IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 13, Issue 4, April 2023, ||Series -I|| PP 29-33

Critical Velocities of Axially Exponentially Graded Long Pipes Conveying Fluid Resting on Winkler Foundation

Chellapilla Kameswara Rao
Director, Innovations and Inventions Hub, Ground Floor, Below ICICI Bank, Akkayyapalem Branch, Vis*akhapatnam, Andhra Pradesh-530016,*
Received 25 April 2023; Accepted 08 May 2023

ABSTRACT
The present paper deals with the problem of computing critical velocities of axially exponentially graded fluid-conveying long pipes resting on a Winkler foundation with simply supported boundary conditions. Generally long pipes are used extensively in fluid transporters, mass power plant fluid delivery pipe systems. The velocity of the fluid in the pipe imparts energy to the piping system making it to vibrate. It is well established from published literature that there exists a critical velocity of the conveying fluid near which the natural frequency of the long pipes tends to zero. This is the required condition for buckling of the long pipes. Literature abounds with analyses, which give information on the influence of boundary conditions on the stability of long pipes conveying fluids.
However, much of these studies have been carried out for classical long pipe resting on Winkler type of continuous foundations. This has provided the motivation for studying the critical velocities of axially exponentially graded fluid-conveying long pipes resting on Winkler foundation as an essential and useful case worth investigating. The foundation considered in this study is a one-parameter Winkler foundation model. Expression is derived for the critical flow velocity by utilizing Galerkin's method for the simply supported boundary conditions. The effect of varying values of the exponential graded material factor and the Winkler foundation parameter on the critical flow velocity of the long pipe can be easily drawn from the closed-form expression derived in this paper for the case of long pipe simply supported at both the ends.
Keywords: Critical velocity, Exponentially Graded long pipes, Winkler foundation.

I. INTRODUCTION

It is a well-known fact that the velocity of the fluid has considerable effect on the natural frequencies of the straight pipe conveying the fluid. There have been many investigations on the effect of flow velocity on the natural frequencies of straight pipes conveying fluid with simple boundary conditions such as hinged-hinged, hinged-fixed and fixed-fixed. These boundary conditions are idealised cases and are difficult to realise practically. A more realistic boundary condition is the rotational restraint at both ends of the pipe. However, results are not available for rotationally restrained straight Bernoulli-Euler fluid conveying pipes resting on soil medium in the published literature.

Since 1947, many investigations [1 - 7] have been carried out for studying the vibration behaviour of fluid conveying pipes utilising various methods such as Galerkin, Rayleigh-Ritz and Fourier series solutions. Stein and Tobriner[8], Dermendjian-Ivanova[9]and Raghava Chary, Kameswara Rao and Iyengar[10], have devoted their studies on fluid conveying pipes resting on elastic foundation.

As can be seen from the above discussion, many methods have been utilised by investigators in solving this problem, but the finite element method was utilised by only a few investigators such as Kohli and Nakra[11]and Pramila, Laukkanen and Liukkonen[12]. However, the effect of foundation modulus is not included in both these studies. In earlier papers [13], the present authors have presented numerical results for the first three natural frequencies obtained from the mass and stiffness matrices developed for straight Bernoulli-Euler fluid conveying pipes resting on soil medium. These results were however are for the three classical boundary conditions of hinged-hinged, hinged-fixed, fixed-fixed and in the recent paper [14], the case of guided-guided conditions is dealt with.

The present paper deals with development of a finite element analysis for rotationally restrained long exponentially graded long pipes with internal flow and resting on Winkler foundation. Different values of pipe rotational restraint parameter are considered in generating results for the natural frequency of the piping system with variations in the non-dimensional parameters defining the flow velocity and foundation stiffness. For extreme values of the rotational restraint parameter, the results are in good agreement with those available in published literature. The effects of soil inertia along with shear deformation are assumed to be neglisible in long pipes and hence not considered in the present study.

QUATIONS OF MOTION AND SOLUTION

Mathematical model of the exponentially guided pipe conveying fluid

To derive the equation of motion by the Newtonian approach, we first consider the beam equations,

$$\epsilon_{xx} = z\frac{\partial^2 \sigma_{xx}}{\partial x^2}; M_b(x,t) = \int_A z\,\sigma_{xx} dA; \int_A z^2\, dA = I; Q = -\frac{\partial M_b}{\partial x} \qquad (1)$$

where, M_b(x) is the bending moment, Q (x) is the shearing force. Using the Bending Moment Equation in (1) and considering the pipe material to be exponentially graded [18], we have

$$M_b(x,t) = E(x) I \frac{\partial^2 w}{\partial x^2} \text{ where } E(x) = E_0\, e^{2\beta(x/l)} \qquad (2)$$

Differentiation Equation (5) twice w.r.t. X

$$\frac{\partial M_b}{\partial x} = E_0\, e^{2\beta(x/l)} I \frac{\partial^3 w}{\partial x^3} + 2\beta E_0\, e^{2\beta(x/l)} I \frac{\partial^2 w}{\partial x^2} \qquad (3a)$$

$$\frac{\partial^2 M_b}{\partial x^2} = E_0\, e^{2\beta(x/l)} I \frac{\partial^4 w}{\partial x^4} + 4\beta E_0\, e^{2\beta(x/l)} I \frac{\partial^3 w}{\partial x^3} + 4\beta^2\, E_0\, e^{2\beta(x/l)} I \frac{\partial^2 w}{\partial x^2} \qquad (3b)$$

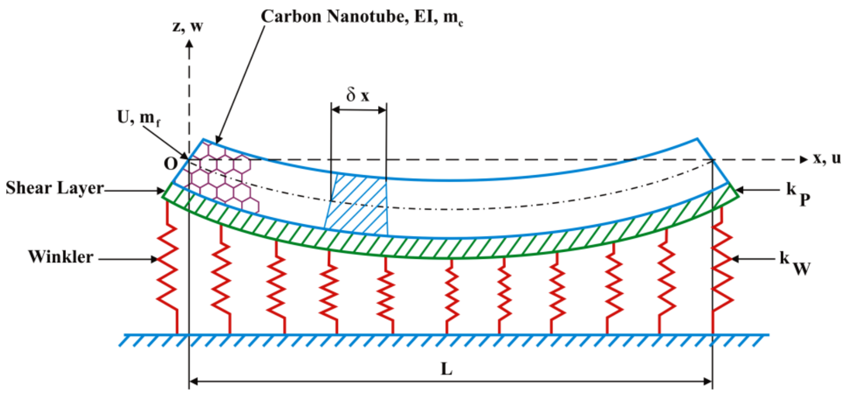

Figure 1 Model of a Pipe Conveying Fluid and Embedded in elastic medium modeled as a Two-Parameter Winkler-Pasternak Foundation (in *this study* k_p the Pasternak foundation factor is taken as negligibly small).

The equation of motion for an exponentially guided fluid conveying long pipe, embedded in a Winkler foundation medium has been derived earlier by many researchers as

$$-\frac{\partial^2 M_b}{\partial x^2} = -R_P + m_f a_{fz} \qquad (4)$$

In the above equation, the inertial force due to the SWCNT element acceleration in z-direction is given as $m_c a_{cz}$. The inertia forces due to the fluid acceleration in the z-direction, $m_f a_{fz}$. The fluid flow is considered to be a simple plug flow.

Long Pipe and fluid acceleration terms in Equation (7) have been derived by many researchers (see [8] for details)

$$a_{fz} = U^2 \frac{\partial^2 w}{\partial x^2} \qquad (5)$$

Substituting Equation (6) in Equation (7), using Equations (8) and Equation (3) and rearranging, the final non-local equation of motion for an exponentially graded fluid conveying long pipe is obtained:

$$E_0\, I\, e^{2\beta x}\left(\frac{\partial^4 w}{\partial x^4} + 4\beta \frac{\partial^3 w}{\partial x^3} + 4\beta^2 \frac{\partial^2 w}{\partial x^2} + M \frac{\partial^2 w}{\partial t^2} + (m_f U^2 - {k_P}^2)\right) \frac{\partial^2 w}{\partial x^2} + 2 m_f U \frac{\partial^2 w}{\partial x \partial t} + k_W w = 0$$

For natural frequency of the fluid conveying long pipe to become zero, and in order to obtain the critical velocity of the fluid, we need to solve the following differential equation:

$$E_0\, I\, e^{2\beta x}\left(\frac{\partial^4 w}{\partial x^4} + 4\beta \frac{\partial^3 w}{\partial x^3} + 4\beta^2 \frac{\partial^2 w}{\partial x^2}\right) + (m_f U^2 - {k_P}^2) \frac{\partial^2 w}{\partial x^2} + k_W \mathrm{w} = 0 \qquad (6)$$

Solution for the Simply Supported pipe case:

The solution of Equation (6) is considered as

$$w(x) = A\ \sin\frac{\pi x}{L} \qquad (7)$$

where Equation (7) must satisfy the boundary conditions represented in Equations (8a) and (8b) given below:

$$w(0) = w(L) = 0 \qquad (8a)$$

$$\frac{\partial^2 w(0)}{\partial x^2} = \frac{\partial^2 w(L)}{\partial x^2} = 0 \qquad (8b)$$

By substituting Equation (7) in Equation (6) an expression for computing the flow velocity parameter, called the critical flow velocity parameter, V_{cr} can be derived as follows:

$$V_{c\,r}{}^2 = \left[e^{2(\beta l)}\left(\pi^2 - 4(\beta l)\pi - 4(\beta l)^2\right) + \left(\frac{\gamma_W^2}{\pi^2}\right)\right] \qquad (9)$$

In Equation (13), the following non-dimensional parameters have been used:

$$V = UL\sqrt{\frac{m_f}{E_0\, I}};\ \gamma_W^2 = \frac{k_W L^4}{E_0\, I} \qquad (10)$$

We can obtain the critical flow velocity, $V_{c\,r}\ for$ the simply supported case by solving Equation (9) for various values of exponential grade parameter (βl) and Winkler foundation parameter γ_W.

II. CONCLUSIONS

This study has attempted to address the gaps in the literature by presenting the closed-form expression for computing the critical velocity of a exponentially graded fluid conveying long pipes, embedded in a one-parameter elastic medium like the Winkler foundation. The governing equations have been formulated in the present paper duly taking into account the concept of exponentially graded material property variation using the Galerkin's method.

It has been very well established that when the flow velocity reaches a certain value, called the critical flow velocity, the frequency becomes zero, leading to instability. Simple closed-form expression derived in this paper for the critical velocity parameter is presented here for the first time. The critical flow velocity parameter obtained for both the simply-supported exponentially graded fluid conveying long pipe, embedded in a Winkler-type elastic foundation medium are presented in explicit form in this paper.

References

[1]. J. Yoon, C.Q. Ru, and A. Mioduchowski, Vibration and instability of carbon nanotubes conveying fluid, Composites Science and Technology, 65, 1326–1336 (2005).

[2]. C.D. Reddy,C.Lu, S.Rajendran, S. and K.M. Liew, Free Vibration Analysis of Fluid-conveying Single-walled Carbon Nanotubes, Applied Physics Letters, 90, 133122 (2007).

[3]. W.J. Chang, and H.L. Lee, Free Vibration of a Single-walled Carbon Nanotube Containing a Fluid Flow using the Timoshenko Beam Model, Physics Letters A, 373, 982-985(2009).

[4]. A.C. Eringen, andG.B. Edelen, On Nonlocal Elasticity, International Journal of Engineering Science, 10, 233-248 (1972).

[5]. A.C. Eringen, Nonlocal Continuum Field Theories, Springer-Verlag, NY (2002).

[6]. H.L. Lee, and W.J. Chang, Free Transverse Vibration of the Fluid-conveying Single-walled Carbon Nanotube Using Nonlocal Elastic Theory, Journal of Applied Physics, 103, 24302 (2008).

[7]. H.L. Lee, and W.J. Chang, Vibration Analysis of a Viscous Fluid-conveying Single-walled Carbon Nanotube Embedded in an Elastic Medium, Physica E, 41, 529-532 (2009).

[8]. H.S. Simha, Vibrations and Stability of Fluid Conveying Pipes Resting on Elastic Media, *Ph. D. Thesis*, Department of Mechanical Engineering, Osmania University, Hyderabad, India (2013).

[9]. A. Tounsi, H.Heireche, and E.A.A. Bedia, Comment on 'Free transverse vibration of the fluid-conveying single-walled carbon nanotube using nonlocal elastic theory, Journal of Applied Physics, 105, 126105 (2009).

[10]. L. Wang, Vibration and instability analysis of tubular nano- and micro-beams conveying fluid using nonlocal elastic theory, Physica E, 41**,** 1835–1840 (2009).

[11]. A. Farshidianfar, A.A. Ghassabi, and M.H. Farshidianfar, Transverse vibration of fluid conveying carbon nanotubes embedded in two-parameter elastic medium, Proceedings of the18th International Congress on Sound and Vibration, Rio de Janeiro, Brazil (2011).

[12]. Feng Liang and Bao Ridong, Stability Analysis of a Fluid-conveying Carbon Nanotube with consideration of Nonlocal and Surface Effects, (In Chinese), Mechanics in Engineering, 36(1) 48-53 (2014).

[13]. A.D. Kerr, Elastic and Viscoelastic Foundation Model, Transactions of the ASME, Journal of Applied Mechanics, September, 491-498 (1964).

[14]. Chellapilla Kameswara Rao and H.S. Simha, Critical Velocity of fluid conveying pipes resting on two-parameter foundation, Journal of Sound and Vibration, 302, 387-397 (2007).

For $(\beta l) = 0, the\ values\ of\ \ {V_c}_r{}^2$ are given in Table 1

Table 1 – Values of the critical velocity parameter for various values of g1 and g2 for the three boundary conditions

Pinned-pinned pipe			Pinned-fixed Pipe			Fixed-fixed pipe		
g1	g2	VCr	g1	g2	VCr	g1	g2	VCr
1.00E-06	1.00E-06	3.14159	1.00E-06	1.00E-06	4.49975	1.00E-06	1.00E-06	5.43433
1.00E-05	1.00E-06	3.14159	1.00E-05	1.00E-06	4.49975	1.00E-05	1.00E-06	5.43433
1.00E+02	1.00E-06	4.47233	1.00E+02	1.00E-06	5.32942	1.00E+02	1.00E-06	5.95246
1.00E+03	1.00E-06	8.05039	1.00E+03	1.00E-06	8.66386	1.00E+03	1.00E-06	9.40901
1.00E+04	1.00E-06	17.11086	1.00E+04	1.00E-06	16.91955	1.00E+04	1.00E-06	16.00158
1.00E-06	1.00E-05	3.14159	1.00E-06	1.00E-05	4.49975	1.00E-06	1.00E-05	5.43433
1.00E-05	1.00E-05	3.14159	1.00E-05	1.00E-05	4.49975	1.00E-05	1.00E-05	5.43433
1.00E+02	1.00E-05	4.47233	1.00E+02	1.00E-05	5.32942	1.00E+02	1.00E-05	5.95246
1.00E+03	1.00E-05	8.05039	1.00E+03	1.00E-05	8.66386	1.00E+03	1.00E-05	9.40901
1.00E+04	1.00E-05	17.11086	1.00E+04	1.00E-05	16.91955	1.00E+04	1.00E-05	16.00158
1.00E-06	1.00E+01	4.45753	1.00E-06	1.00E+01	5.4998	1.00E-06	1.00E+01	6.28745
1.00E-05	1.00E+01	4.45753	1.00E-05	1.00E+01	5.4998	1.00E-05	1.00E+01	6.28745
1.00E+02	1.00E+01	5.47738	1.00E+02	1.00E+01	6.19699	1.00E+02	1.00E+01	6.74031
1.00E+03	1.00E+01	8.6492	1.00E+03	1.00E+01	9.22293	1.00E+03	1.00E+01	9.9262
1.00E+04	1.00E+01	17.40061	1.00E+04	1.00E+01	17.21253	1.00E+04	1.00E+01	16.31105
1.00E-06	1.00E+02	10.48187	1.00E-06	1.00E+02	10.96575	1.00E-06	1.00E+02	11.38121
1.00E-05	1.00E+02	10.48187	1.00E-05	1.00E+02	10.96575	1.00E-05	1.00E+02	11.38121
1.00E+02	1.00E+02	10.95453	1.00E+02	1.00E+02	11.33149	1.00E+02	1.00E+02	11.63751
1.00E+03	1.00E+02	12.83778	1.00E+03	1.00E+02	13.23111	1.00E+03	1.00E+02	13.7306
1.00E+04	1.00E+02	19.81871	1.00E+04	1.00E+02	19.65379	1.00E+04	1.00E+02	18.8693
1.00E-06	1.00E+03	31.77845	1.00E-06	1.00E+03	31.94132	1.00E-06	1.00E+03	32.08632
1.00E-05	1.00E+03	31.77845	1.00E-05	1.00E+03	31.94132	1.00E-05	1.00E+03	32.08632
1.00E+02	1.00E+03	31.93747	1.00E+02	1.00E+03	32.06872	1.00E+02	1.00E+03	32.17812
1.00E+03	1.00E+03	32.63141	1.00E+03	1.00E+03	32.78814	1.00E+03	1.00E+03	32.99287
1.00E+04	1.00E+03	35.95527	1.00E+04	1.00E+03	35.86462	1.00E+04	1.00E+03	35.4408
1.00E-06	5.00E+03	70.78043	1.00E-06	5.00E+03	70.85371	1.00E-06	5.00E+03	70.91919
1.00E-05	5.00E+03	70.78043	1.00E-05	5.00E+03	70.85371	1.00E-05	5.00E+03	70.91919
1.00E+02	5.00E+03	70.85197	1.00E+02	5.00E+03	70.91123	1.00E+02	5.00E+03	70.96078
1.00E+03	5.00E+03	71.16747	1.00E+03	5.00E+03	71.23947	1.00E+03	5.00E+03	71.33393
1.00E+04	5.00E+03	72.7515	1.00E+04	5.00E+03	72.70675	1.00E+04	5.00E+03	72.49862
1.00E-06	8.00E+03	89.49787	1.00E-06	8.00E+03	89.55584	1.00E-06	8.00E+03	89.60766
1.00E-05	8.00E+03	89.49787	1.00E-05	8.00E+03	89.55584	1.00E-05	8.00E+03	89.60766
1.00E+02	8.00E+03	89.55446	1.00E+02	8.00E+03	89.60135	1.00E+02	8.00E+03	89.64057

1.00E+03 8.00E+03 89.80428 1.00E+03 8.00E+03 89.86135 1.00E+03 8.00E+03 89.93625
1.00E+04 8.00E+03 91.06471 1.00E+04 8.00E+03 91.02896 1.00E+04 8.00E+03 90.86281

1.00E-06 1.00E+04 100.0493 1.00E-06 1.00E+04 100.1012 1.00E-06 1.00E+04 100.1476
1.00E-05 1.00E+04 100.0493 1.00E-05 1.00E+04 100.1012 1.00E-05 1.00E+04 100.1476

1.00E+02 1.00E+04 100.1 1.00E+02 1.00E+04 100.1419 1.00E+02 1.00E+04 100.177
1.00E+03 1.00E+04 100.3235 1.00E+03 1.00E+04 100.3746 1.00E+03 1.00E+04 100.4417

1.00E+04 1.00E+04 101.4534 1.00E+04 1.00E+04 101.4213 1.00E+04 1.00E+04 101.2722

Chellapilla Kameswara Rao. "Critical Velocities of Axially Exponentially Graded Long Pipes Conveying Fluid Resting on Winkler Foundation." *IOSR Journal of Engineering (IOSRJEN),* 13(4), 2023, pp. 29-33.