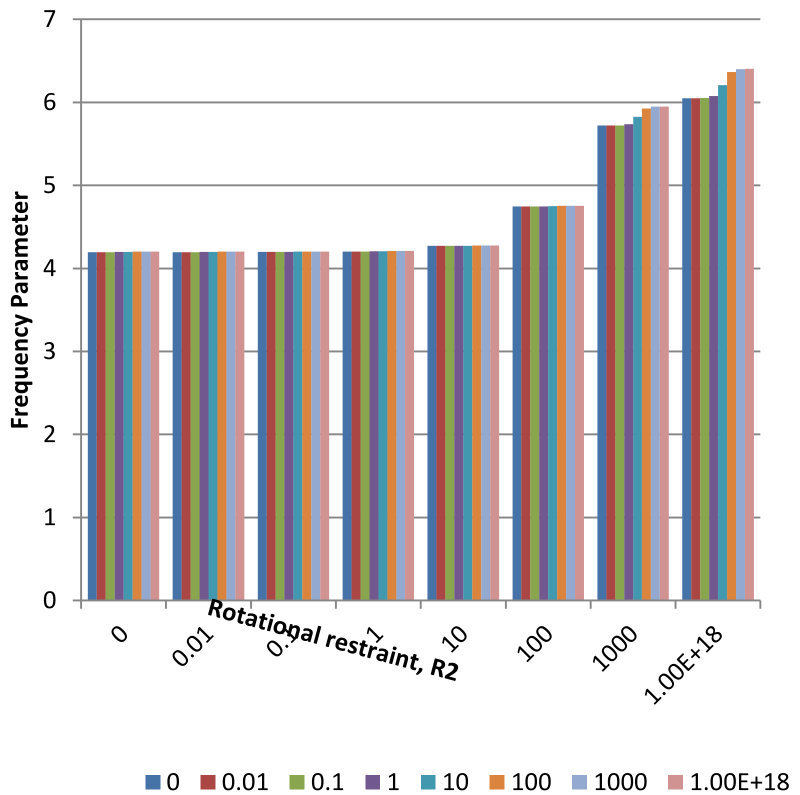


(c)

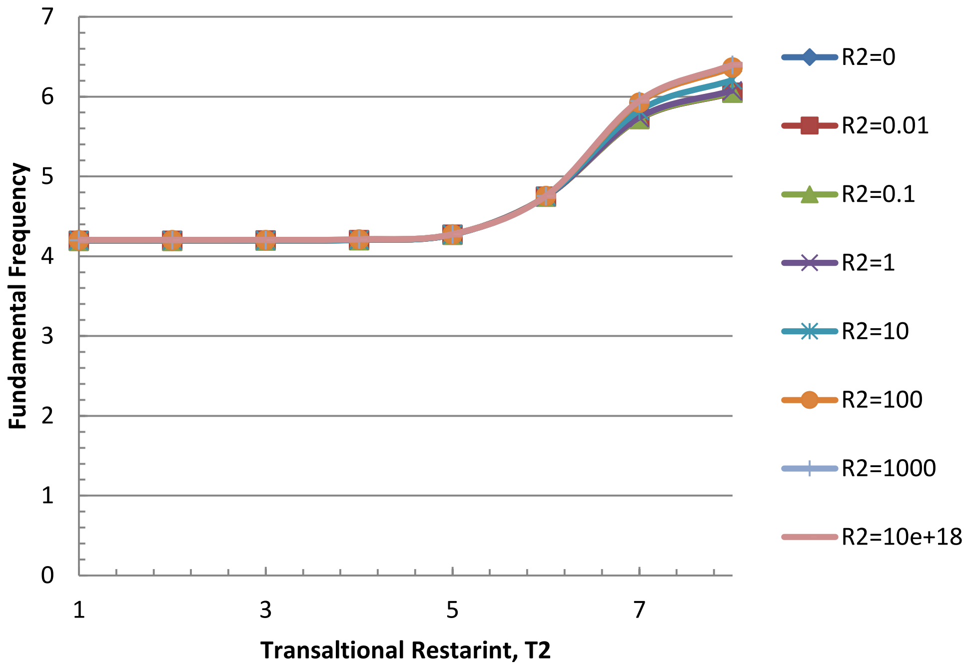
Fig. 7(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=4).

Table.7(5)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter $K = 10.0$

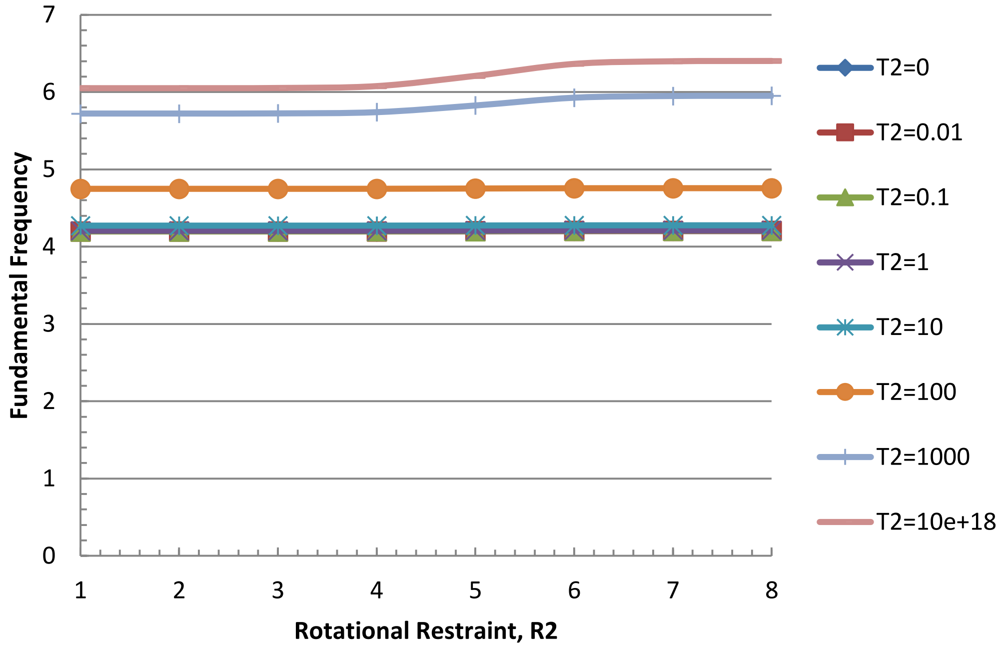
$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	4.1965	4.1966	4.1973	4.2043	4.2715	4.7462	5.7208	6.0483
0.01	4.1965	4.1966	4.1973	4.2043	4.2715	4.7462	5.7210	6.0486
0.1	4.1966	4.1966	4.1973	4.2043	4.2715	4.7463	5.7227	6.0512
1	4.1971	4.1972	4.1979	4.2049	4.2718	4.7470	5.7384	6.0754
10	4.1999	4.2000	4.2007	4.2075	4.2732	4.7508	5.8251	6.2091
100	4.2027	4.2028	4.2035	4.2101	4.2746	4.7548	5.9259	6.3648
1000	4.2033	4.2033	4.2040	4.2107	4.2749	4.7556	5.9479	6.3984
10^{18}	4.2033	4.2034	4.2041	4.2107	4.2750	4.7557	5.9506	6.4026



(a)



(b)

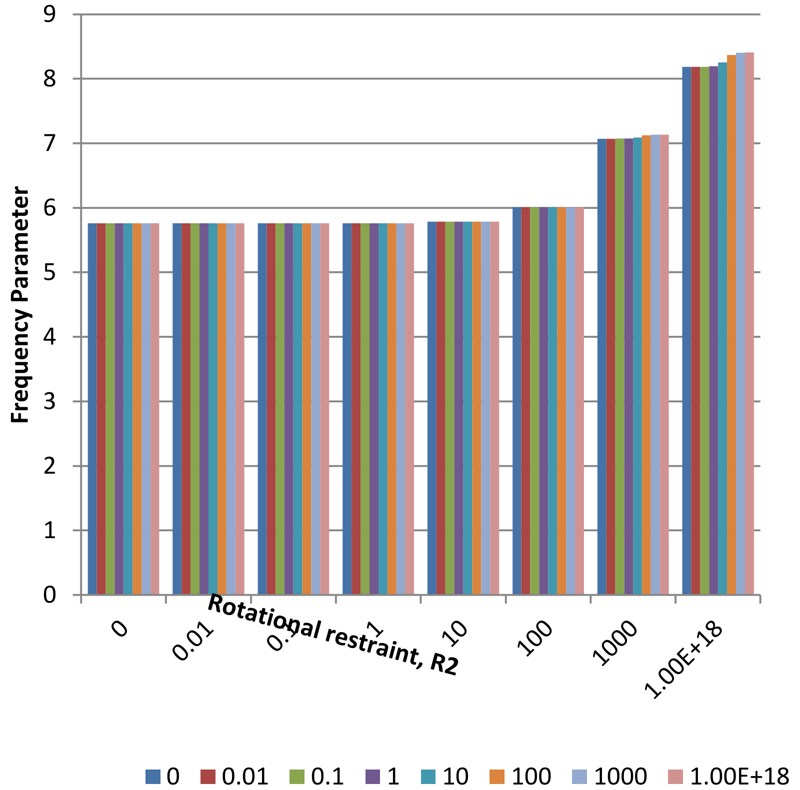


(c)

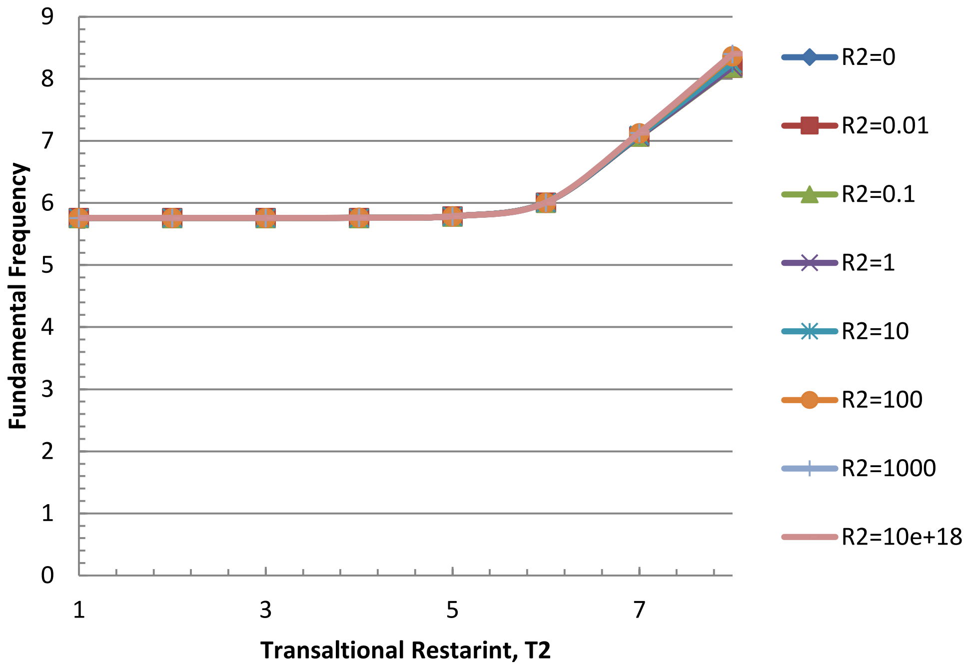
Fig. 8(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=10).

Table.8(6)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 20.0

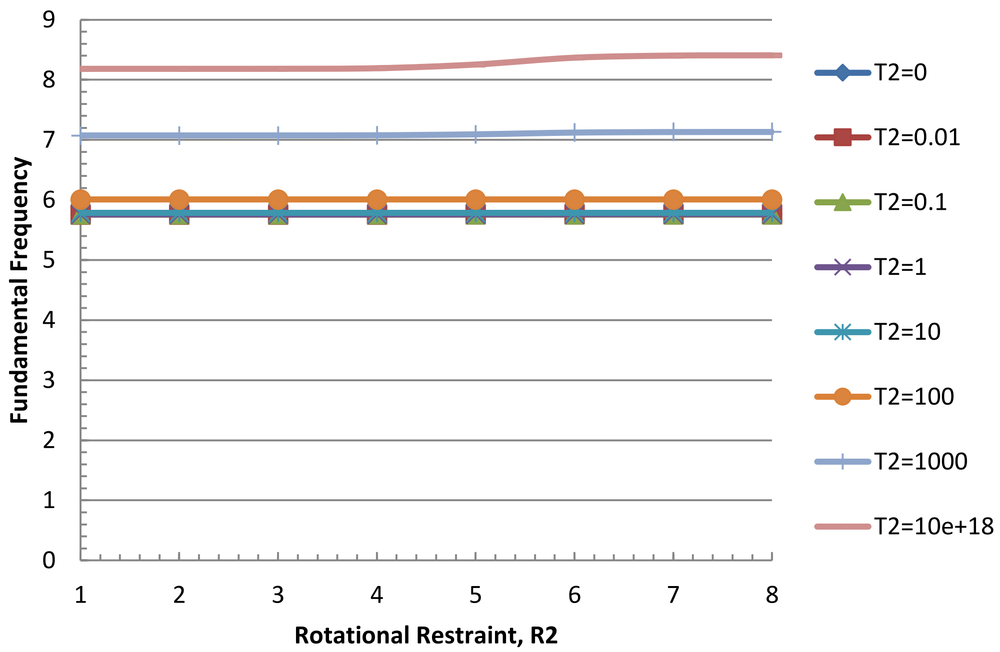
$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	5.7585	5.7585	5.7588	5.7613	5.7861	6.0080	7.0717	8.1827
0.01	5.7585	5.7585	5.7588	5.7613	5.7861	6.0080	7.0717	8.1829
0.1	5.7585	5.7585	5.7588	5.7613	5.7861	6.0080	7.0720	8.1838
1	5.7586	5.7586	5.7588	5.7613	5.7861	6.0080	7.0745	8.1927
10	5.7589	5.7589	5.7591	5.7616	5.7863	6.0081	7.0915	8.2541
100	5.7594	5.7594	5.7596	5.7621	5.7867	6.0083	7.1226	8.3673
1000	5.7595	5.7595	5.7598	5.7623	5.7868	6.0083	7.1321	8.4020
10^{18}	5.7595	5.7596	5.7598	5.7623	5.7868	6.0083	7.1334	8.4067



(a)



(b)

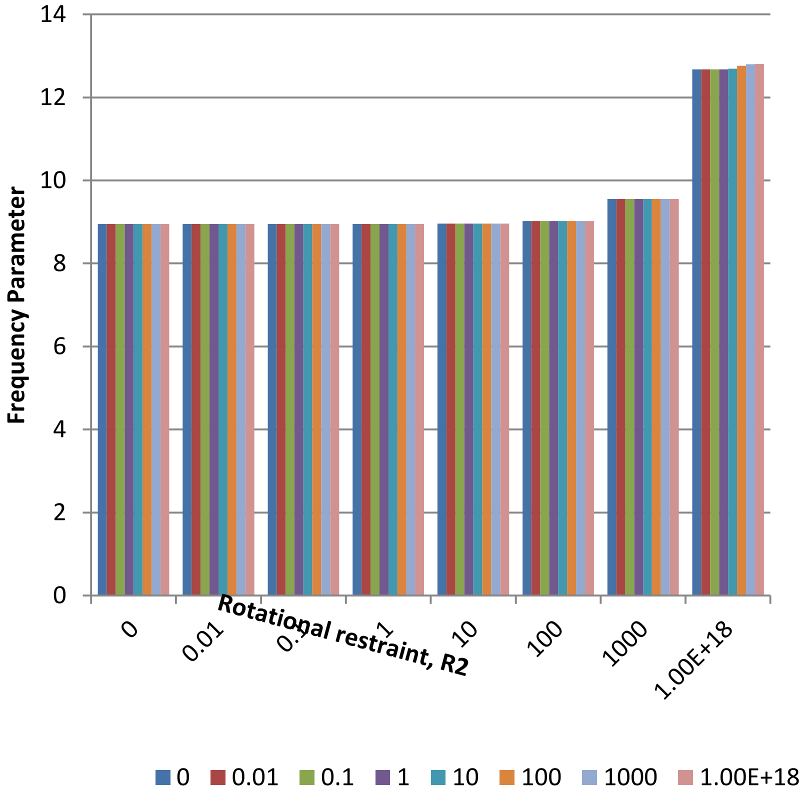


(c)

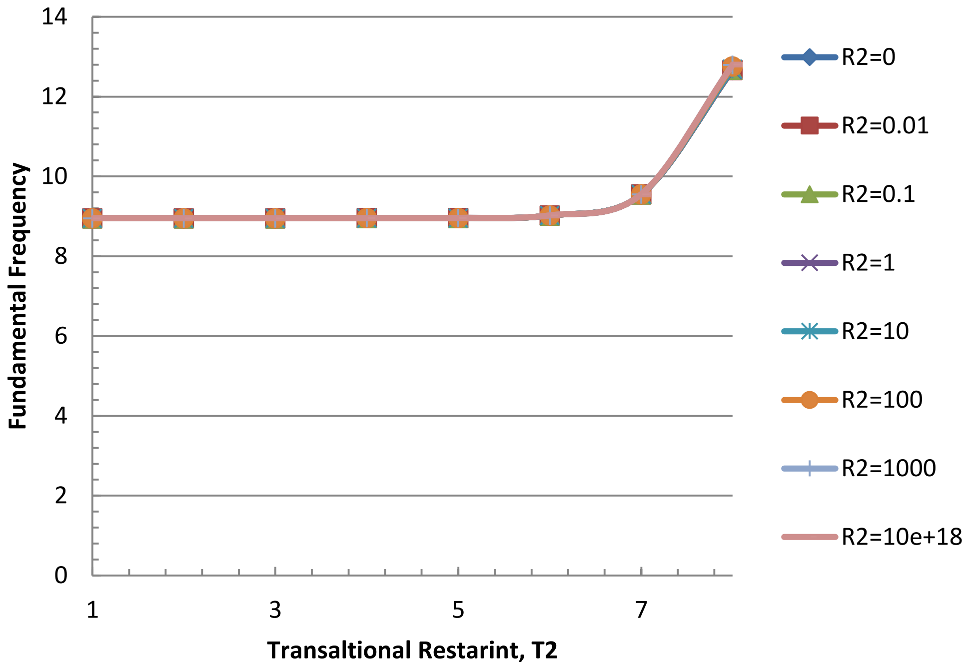
Fig. 9(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=20).

Table.9(7)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 50.0

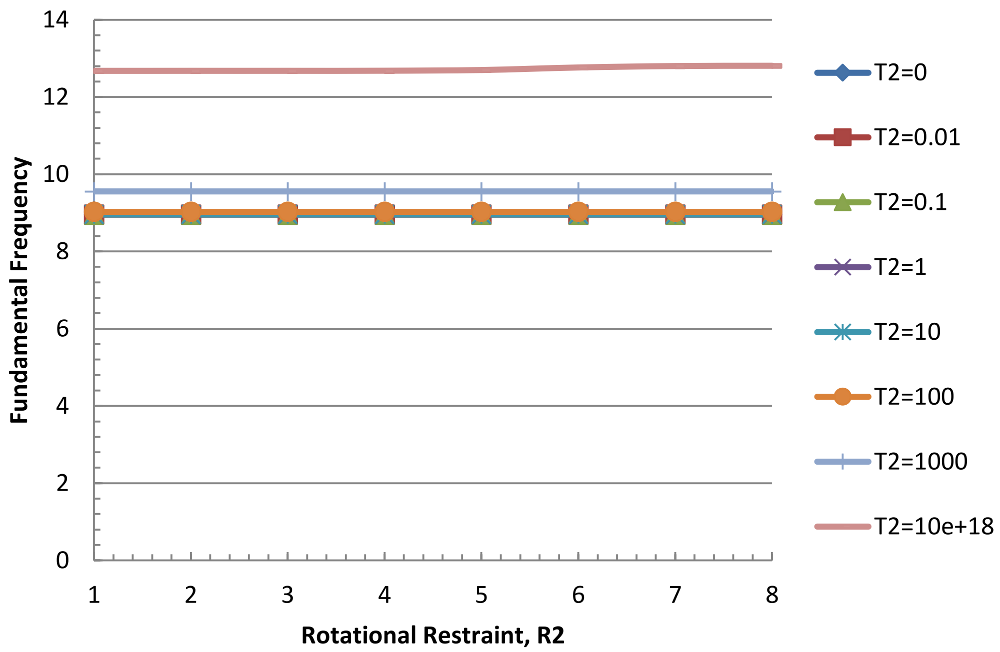
$R2$	$T2 = 0$	$T2 = 0.01$	$T2 = 0.1$	$T2 = 1$	$T2 = 10$	$T2 = 100$	$T2 = 1000$	$T2 = 10^{18}$
0	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5508	12.6729
0.01	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5508	12.6730
0.1	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5508	12.6732
1	8.9544	8.9544	8.9544	8.9551	8.9615	9.0242	9.5509	12.6755
10	8.9544	8.9544	8.9545	8.9551	8.9615	9.0242	9.5513	12.6945
100	8.9544	8.9544	8.9545	8.9551	8.9615	9.0242	9.5527	12.7600
1000	8.9545	8.9545	8.9545	8.9552	8.9616	9.0242	9.5535	12.7979
10^{18}	8.9545	8.9545	8.9545	8.9552	8.9616	9.0242	9.5536	12.8043



(a)



(b)

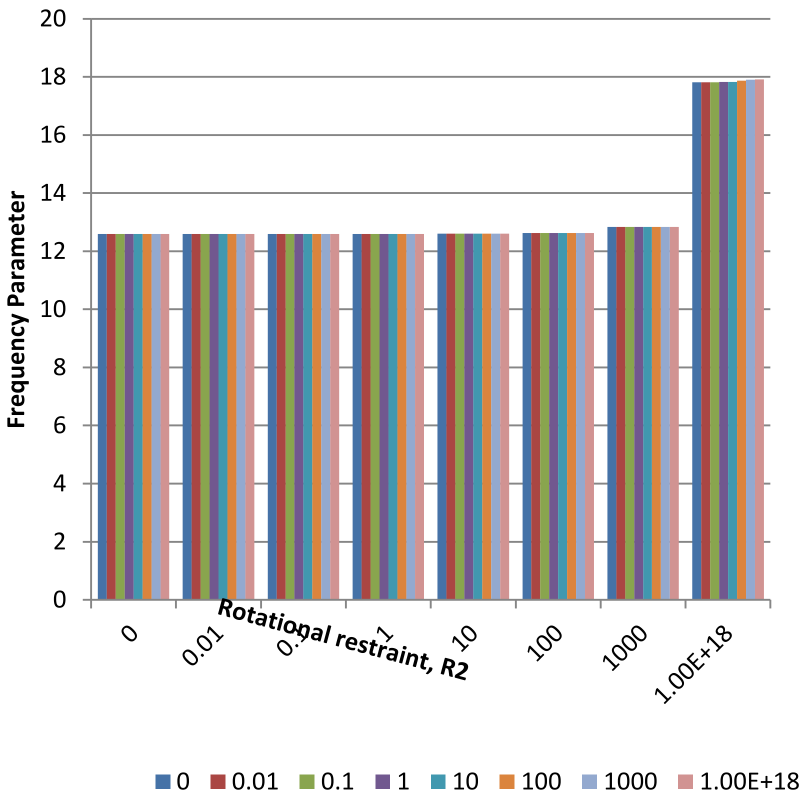


(c)

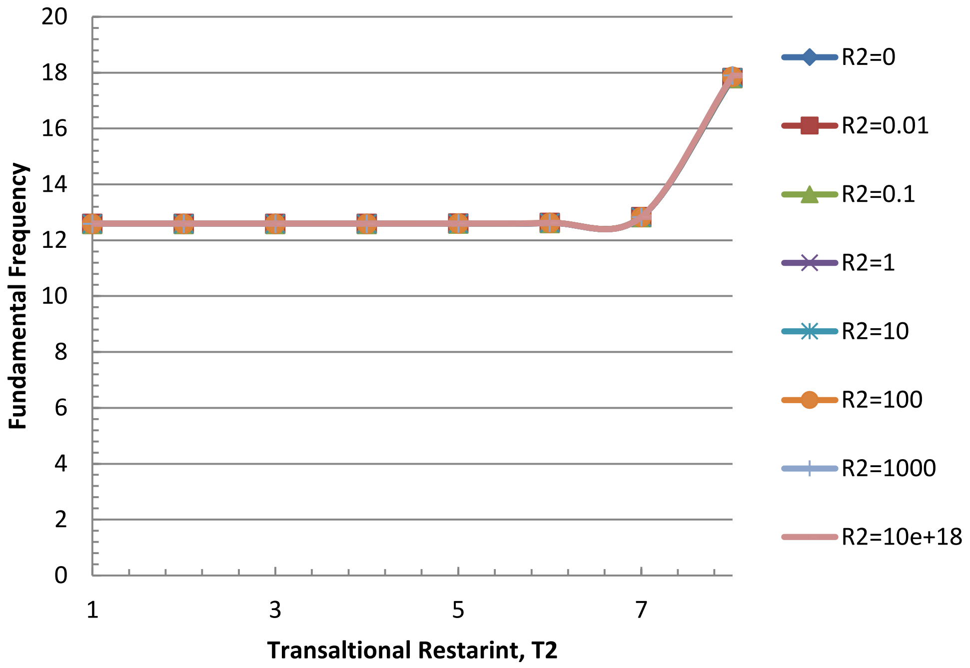
Fig. 10(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=50).

Table.10(8)First mode natural frequencies for various values of rotational and translational restraint parameters R2 and T2 and for warping parameter K= 100.0

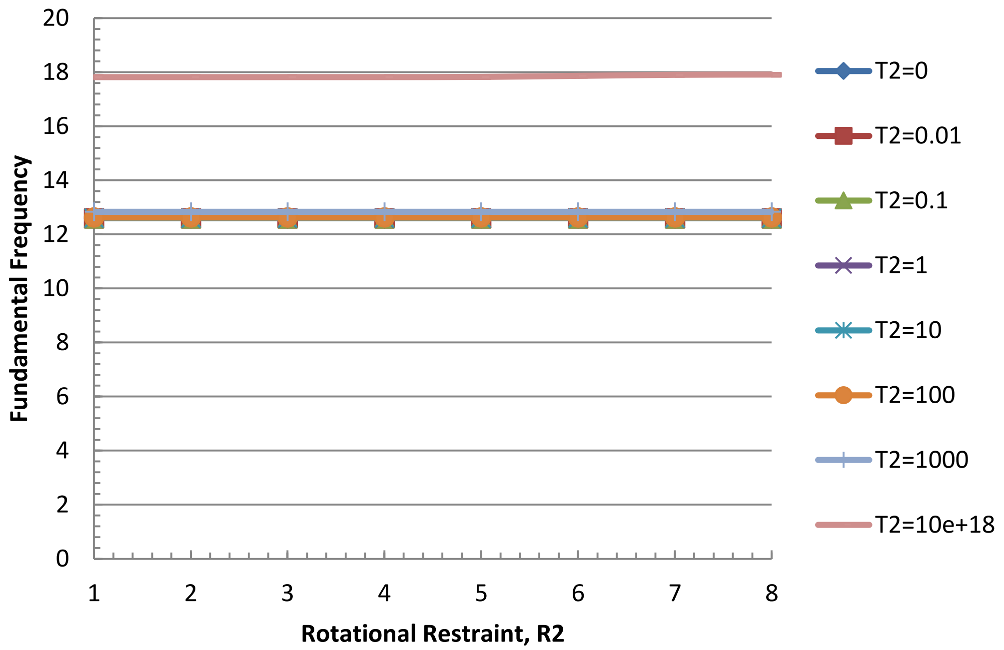
$R2$	$T2=0$	$T2=0.01$	$T2=0.1$	$T2=1$	$T2=10$	$T2=100$	$T2=1000$	$T2=10^{18}$
0	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8182
0.01	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8182
0.1	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8183
1	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8191
10	12.5970	12.5970	12.5971	12.5973	12.5996	12.6222	12.8378	17.8264
100	12.5971	12.5971	12.5971	12.5973	12.5996	12.6222	12.8379	17.8634
1000	12.5971	12.5971	12.5971	12.5973	12.5996	12.6222	12.8379	17.9006
10^{18}	12.5971	12.5971	12.5971	12.5973	12.5996	12.6222	12.8380	17.9089



(a)



(b)



(c)

Fig. 11(a),(b) and (c). Variation of frequency parameter with rotational restraints (R2&T2=0 to 10^{18}) for a given Warping parameter (K=100).

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