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Scenario of SEIR Epidemic Model for COVID-19 Pandemic Using Fractional Order Derivative

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Abstract: - This research paper develops a new version of the well known epidemic mathematical SEIR model to analyze the dynamics of COVID-19 (Coronavirus Disease-2019) transmission in India and Brazil. The equations of the proposed SEIR model are extended to incorporate fraction-order differential equations. After determine the equilibrium points, the stability analysis of this model is also performed. The numerical simulations are displaced graphically and theoretically for the various values of fractional parameter p . We also fitted the SEIR model with actual data to verify the accuracy of our mathematical study. Finally, we include a discussion and concluding remarks.

Keywords: -COVID-19, Fractional calculus, SEIR model, stability analysis, Adam-Bashforth-Moulton method.

I. INTRODUCTION

An highly contagious infection disease Coronavirus pandemic (COVID-19) has spread worldwide and generated the fifth documented pandemic since 1918 flu pandemic. The virus was officially named severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) by the International Committee on Taxonomy of Virus base on phylogenetic analysis and the disease was declared to be COVID-19 [1]. It is believed that SARS-CoV-2 was a spillover of an animal coronavirus and later adapted the ability of human-to-human transmission. Due to its extremely contagious high fatality rate in worldwide, the World Health Organization (WHO) declared the outbreak a 'Public Health Emergency of International Concern', on 30th January and a 'Global Pandemic' on 11th March 2020. Direct transmission occurs through individual-to-individual contact, through a sneeze or cough, or through skin-skin contact [3]. The outbreak of COVID-19 started in Wuhan, Hubei Province, China in 2019 [4], which has so far killed 63.1 L people and 53.7 Cr. people infected across the globe. In India, 42674712 confirmed cases and 524803 deaths (as on June 16, 2022) have been reported so far according to official figures released by the Ministry of Health and Family Welfare (MoHFW).

Therefore, in the present scenario, a suitable mathematical model would not only represent the whole disease system but also study the transmission of the disease, the effect of preventive measures, and future outbreaks prediction. Among the initial works on mathematical disease model, Kermack and Mckendrick [5] developed the epidemic model in 1927. To understand the transmission dynamics of peculiar epidemiological traits of COVID-19, several mathematical models such as SIR, SIQR [8] have already been conducted. Some of these are listed in our references [9, 10, 11] invented the SEIR model where the basic reproduction was estimated. To study such a mathematical model, the entire population (N) is primarily divided in subclasses: the susceptible individuals (S), the exposed individuals (E), the infected individuals (I) and the recovered individuals (R). Zhu et al.[7] proposed a framework to predict the prevalence of COVID-19 in different countries and cities.

Fractional calculus is a significant tool for mathematical modeling of numerous issues in physics and mathematics due to its conciseness of the definition and its strong expressive force. It is observed that the fractional order derivative is more intuitionistic in comparison with integral order of the SEIR model. In recent

years, many biological and physical problems have been solved by fractional order derivatives. The Fractional order differential equations (FODEs) and their applications have an numerous uses in the field of biology, physics, chemistry, biochemistry, solid mechanics, engineering and medicine. Several researcher, including Ucar [12], Saleem [13], chan [14] have used Caputo-Fabrizio fractional derivative to solve several problems.

In the present work, we intend to investigate the spread of COVID-19 disease using the SEIR mathematical model in the context of fractional order derivative. First, we analyze the model mathematically, and then, in numerical section, we present simulations for the Covid-19 transmission in India and Brazil. Also, to evaluate the advantage of fractional order derivatives, we calculate the results for different orders of fractional derivatives with the help of estimated parameters and investigate the effect of derivative order on the SEIR model.

The paper is organized as follows: In section 2, the fundamental concept of the fractional operator is recalled. A brief study of SEIR model with fractional derivative operators for COVID-19 is described and the study of the equilibrium states of the system is also mentioned in section 3. In section 4, we discussed the local stability as well as global stability and stability criterion of the system. In section 5, the numerical simulation is carried out to validate analytic results via Adam-Bashforth-Moulton predictor-corrector scheme for SEIR model. Some numerical discussion using MATLAB-21a software are presented in section 6. In section 7, based on some real-world data, we infer the fractional order, time dependent parameters, as well as future predictions using the fractional SEIR model. Finally, in section 8 we constitutes the conclusion of the paper.

II. PRELIMINARIES

In this part, we recalled some of the fundamental concept of fractional differential and integral operators [15].

Definition 2.1 A function $f : \mathbf{R} \to \mathbf{R}$ with fractional order $0 < p \le 1$ is defined as

$$ {}^{C}I_t^p(f(t)) = \frac{1}{\Gamma(p)} \int_0^t (t-x)^{p-1} f(x)dx, \tag{2.1} $$

where $\Gamma()$ is the Gamma function.

Definition 2.2 The Caputo fractional derivative operator of order $0 < p \le 1$ is defined as

$$ {}^{C}D_t^p(f(t)) = I^{n-p}D_t^p(f(t)) = \frac{1}{\Gamma(n-p)} \int_0^t (t-x)^{n-p-1} \frac{d^n}{dx^n} f(x)dx, \tag{2.2} $$

Definition 2.3 The Caputo-Fabrizio fractional derivative operator of order $0 < p \le 1$ is defined as

$$ {}^{CF}D_t^p(f(t)) = \frac{M(p)}{(n-p)} \int_0^t \exp[-\frac{p(1-x)}{(1-p)} \frac{d}{dx}(f(x))dx, \tag{2.3} $$

Definition 2.4 The Caputo-Fabrizio fractional integral operator of order $0 < p \le 1$ is defined as

$$ {}^{CF}I_t^p(f(t)) = \frac{2(1-p)}{(2-p)M(p)} f(t) + \frac{2(1-p)}{(2-p)M(p)} \int_0^t f(x)dx, \tag{2.4} $$

Here $M(p)$ is a normalization function.

III. MODEL FORMATION

In compartmental models, based on the epidemiology of COVID-19 and the control measures taken by the Government (India and Brazil), we divided the population into compartments with labels, such as S, E, I and R corresponding to susceptible, exposed, infectious, recovered compartments. At any time $t \ge 0$, the population (N) can be written as:

$$ N(t) = S(t) + E(t) + I(t) + R(t). \tag{3.1} $$

The differential equations of the SEIR model are given as:

$$\begin{aligned} D_t S(t) &= \lambda - \beta SI - \mu S, \\ D_t E(t) &= \beta SI - (\mu + k)E, \\ D_t I(t) &= kE - (\mu + \gamma)I, \\ D_t R(t) &= \gamma I - \mu R, \end{aligned} \tag{3.2}$$

with the initial condition: $S(0) = S_0 > 0, E(0) = E_0 > 0, I(0) = I_0 > 0, R(0) = R_0 > 0$. Where the parameters $\lambda, \beta, \gamma, k$ and μ represent the natural birth rate, contact rate between susceptible and infected, recovery rate, progression rate exposed to infected, and mortality rate respectively.

The SEIR model in (3.2) having Caputo-Fabriozio operator on fractional order derivatives has been proposed as follows:

$$\begin{aligned} {}^{CF}D_t S(t) &= \lambda - \beta SI - \mu S, \\ {}^{CF}D_t E(t) &= \beta SI - (\mu + k)E, \\ {}^{CF}D_t I(t) &= kE - (\mu + \gamma)I, \\ {}^{CF}D_t R(t) &= \gamma I - \mu R, \end{aligned} \tag{3.3}$$

where $0 < p \leq 1$.

3.1 Non-negativity and boundedness of solutions

Theorem: All the variables are non-negative for $t \geq 0$. The closed region $\Omega = \{(S, E, I, R) \in R^4 : 0 < N \leq \frac{\lambda}{\mu}\}$ is positive invariant for the model (3.3).

Proof: From the model given in (3.3),

$${}^{CF}D_t^p S(t) = \lambda - \beta SI - \mu S \geq -(\beta I + \mu)S.$$

Therefore we have, $S(t) \geq S(0)\exp(-\int_0^t (\beta I + \mu)dp) > 0$.

Now,

$${}^{CF}D_t^p E(t) = \beta SI - (\mu + k)E \geq -(\mu + k)E.$$

Thus we have $E(t) \geq E(0)\exp(-\int_0^t (k + \mu)dp) > 0$.

Again, ${}^{CF}D_t^p I(t) = kE - (\mu + \gamma)I \geq -(\mu + \gamma)I$.

Thus we have $I(t) \geq I(0)\exp(-\int_0^t (\gamma + \mu)dp) > 0$.

Similarly ${}^{CF}D_t^p R(t) = \gamma I - \mu R \geq -\mu R$.

Thus we have $R(t) \geq R(0)\exp(-\int_0^t (\mu)dp) > 0$.

Again ${}^{CF}D_t^p (S + E + I + R) = \lambda - \mu(S + E + I + R)$.

Therefore ${}^{CF}D_t^p N(t) = \lambda - \mu N$.

Thus, $\limsup_{t\to\infty} N(t) \leq \frac{\lambda}{\mu}$.

Therefore, the model (3.3) is bounded by $\frac{\lambda}{\mu}$.

Then we get S, E, I, R are positive functions and Ω is positively consistent of the system (3.3).

3.2 Basic reproduction number, Disease-free equilibrium, Epidemic equilibrium state:

The basic reproduction number denotes by R_0 , R_0 means the average number of secondary infections that produced by infected individuals in a fully susceptible population through the duration of infectiousness.

R_0 indicates a threshold condition for the stability of the disease-free equilibrium point. The disease-free equilibrium point is locally asymptotically stable if $R_0 < 1$ i.e the disease outbreaks. The disease-free equilibrium point is unstable when $R_0 > 1$ i.e an epidemic occurs.

Here 'Next Generation Matrix' method [16] is used to derive the basic reproduction number (R_0) for the COVID-19 model. Now, the next generation matrix be, $G = FV^{-1}$.

Since R_0 is the dominant eigen value of the matrix, $G = FV^{-1}$.

where, $F = \begin{bmatrix} \beta\dfrac{\lambda}{\mu} & 0 \\ 0 & 0 \end{bmatrix}$ and $V = \begin{bmatrix} 0 & (k+\mu) \\ (\mu+\gamma) & -k \end{bmatrix}$.

Therefore the Reproduction number

$$(R_0) = \frac{k\beta\lambda}{\mu(\mu+\gamma)(k+\mu)}. \tag{3.4}$$

IV. STABILITY ANALYSIS

To obtain the equilibrium points of the system (3.3) we solve the following system of equations:

$$^{CF}D_t^p S(t) = {}^{CF}D_t^p E(t) = {}^{CF}D_t^p I(t) = {}^{CF}D_t^p R(t) = 0 \tag{4.1}$$

We have two equilibrium points, given by $E_0 = \left(\frac{\lambda}{\mu}, 0, 0, 0\right)$ and $E_1 = \left(S^*, E^*, I^*, R^*\right)$, where E_0 is the disease-free equilibrium and E_1 is the unique epidemic point of the system (3.3), where

$$S^* = \frac{(\mu+k)(\mu+\gamma)}{k\beta}$$

$$E^* = \frac{\mu(\mu+k)(\mu+\gamma)}{k\beta(\mu+k)}(R_0 - 1),$$

$$I^* = \frac{\lambda k}{(\mu+k)(\mu+\gamma)} - \mu,$$

$$R^* = \frac{\gamma k E^*}{\mu(\mu+\gamma)}.$$

Where R_0 is given by equation (3.4). In case of an epidemic, E^* will exist only when $R_0 > 1$.

4.1 Theorem: The disease-free equilibrium of the system (3.3) is locally stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof: From system (3.3) we consider,

$$F_1 = {}^{CF}D_t^p S(t) = \lambda - \beta SI - \mu S,$$

$$F_2 = {}^{CF}D_t^p E(t) = \beta SI - (\mu+k)E,$$

$$F_3 = {}^{CF}D_t^p I(t) = kE - (\mu+\gamma)I,$$

$$F_4 = {}^{CF}D_t^p R(t) = \gamma I - \mu R.$$

The Jacobian matrix is, $J = \begin{bmatrix} -(\beta I + \mu) & 0 & -\beta S & 0 \\ \beta I & -(\mu + K) & \beta S & 0 \\ 0 & K & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}$.

At the equilibrium point $E_0 = \left(\frac{\lambda}{\mu}, 0, 0, 0\right)$, the Jacobian matrix become

$$J(E_0) = \begin{bmatrix} -\mu & 0 & -\beta\frac{\lambda}{\mu} & 0 \\ 0 & -(\mu + K) & \beta\frac{\lambda}{\mu} & 0 \\ 0 & K & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}.$$

Therefore we get the characteristic equation: $(\mu + x)^2(x^2 + Ax + B) = 0$,

where, $A = (2\mu + \gamma + K) > 0$ and $B = (1 - R_0)(\mu + K)(\mu + \gamma) > 0$.

Since all the coefficients are positive, the roots are surely negative. It depends on the value of R_0.

So the characteristic roots are $-\mu, -\mu$ and the last two roots are obtained from the quadratic equation, and the roots of the equation is negative provided $R_0 < 1$ and positive roots provide $R_0 > 1$.

Hence the equilibrium point E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

4.2 Theorem: If $R_0 > 1,$, the epidemic equilibrium $E_1 = (S^*, E^*, I^*, R^*)$ is locally asymptotically stable.

Proof: At the equilibrium point $E_1 = (S^*, E^*, I^*, R^*)$, the Jacobian matrix becomes,

$$J(E_1) = \begin{bmatrix} -(\beta I^* + \mu) & 0 & -\beta S^* & 0 \\ \beta I^* & -(\mu + K) & \beta S^* & 0 \\ 0 & K & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix}.$$

Now, the characteristic equation is of the form: $(\mu + x)(x^3 + ax^2 + bx + c) = 0,$

where,

$$a = (\beta I^* + 3\mu + K + \gamma),$$

$$b = (\beta I^* + \mu)(2\mu + K + \gamma) + (1 - R_0)(\mu + K)(\mu + \gamma) + \frac{\beta K E^*(\mu + K)}{\mu}$$

$$c = (\beta I^* + \mu)(\mu + K)(\mu + \gamma) - \mu\beta K S^*.$$

By Routh-Hurwitz Criterion, the system (3.3) is locally asymptotically stable if $a > 0, c > 0, ab > c.$

Thus $E_1 = (S^*, E^*, I^*, R^*)$ is a locally asymptotically stable equilibrium point.

4.3 Theorem: The disease-free equilibrium of the system (3.3) is globally asymptotically stable if $R_0 < 1.$

Proof: Consider, the following linear Lyapunov function

$$L = B_1 E + B_2 I.$$

Calculative the time fractional derivative of the above function

$${}^{CF}D_t^p L(t) = {}^{CF}D_t^p (B_1 E(t) + B_2 I(t)).$$

Substituting with the values of ${}^{CF}D_t^p E(t)$ and ${}^{CF}D_t^p I(t)$ [from system (3.3)] we get,

$$\Rightarrow {}^{CF}D_t^p L(t) = I[B_1\beta S + B_2(\mu + \gamma)] + E[B_2 K + B_1(\mu + K)]. \quad (4.2)$$

A little perturbation from equation (4.2) we get,

$$B_1 = K, B_2 = (\mu + K).$$

Now substituting the expression B_1, B_2 in equation (4.2), we get

$$^{CF}D_t^p L(t) = I\beta SK - (\mu + K)(\mu + \gamma)I,$$

$$\Rightarrow (\mu + K)(\mu + \gamma)I\left[\frac{\beta SK}{(\mu + K)(\mu + \gamma)} - 1\right].$$

Since $S = \frac{\lambda}{\mu} \leq N$, it follows that

$$^{CF}D_t^p L(t) = (\mu + K)(\mu + \gamma)I\left[\frac{\beta\lambda K}{\mu(\mu + K)(\mu + \gamma)} - 1\right],$$

$$= (\mu + K)(\mu + \gamma)I(R_0 - 1).$$

Hence, if $R_0 < 1$, then $^{CF}D_t^p E(t) < 0$. By LaSalle's extension to Lyapunov's principle [17, 18], the disease-free equilibrium points are globally asymptotically stable.

4.4 Theorem: If $R_0 > 1$, the epidemic equilibrium E_1 is globally asymptotically stable.

Proof: Consider the system (3.3) and $R_0 > 1$, so that the epidemic equilibrium E_1 of system exists.

Now, consider the following non-linear Lyapunov function of Goh-Volterra type:

$$V = \left(S - S^* - S^* \log\frac{S}{S^*}\right) + \left(E - E^* - E^* \log\frac{E}{E^*}\right) + Q\left(I - I^* - I^* \log\frac{I}{I^*}\right).$$

With Lyapunov fractional derivative with respect to time we have,

$$^{CF}D_t^p V(t) = \left(^{CF}D_t^p S(t) - \frac{^{CF}D_t^p S(t)S^*}{S}\right) + \left(^{CF}D_t^p E(t) - \frac{^{CF}D_t^p E(t)E^*}{E}\right) + Q\left(^{CF}D_t^p I(t) - \frac{^{CF}D_t^p I(t)I^*}{I}\right). \quad (4.3)$$

Substituting the values of $^{CF}D_t^p S(t), ^{CF}D_t^p E(t), ^{CF}D_t^p I(t)$ from (3.3) into(4.3) we get

$$^{CF}D_t^p V(t) = \left(\lambda - \beta SI - \mu S - \frac{(\lambda - \beta SI - \mu S)S^*}{S}\right) + \left(\beta SI - (\mu + k)E - \frac{(\beta SI - (\mu + k)E)E^*}{E}\right) \quad (4.4)$$

$$+ Q\left(KE - (\mu + \gamma)I - \frac{(KE - (\mu + \gamma)I)I^*}{I}\right).$$

At steady state, from equation (3.3) we have

$$^{CF}D_t^p S(t) = 0 = \lambda - \beta S^* I^* - \mu S^*$$

$$\Rightarrow \lambda = \beta S^* I^* + \mu S^* \quad (4.5)$$

Substituting the equation (4.5) into (4.4) gives

$$^{CF}D_t^p V(t) = \beta S^* I^* + \mu S^* - \mu S - \frac{(\beta S^* I^* + \mu S^* - \beta SI - \mu S)S^*}{S} + [-(\mu + K)E] - \frac{(\beta SI - (\mu + K)E)E^*}{E} \quad (4.6)$$

$$Q\left(KE - (\mu + \gamma)I - \frac{I^*(KE - (\mu + \gamma)I)}{I}\right).$$

Collecting all infected classes without a single star (*) from (4.6) and equating to zero:

$$S\beta I - (\mu + K)E + Q[KE - (\mu + \gamma)] = 0.$$

A little perturbation of steady state from (3.3) and (4.7) resulted into:

$$Q = \frac{\beta S^*}{\mu + \gamma}, (\mu + K) = \frac{\beta S^* I^*}{E^*}, K = \frac{(\mu + \gamma)I^*}{E^*}. \quad (4.8)$$

Substituting the expression from (4.8) into (4.6) we have

$$^{CF}D_t^pV(t) = \beta S^*I^* + \mu S^* - \mu S - \frac{(\beta S^*I^* + \mu S^* - \mu S)S^*}{S} - \frac{SI^*\beta EI^*}{E} + \beta S^*I^* - \frac{\beta SIE^*}{E} + I^*S^*\beta$$

$$= \beta S^*I^*\left(3 - \frac{S^*}{S} - \frac{I^*E}{IE^*} - \frac{SIE^*}{S^*I^*E}\right) + \mu S^*\left(2 - \frac{S}{S^*} - \frac{S^*}{S}\right).$$

Finally, since the arithmetic mean exceeds the geometric mean, we have

$$\left(3 - \frac{S^*}{S} - \frac{I^*E}{IE^*} - \frac{SIE^*}{S^*I^*E}\right) \le 0, \left(2 - \frac{S}{S^*} - \frac{S^*}{S}\right) \le 0.$$

Then, $^{CF}D_t^pV(t) \le 0,$ for $R_0 > 1.$

Hence, V is a Lyapunov function, by Lasalle's Invarience Principle [18], the epidemic equilibrium E_1 is globally asymptotically stable.

V. THE SEIR MODEL'S ADAMS-BASHFORTH-MOULTAN PREDICTOR CORRECTOR SCHEME:

For fractional order initial value problems of any variety, the Adams-Bashforth-Moulton technique is the most used numerical method. Consider the fractional differential equation as follows:

$$D_t^pW_j(t) = f_j(t, W_j(t)), W_{j0}^r, r = 0,1,2,3,....., [p], j \in N,$$

where W_{j0}^r is the arbitrary real number, $\alpha > 0,$ and in the Caputo interpretation, D_t^α is the fractional differential operator,

$$W_j(t) = \sum_{n=0}^{[p]-1} W_{j0}^r \frac{t^n}{n!} + \frac{1}{\Gamma(p)}\int_0^t (t-\upsilon)^{p-1} f_j(\upsilon, W_j(\upsilon))d\upsilon, j \in N.$$

We use Adam-Bashforth-Moulton predictor-corrector technique to understand the numerical solution of a fractional SEIR model. The algorithm is represented in the following way.

Let $h = \frac{T}{\tilde{m}}, t_n = nh, n = 0,1,2,....,\tilde{m}.$

The corrector values are defined as,

$$S_{n+1} = S_0 + \frac{h^{p_1}}{\Gamma(p_1+2)}(\lambda - \beta S_{n+1}^Q I_{n+1}^Q - \mu S_{n+1}^Q) + \frac{h^{p_1}}{\Gamma(p_1+2)}\sum_{j=0}^{n} p_{1,j,n+1}(\lambda - \beta S_j I_j - \mu S_j),$$

$$E_{n+1} = E_0 + \frac{h^{p_2}}{\Gamma(p_2+2)}(\beta S_{n+1}^Q I_{n+1}^Q - (\mu+k)E_{n+1}^Q) + \frac{h^{p_2}}{\Gamma(p_2+2)}\sum_{j=0}^{n} p_{2,j,n+1}(\beta S_j I_j - (\mu+k)E_j),$$

$$I_{n+1} = I_0 + \frac{h^{p_3}}{\Gamma(p_3+2)}(kE_{n+1}^Q - (\mu+\gamma)I_{n+1}^Q) + \frac{h^{p_3}}{\Gamma(p_3+2)}\sum_{j=0}^{n} p_{3,j,n+1}(kE_j - (\mu+\gamma)I_j),$$

$$R_{n+1} = R_0 + \frac{h^{p_4}}{\Gamma(p_4+2)}(\gamma I_{n+1}^Q - \mu R_{n+1}^Q) + \frac{h^{p_4}}{\Gamma(p_4+2)}\sum_{j=0}^{n} p_{4,j,n+1}(\gamma I_j - \mu R_j).$$

The corresponding predictor values are given by

$$S^Q{}_{n+1} = S_0 + \frac{1}{\Gamma(p_1)}\sum_{j=0}^{n} \beta_{1,j,n+1}(\lambda - \beta S_j I_j - \mu S_j),$$

$$E^Q{}_{n+1} = E_0 + \frac{1}{\Gamma(p_2)}\sum_{j=0}^{n} \beta_{2,j,n+1}(\beta S_j I_j - (\mu+k)E_j),$$

$$I^Q{}_{n+1} = I_0 + \frac{1}{\Gamma(p_3)}\sum_{j=0}^{n} \beta_{3,j,n+1}(kE_j - (\mu+\gamma)I_j),$$

$$R^Q{}_{n+1} = R_0 + \frac{1}{\Gamma(p_4)}\sum_{j=0}^{n} \beta_{4,j,n+1}(\gamma I_j - \mu R_j).$$

Where

$$p_{i,j,n+1} = \begin{cases} n^{p+1} - (n-p)(n+1)^p, j = 0, \\ (n-j+2)^{p+1} + (n-j)^{p+1} - 2(n-j+1)^{p+1}, 0 \le j \le n, \\ 1, j = 1, \end{cases}$$

and $\beta_{i,j,n+1} = \frac{h^{p_i}}{p}[(n+1-j)^{p_i} - (n-j)^{p_i}], 0 \le j \le n; i = 1,2,3,4.$

VI. NUMERICAL SIMULATION AND DISCUSSION:

In this session, we implement meticulous numerical validation of the Caputo-Fabrizio COVID-19 model obtained analytically in a certain period of time. We considered the fraction order parameter $p = 1, 0.95, 0.90, 0.85$ To find the numerically approximate solution to our model, we have used mathematical software MATLAB (2021a).

Case 1: India

In the first case, we provide a numerical simulation using real data for our proposed transmission model of COVID-19 in India from 1st December 2021 to 5th June 2022. The estimated parametric values are obtained from the official site of WHO in case of COVID-19 in India are as follows:

Table 1

Parameter	Value	Reference
λ	9703	Estimated
β	0.0000004	Estimated
γ	0.0691	Estimated
k	0.00127	Estimated
μ	0.04	Estimated
R_0	$3.1988 > 1$	Estimated

A comparison of various categories of population for different values of p for 180 days is displaced in Figure 1.

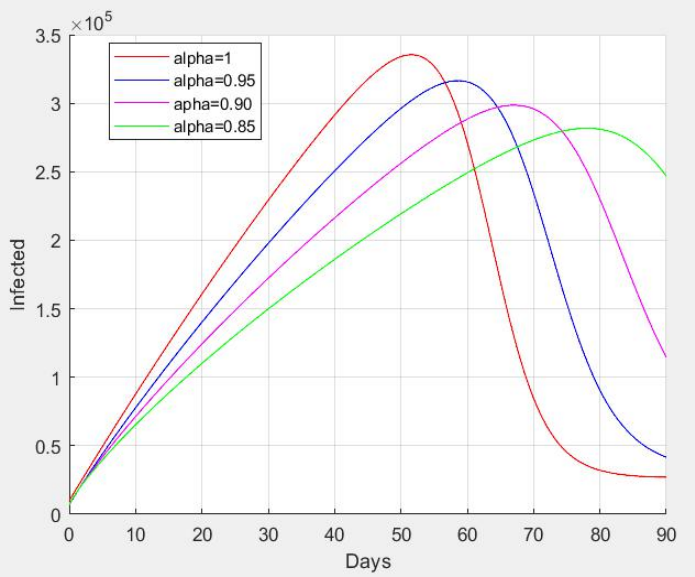

Figure 1a: Infected

In Figure 1a, we observe that the evidence of infection starts rising from the starting days of our observation i.e, 1st December 2021. For $p=1$, not only was the peak of the infection attained very fast, but also the decline was also reported quickly. As the value of fractional derivative decline from 1 to 0.85 gradually, the maximum number of infected individuals is less, but the infection stays for a longer time.

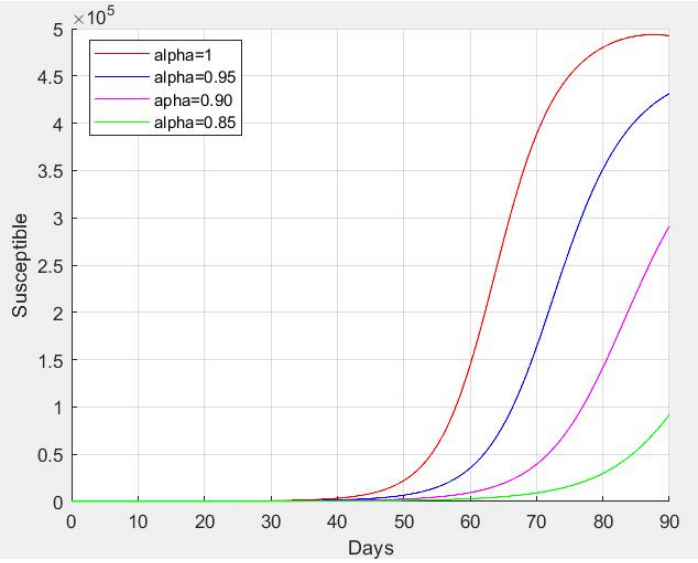

Figure 1b: Susceptible

In Figure 1b, the susceptible population is increasing with the increment of time to a certain point. After that point, even though the infection reduces, the susceptible also shows a declining trend. With the increment of fractional order p, the maximum number of susceptible population gives a higher numerical value.

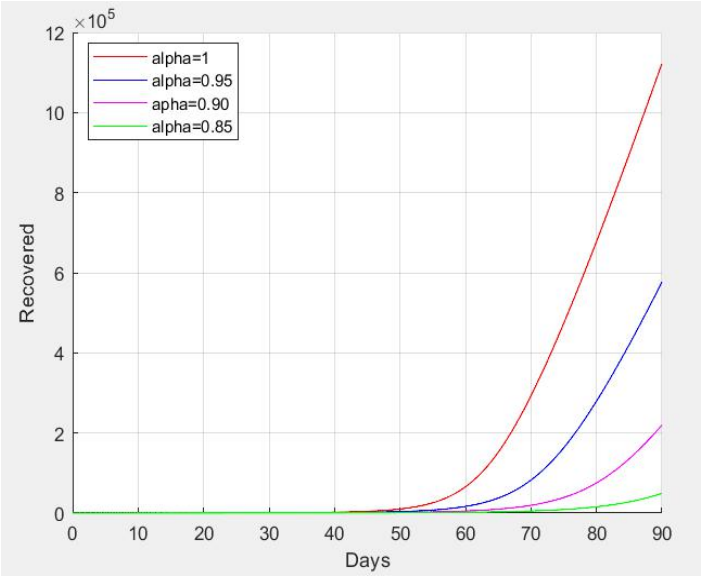

Figure 1c: Recovered

Figure 1c depicts the variation of the recovered population over time. We observed that after a certain time, the rate of recovered population started to increase with increased time. The peak of the recovered population for $p=1$ is higher than that for other values of p.

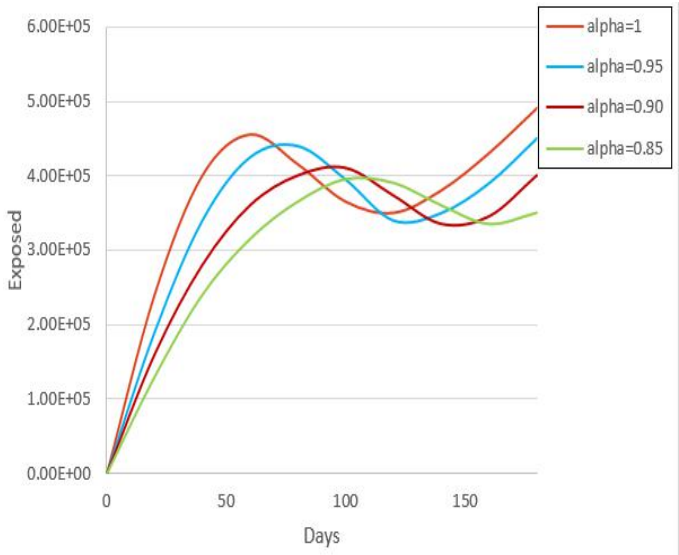

Figure 1d: Exposed

In Figure 1d depicts that, the behavior of the exposed individuals with respect to time. They start rising from the initial days of our observation. When $p=1$ changes, not only the peak of the infection attained very fast, but also the decline was also reported quickly.

Case 2: Brazil

In this portion, we present the effect of the COVID-19 pandemic on the Brazilian population. The information provided by WHO of the reported cases of COVID-19 in Brazil from 5th December 2021 to 5th March 2022[19], the estimated parametric values are as follows:

Table 2

Parameter	Value	Reference
λ	8067	Estimated
β	0.0000006	Estimated
γ	0.08	Estimated
k	0.002	Estimated
μ	0.006	Estimated
R_0	$2.3451 > 1$	Estimated

In figure 2, we investigate the impact of the transmission rate of 90 days on various populations for different values of fractional parameter p .

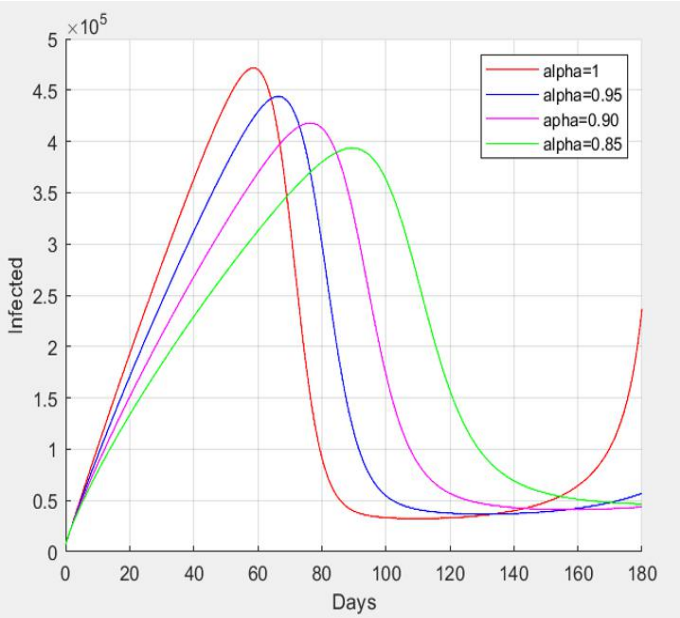

Figure 2a: Infected

In Figure 2a, we observe that an increase in the rate of disease transmission speeds up the increase of infection and attends its peak after 52 days from the starting date of our observation i.e, 5th December 2021 for $p=1$. Moreover, the attend peak for $p=1$ is significantly higher than that for other values of p.

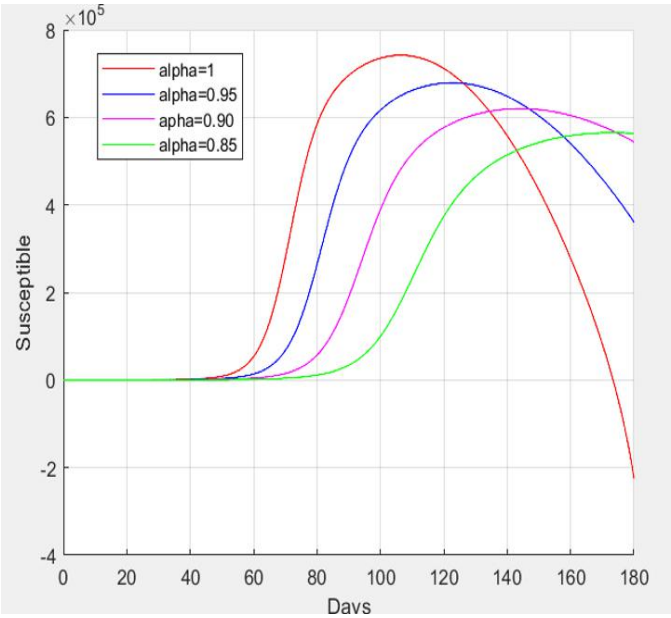

Figure 2b: Susceptible

In Figure 2b, the susceptible population is increasing with the increment of time after a certain period of our observation. After that point, even though the infection reduces, the susceptible also shows a declining trend. As the values of p decline from 1 to 0.85, the maximum number of susceptible populations is getting lower.

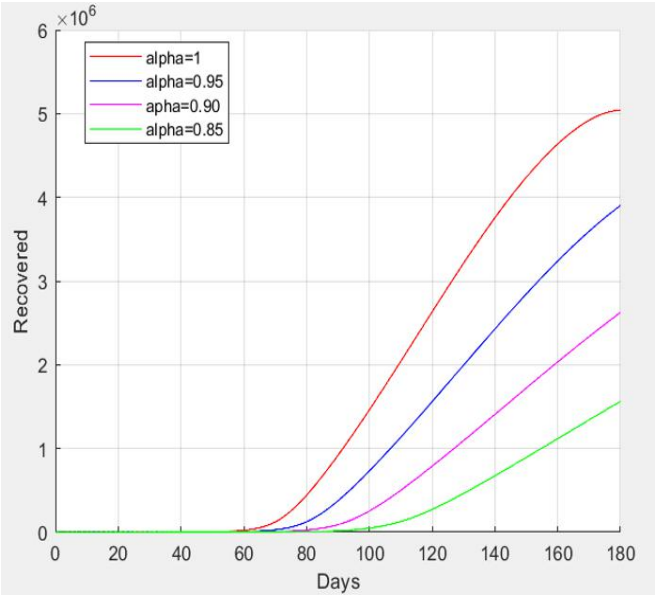

Figure 2c: Recovered

Figure 2c shows that the obtained plot for different values of fractional order parameter are different in quantity though they have same behavior. It is also observed that by decreasing the fractional order parameter $p = 1, 0.95, 0.90, 0.85$, the population of recovered individuals also decreases.

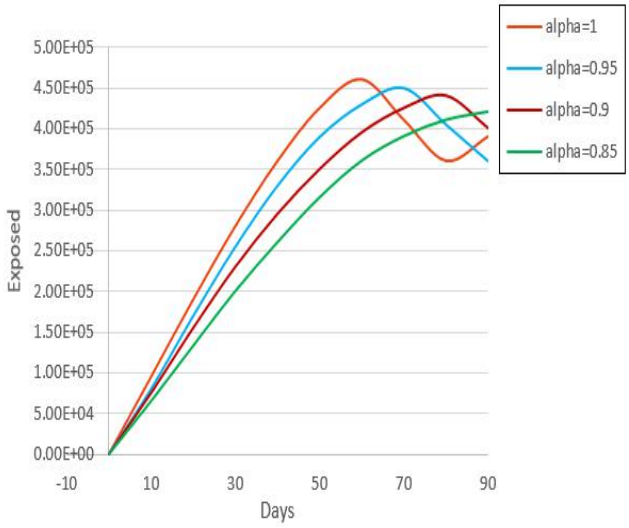

Figure 2d: Recovered

In Figure 2d shows that the rate of exposed individuals increases with the increment of time and attends its peak after a certain period for $p=1$. Moreover, the attend peak for $p=1$ is significantly higher than that for others values of α.

VII. NUMERICAL APPLICATION FOR REAL-WORLD DATA:

In this section, we predict the COVID-19 situation in India and Brazil after the third wave (late December 2021 to late February 2022) using the system (3.3).

We collect the average data of 7 days of active infected of India and Brazil. For India, we use the data of a certain period i.e, from 1st December 2021 to 5th June 2022 from the official website [20]. And for Brazil, the data was collected from 5th December 2021 to 5th March 2022 [19].

In this session we use MATLAB-21a software to draw the prediction graph for India in Figure 3 and for Brazil in Figure 4 and fit the real data with our system of the model (3.3) by taking the parameter set for India and Brazil from table 1 and table 2 respectively.

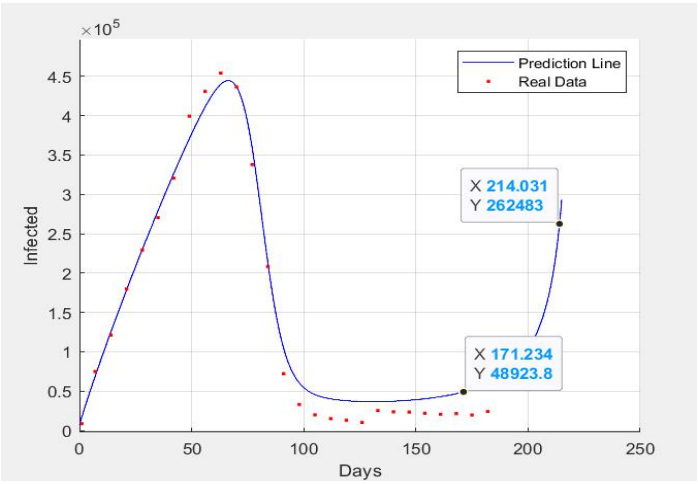

Figure 3: Future prediction of daily active infected of COVID-19 cases in India

In Figure 3, the prediction graph is drawn up to July 2022 for India and we see that the number of daily active infected will be started increasing at 2nd week of June, 2022 and at 1st week of July, 2022, the number of active infected will be approximately 262000. From this graph, we can predict that the fourth wave will be started on 2nd week of July 2022 and which was actually very close to the real figure.

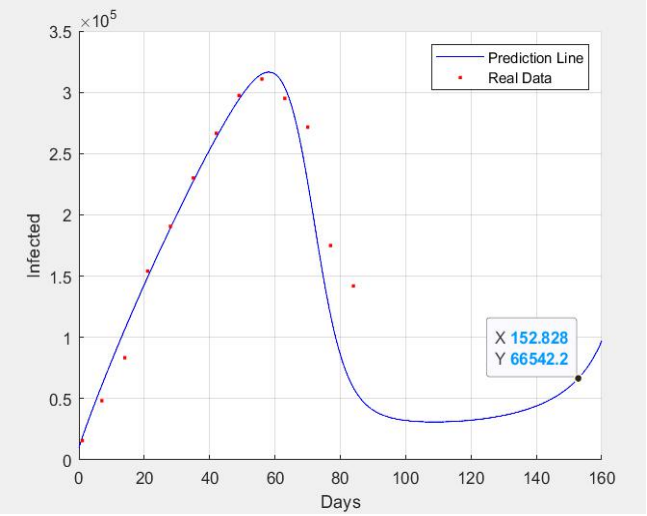

Figure 4: Future prediction of daily active infected of COVID-19 cases in Brazil

Figure 4 shows that the prediction graph of Brazil is drawn up to 2nd week of May 2022, when the number of daily active infected is started increasing again, the number of infected would be approximately 66500. So we can predict the daily infected of any day between this period of time.

VIII. CONCLUSION:

In this work, the Susceptible-Exposed-Infection-Recovered (SEIR) epidemic model for the transmission of COVID-19 using the Caputo-fractional derivative has been presented.The fractional approach is much better than the integer order model because it makes the good fitting with real data and the fractional order p works as a control parameter on the epidemic model. We have used the Adams-Bashforth-Moulton predictor-corrector technique to obtain the numerical solution to our proposed problem. Based on the COVID-19 cases in India, we collected the data from 1st December 2021 to 5th June 2022. For Brazil, the data was collected from 5th December 2021 to 5th March 2022. We estimated the essential parameters namely effective contact rate (β), birth rate of the susceptible (λ), mortality rate (μ), recovery rate (γ) and progression rate exposed to infected (k) are shown in table 1 and table 2 for India and Brazil respectively. Also in the numerical session, we have examined the effectiveness of using the fractional order derivative instead of integral order one. The numerical analysis of our investigation shows that the incidence of disease transmission needs to be controlled to prevent outbreaks.

Lastly we have draw a prediction graph of daily active infected for India and Brazil after third wave. For India, we draw the prediction graph from 1st December 2021 to 1st week of July 2022, i.e., of 6 months with help of the existing data from 1st December 2021 to 5th June 2022. From this prediction graph, we can conclude that if the present situations remain same, there will be a chance of fourth wave started at the 2nd week of July 2022 in India and in Brazil we observed that the infected cases are also arising in the middle week of May 2022 with the help of 3 months existing data i.e, from 5th December 2021 to 5th March 2022. As compared to the real data, we can say in near future the effect of COVID-19 is not as horrible as in the year 2020-2021 because of the success of vaccination throughout the world and the advances in health science.

IX. DATA AVAILABILITY:

For graphical representation, we collected our data from 'https://www.covid19india.org/2020'for India and https://www.who.int/emergencies/diseases/novel-coronavirus-2019 for Brazil.

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