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Longitudinal Vibrations In A Rotating Micropolar Elastic Solid Having A Cylindrical Hole

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Abstract: - In this paper, the effect of rotation on longitudinal vibrations in a micro polar elastic solid with a cylindrical hole have been studied. The frequency equation for longitudinal vibrations is derived. Micro rotations, radial and axial displacements connecting angular rotations are derived interms of Bessel polynomials for symmetric and skew-symmetric modes. For investigating the rotation effect, the numerical example is considered and using MATLAB programme the displacements are plotted against the wave number of the solid.

Keywords: - *Cylindrical hole, Longitudinal vibrations, Micro polar elastic solid, Rotation*

I. INTRODUCTION

The rotation effect on longitudinal vibrations in a cylindrical hole solids are huge important due to its manifold applications. Cylindrical holes are treated as bore hole or a minegallery. Generally bore hole studies are applicable in investigation of gas exploration, hydrocarbons and oils. Chree [1] and Pochammer [2] are studied the longitudinal wave propagation in a classical theory of elasticity. Bancroft [3] derived the velocity equation for longitudinal waves in cylindrical bars. Wave propagation in a metal walled liquid cylinder was studied by Lamb [4] . Many researchers like S.R.Mahmoud [5], Vashishth and Khurana [6], Kumar and Deswal [7], Arora and Tomar [8] are studied the cylindrical bore problems. Elastic waves in rotating solids was discussed by some author’s named as Clarke and Burdess [9], Saderkvist [10] etc. Longitudinal waves in a rotating elastic metarial which having a cylindrical hole was investigated by Sreelakshmi, Rama and Somaiah et.al. [11]. In this article, the effect of rotation on longitudinal vibrations in a micro polar elastic solid having a cylindrical hole is studied interms of Bessel functions. Using appropriate boundary conditions, the dispersion equation connecting with angular velocity of the solid is derived.

II. BASIC EQUATIONS

The equations of macro displacement vector $\vec{u}$, micro rotational vector $\vec{\Phi}$ and the constitutive relations under the absence of body forces and body couples for rotating medium are presented by [12] as:

$$(\lambda+2\mu+K)\nabla(\nabla\cdot\vec{u})-(\mu+K)\nabla\times\nabla\times\vec{u}+K\nabla\times\vec{\phi}=\rho\left[\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})\right] \quad (1)$$

$$(\alpha+\beta+\gamma)\nabla(\nabla\cdot\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+K\nabla\times\vec{u}-2K\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2} \quad (2)$$

$$t_{kl}=\lambda u_{r,r}\delta_{kl}+\mu(u_{k,l}+u_{l,k})+K(u_{l,k}-\varepsilon_{klr}\phi_r) \quad (3)$$

$$m_{kl}=\alpha\phi_{r,r}\delta_{kl}+\beta\phi_{k,l}+\gamma\phi_{l,k} \quad (4)$$

where $\vec{u}=(u_r,u_\theta,u_z)$ is the macro displacement vector, $\vec{\phi}=(\phi_r,\phi_\theta,\phi_z)$ is the micro rotation vector $\lambda,\mu,\alpha,\beta,\gamma,K$ are material constants, ρ is the density, j is the micro inertia, t_{kl} and m_{kl} are respectively the components of force stress and couple stress, δ_{kl} is the Kronecker's delta and $\vec{\Omega}$ is angular velocity of the solid.

III. FORMULATION OF THE PROBLEM AND ITS SOLUTIONS

We consider an infinite homogeneous elastic solid with a cylindrical hole which having a circular cross section of radius $r = d$. Let us consider the cylindrical coordinate system of coordinate point (r, θ, z). The z -axis is taken along the cylindrical hole.

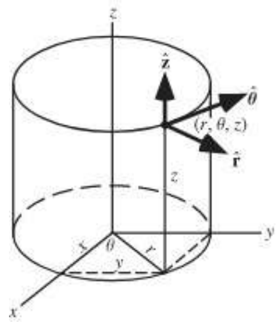

Figure1: Geometry of the Problem

Choose that the continuum is rotating about z -axis with a constant rate of angular velocity $\vec{\Omega} = (0, 0, \Omega)$. For axial symmetry all the displacements and stresses are independent of θ. The macro displacement vector $\vec{u}$ and the micro-rotation vector Φ taken as $\vec{u} = (U_r, 0, U_z); \vec{\phi} = (0, \Phi_\theta, 0)$ *where* U_r, U_z and Φ_θ are functions of r, z and t.

With these assumptions the equations of motion (1) and (2) reduces to

$$\left(\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r}\frac{\partial U_r}{\partial r} + \frac{\partial^2 U_z}{\partial z \partial r} - \frac{U_r}{r^2}\right) + \varepsilon_1\left(\frac{\partial^2 U_z}{\partial r \partial z} - \frac{\partial^2 U_r}{\partial z^2}\right) + \varepsilon_2 \frac{\partial \Phi_\theta}{\partial z} = \varepsilon_3\left(\frac{\partial^2 U_r}{\partial t^2} - \Omega^2 U_r\right) \quad (5)$$

$$\left(\frac{\partial^2 U_r}{\partial z \partial r} + \frac{1}{r}\frac{\partial U_r}{\partial z} + \frac{\partial^2 U_z}{\partial z^2}\right) + \varepsilon_1\left(\frac{\partial^2 U_r}{\partial r \partial z} - \frac{\partial^2 U_z}{\partial r^2}\right) - \varepsilon_2 \frac{\partial \Phi_\theta}{\partial r} = \varepsilon_3\left(\frac{\partial^2 U_z}{\partial t^2}\right) \quad (6)$$

$$\left(\nabla^2 - \frac{1}{r^2}\right)\Phi_\theta + \delta\left(\frac{\partial U_r}{\partial z} - \frac{\partial U_z}{\partial r}\right) - 2\delta\Phi_\theta = \delta_1 \frac{\partial^2 \Phi_\theta}{\partial t^2} \quad (7)$$

where

$$\varepsilon_1 == \frac{-(\mu + K)}{(\lambda + 2\mu + K)}; \varepsilon_2 = \frac{-K}{(\lambda + 2\mu + K)}; \varepsilon_3 = \frac{\rho}{(\lambda + 2\mu + K)}$$
$$\delta = \frac{K}{\gamma}; \delta_1 = \frac{\rho j}{\gamma}; \nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad (8)$$

$$t_{rr} = (\lambda + 2\mu + K)\frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \lambda \frac{\partial U_z}{\partial z} \quad (9)$$

$$t_{rz} = \mu\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right) + K\left(\frac{\partial U_r}{\partial z} - \Phi_\theta\right) \quad (10)$$

$$m_{r\theta} = \gamma \frac{\partial \Phi_\theta}{\partial r} \quad (11)$$

Taking the potential functions Φ and Ψ in equations (5) and (7) of the form

$$U_r = \Phi_{,r} + \Psi_{,z}; U_z = \Phi_{,z} - \Psi_{,r} - r^{-1}\Psi$$

We obtain

$$\nabla^2 \Phi = \varepsilon_3\left[\frac{\partial^2 \Phi}{\partial t^2} - \frac{\Omega^2}{2}\Phi\right] \quad (12)$$

$$\varepsilon_1\left(\nabla^2\psi-\frac{\psi}{r^2}\right)-\varepsilon_2\Phi_\theta=\varepsilon_3\left[\frac{\Omega^2}{2}\psi-\frac{\partial^2\psi}{\partial t^2}\right] \tag{13}$$

And

$$\left[\nabla^2\Phi_\theta-\frac{\Phi_\theta}{r^2}\right]+\delta\left(\nabla^2\psi-\frac{\psi}{r^2}\right)-2\delta\Phi_\theta=\delta_1\frac{\partial^2\Phi_\theta}{\partial t^2} \tag{14}$$

The solutions of the equations (12) to (14) in the following passion

$$\left(\Phi,\Psi,\Phi_\theta\right)=\left[F(z)J_0(kr),G(z)J_1(kr),H(z)J_1(kr)\right]e^{-i\omega t} \tag{15}$$

where ω is the angular frequency, k is the wave number, $J_0(kr)$, $J_1(kr)$ are respectively the Bessel functions of order zero and one.

On using equation (15) in equation (12), we obtain the following differential equation

$$F''(z)+m^2F(z)=0 \tag{16}$$

Where

$$m^2=k^2\frac{J_0''(kr)}{J_0(kr)}+\frac{k}{r}\frac{J_0'(kr)}{J_0(kr)}+\varepsilon_3\left(\omega^2+\frac{\Omega^2}{2}\right) \tag{17}$$

the solution of equation (16) is,

$$F(z)=A\cos mz+B\sin mz$$

Therefore,

$$\Phi(z,r,t)=\left[A\cos mz+B\sin mz\right]J_0(kr)e^{-i\omega t} \tag{18}$$

Using equation (15) in equation (13) and (14) we obtain,

$$\left(D^2+X_1\right)G(z)-X_2H(z)=0 \tag{19}$$

Where

$$D^2\equiv\frac{\partial^2}{\partial z^2};\ X_2=\frac{\varepsilon_2}{\varepsilon_1}=\frac{K}{\mu+K};$$

$$X_1=k^2\frac{J_1''(kr)}{J_1(kr)}+\frac{k}{r}\frac{J_1'(kr)}{J_1(kr)}-\frac{1}{r^2}+\frac{\varepsilon_3}{\varepsilon_1}\left(\frac{\Omega^2}{2}+\omega^2\right) \tag{20}$$

$$\frac{\varepsilon_3}{\varepsilon_1}=\frac{-\rho}{(\mu+K)}$$

and $\left(D^2+X_1\right)G(z)+\left[\delta^{-1}D^2+X_3\right]H(z)=0$ (21)

where

$$X_3=\left(X_1+\delta_1\omega^2\right)\delta^{-1}-2 \tag{22}$$

Solving simultaneous equations (19) and (21) we get

$$\left(D^4 + aD^2 + b\right)G(z) = 0 \tag{23}$$

where

$$\mathrm{a} = \mathrm{X}_1 + \left(\mathrm{X}_1 + \mathrm{X}_2\right)\delta; \mathrm{b} = \left(\mathrm{X}_2 + \mathrm{X}_3\right)\mathrm{X}_1 \tag{24}$$

The solution of equation (23) is

$$G(z) = C\cos m_1 z + D\sin m_1 z + C^{'}\cosh m_2 z + D^{'}\sinh m_2 z \tag{25}$$

where

$$m_j = \left(\frac{a}{2} + \frac{1}{2}\sqrt{a^2 - 4b}\right); j = 1, 2$$

(or)

$$m_1 = \pm i\left(\frac{a}{2} + \frac{1}{2}\sqrt{a^2 - 4b}\right); m_2 = \pm\left(\frac{a}{2} + \frac{1}{2}\sqrt{a^2 - 4b}\right) \tag{26}$$

From equation (19);

$$H(z) = X_2^{-1}\left[\left(X_1 - m_1^2\right)\left(C\cos m_1 z + D\sin m_1 z\right) + \left(X_1 + m_2^2\right)\left(C^{'}\cosh m_2 z + D^{'}\sinh m_2 z\right)\right] \tag{27}$$

Therefore by equation (25) and (27) we obtain Ψ and Φ_θ by

$$\Psi = [C\cos m_1 z + D\sin m_1 z + C^{'}\cosh m_2 z + D^{'}\sinh m_2 z]J_1(kr)e^{(-i\omega t)} \tag{28}$$

and

$$\begin{aligned}\Phi_\theta = [&X_2^{-1}\left(X_1 - m_1^2\right)\left(C\cos m_1 z + D\sin m_1 z\right) + \\ &X_2^{-1}\left(X_1 + m_2^2\right)\left(C^{'}\cosh m_2 z + D^{'}\sinh m_2 z\right)]J_1(kr)e^{-i\omega t}\end{aligned} \tag{29}$$

By equation (18) and (28) now the displacement components U_r and U_z are given by

$$\begin{aligned}\mathrm{U}_r = [&-\left(A\cos mz + B\sin mz\right) + \left(Dm_1\cos m_1 z - Cm_1\sin m_1 z\right) + \\ &\left(C^{'}m_2\sinh m_2 z + D^{'}m_2\cosh m_2 z\right)]J_1(kr)e^{-i\omega t}\end{aligned} \tag{30}$$

$$\begin{aligned}U_z = [&\left(-Am\sin mz + Bm\cos mz\right) - \left(C\cos m_1 z + D\sin m_1 z\right) + \\ &\left(C^{'}\cosh m_2 z + D^{'}\sinh m_2 z\right)]J_0(kr)e^{-i\omega t}\end{aligned} \tag{31}$$

IV. BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

The non-dimensional mechanical boundary conditions at $r = d$ are stress free and rigidly fixed boundaries respectively given by

$$\begin{aligned} t_{rr} &= 0, t_{rz} = 0, m_{r\theta} = 0 \\ u_r &= 0, u_z = 0, \Phi_\theta = 0 \end{aligned} \tag{32}$$

On inserting equations (9), (10), (11), (30) and (31) in boundary conditions (32), we obtain the following matrix equation in A, B, C, D, C', D'

$$\left[a_{ij}\right]\begin{bmatrix} A \\ B \\ C \\ D \\ C' \\ D' \end{bmatrix} = 0; 1 \le i, j \le 6 \tag{33}$$

Where

$$a_{11} = -(\lambda+2\mu+K)\left[J_0(kd) - \frac{1}{d}J_1(kd)\right]\cos mz - \frac{1}{d}J_1(kd)\cos mz - \lambda m^2 J_0(kd)\cos mz$$

$$a_{12} = -(\lambda+2\mu+K)\left[J_0(kd) - \frac{1}{d}J_1(kd)\right]\sin mz - \frac{1}{d}J_1(kd)\sin mz - \lambda m^2 J_0(kd)\sin mz;$$

$$a_{13} = -(\lambda+2\mu+K)m_1\left[J_0(kd) - \frac{1}{d}J_1(kd)\right]\sin m_1 z - \frac{m_1}{d}J_1(kd)\sin m_1 z + \lambda m_1 J_0(kd)\sin m_1 z;$$

$$a_{14} = (\lambda+2\mu+K)m_1\left[J_0(kd) - \frac{1}{d}J_1(kd)\right]\cos m_1 z + \frac{m_1}{d}J_1(kd)\cos m_1 z - \lambda m_1 J_0(kd)\cos m_1 z;$$

$$a_{15} = (\lambda+2\mu+K)m_2\left[J_0(kd) - \frac{1}{d}J_1(kd)\right]\sinh m_2 z + \frac{m_2}{d}J_1(kd)\sinh m_2 z + \lambda m_2 J_0(kd)\sinh m_2 z;$$

$$a_{16} = (\lambda+2\mu+K)m_2\left[J_0(kd) - \frac{1}{d}J_1(kd)\right]\cosh m_2 z + \frac{m_2}{d}J_1(kd)\cosh m_2 z + \lambda m_2 J_0(kd)\cosh m_2 z;$$

$$a_{21} = (2\mu+K)mJ_1(kd)\sin mz;$$
$$a_{22} = KmJ_1(kd)\cos mz;$$
$$a_{23} = \left\{\mu\left(1-m_1^2\right) - K\left[m_1^2\left(1-X_2^{-1}\right) + X_2^{-1}X_1\right]\right\}J_1(kd)\cos m_1 z;$$
$$a_{24} = \left\{\mu\left(1-m_1^2\right) - K\left[m_1^2\left(1-X_2^{-1}\right) + X_2^{-1}X_1\right]\right\}J_1(kd)\sin m_1 z;$$
$$a_{25} = -\left\{\mu\left(1-m_2^2\right) - K\left[m_2^2\left(1-X_2^{-1}\right) - X_2^{-1}X_1\right]\right\}J_1(kd)\cosh m_2 z;$$
$$a_{26} = -\left\{\mu\left(1-m_2^2\right) - K\left[m_2^2\left(1-X_2^{-1}\right) - X_2^{-1}X_1\right]\right\}J_1(kd)\sinh m_2 z;$$

$$a_{31} = a_{32} = 0;$$
$$a_{33} = \gamma X_2^{-1}\left(X_1 - m_1^2\right)\left[J_0\left(kd\right) - \frac{1}{d}J_1\left(kd\right)\right]\cos m_1 z;$$
$$a_{34} = \gamma X_2^{-1}\left(X_1 - m_1^2\right)\left[J_0\left(kd\right) - \frac{1}{d}J_1\left(kd\right)\right]\sin m_1 z;$$
$$a_{35} = \gamma X_2^{-1}\left(X_1 + m_1^2\right)\left[J_0\left(kd\right) - \frac{1}{d}J_1\left(kd\right)\right]\cos m_2 z;$$
$$a_{36} = \gamma X_2^{-1}\left(X_1 + m_1^2\right)\left[J_0\left(kd\right) - \frac{1}{d}J_1\left(kd\right)\right]\sin m_2 z;$$
$$a_{41} = -J_1\left(kd\right)\cos mz;$$
$$a_{42} = -J_1\left(kd\right)\sin mz;$$
$$a_{43} = -m_1 J_1\left(kd\right)\sin m_1 z;$$
$$a_{44} = m_1 J_1\left(kd\right)\cos m_1 z;$$
$$a_{45} = m_2 J_1\left(kd\right)\sinh m_2 z;$$
$$a_{46} = m_2 J_1\left(kd\right)\cosh m_2 z;$$
$$a_{51} = -mJ_0\left(kd\right)\sin mz;$$
$$a_{52} = mJ_0\left(kd\right)\cos mz;$$
$$a_{53} = -J_0\left(kd\right)\cos m_1 z;$$
$$a_{54} = -J_0\left(kd\right)\sin m_1 z;$$
$$a_{55} = J_0\left(kd\right)\cosh m_2 z;$$
$$a_{56} = J_0\left(kd\right)\sinh m_2 z;$$
$$a_{61} = a_{62} = 0;$$
$$a_{63} = X_2^{-1}\left(X_1 - m_1^2\right)J_1\left(kd\right)\cos m_1 z;$$
$$a_{64} = X_2^{-1}\left(X_1 - m_1^2\right)J_1\left(kd\right)\sin m_1 z;$$
$$a_{65} = X_2^{-1}\left(X_1 + m_2^2\right)J_1\left(kd\right)\cosh m_2 z; \qquad (34)$$
$$a_{66} = X_2^{-1}\left(X_1 + m_2^2\right)J_1\left(kd\right)\sinh m_2 z;$$

Symmetric and Skew-Symmetric waves:

On using equations(29), (30) and (31) we derive the symmetric and skew-symmetric modes for the displacement components and micro rotation components as follows:

$$\left(U_r\right)_{sym} = \left[-\cos mz + Lm_1\cos m_1 z + L' m_2\cosh m_2 z\right]AJ_1\left(kr\right)e^{-i\omega t} \qquad (35)$$

$$\left(U_r\right)_{sksym} = \left[-\sin mz - Mm_1\sin m_1 z + M' m_2\sinh m_2 z\right]BJ_1\left(kr\right)e^{-i\omega t} \qquad (36)$$

$$\left(Uz\right)_{sym} = -\left[m\sin mz + L\sin m_1 z + L'\sinh m_2 z\right]AJ_0\left(kr\right)e^{-i\omega t} \qquad (37)$$

$$\left(Uz\right)_{sksym} = \left[m\cos mz - M\cos m_1 z - M'\cosh m_2 z\right]BJ_0\left(kr\right)e^{-i\omega t} \qquad (38)$$

$$(\Phi_\theta)_{sym} = \left[X_2^{-1}\left(X_1 - m_1^2\right)\cos m_1 z + X_2^{-1}\left(X_1 + m_2^2\right)N\cosh m_2 z\right]CJ_1(kr)e^{-i\omega t} \quad (39)$$

$$(\Phi_\theta)_{sksym} = \left[X_2^{-1}\left(X_1 - m_1^2\right)\sin m_1 z + X_2^{-1}\left(X_1 + m_2^2\right)N'\sinh m_2 z\right]DJ_1(kr)e^{-i\omega t} \quad (40)$$

Where

$$L = \frac{D}{A}; L' = \frac{D'}{A}; M = \frac{C}{B}; M' = \frac{C'}{B}; N = \frac{C'}{C}; N' = \frac{D'}{D}; \quad (41)$$

V. NUMERICAL ILLUSTRATIONS

In order to discuss the displacement, micro rotation components with the effect of rotation numerically, we adopt the following numerical, parameters from [12] as follows; $\lambda = 7.59\times10^{10}\,dyne/cm^2; \mu = 1.89\times10^{10}\,dyne/cm^2; \alpha = 0.0214\times10^{10}\,dyne/cm^2$
$K = 0.014\times10^{10}\,dyne/cm^2; \beta = 0.0226\times10^{10}\,dyne/cm^2; \gamma = 0.0263\times10^{10}\,dyne/cm^2;$
$\rho = 2.192\,gm/cm^3; J = 0.00196cm^2.$

To study the effect of angular rotation on micro-rotations Φ_θ; radial displacements U_r *and axial displacements* U_z of the solid, we take the speed of the angular rotation Ω of the solid as: rotation-*1* $=2\times10^4$ *rps* and rotation-*2* $=5\times10^4$ *rps.* Natural frequency $\omega=10$ and wave number $k = \frac{2\pi}{10}$. Radius r of the solid taken in cm. as $0.001 \le r \le 1$.

Using MATLAB programme, the micro rotations and displacements are plotted against for wave number kd for non-rotation, rotation-*1*,*2* of the solid. The effect of angular rotations on micro rotations are shown in figures 2 *to* 7. The Symmetric micro rotations(SMR);micro rotations (MR) and Skew-symmetric micro rotations(SSMR) for all angular rotational speeds of the solid are related as SMR< MR< SSMR and they are vanishes in the range of wave number kd with . $0 \le kd \le 0.2$

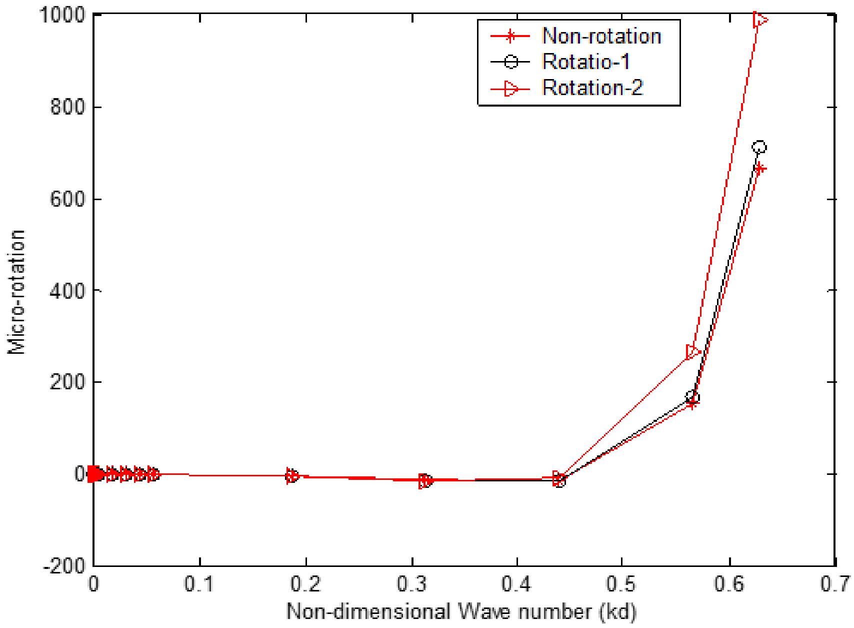

Figure 2: Wave number versus Micro-rotation

The effect of angular rotations on radial displacements (RD), radial symmetric displacements (RSD) and radial skew-symmetric displacements (RSSD) are shown from figure 8 to figure 13. RSSD are coincides for all angular rotations. RD and RSD are proportional to the angular rotations of the solid in the range of kd with $0.4 \le kd \le 1$. and RSD are coincides in the range of kd with $0 \le kd \le 0.2$. RD are coincides the range of kd with $0 \le kd \le 0.35$. RD are more than to RSD in non-rotating and rotating solid.

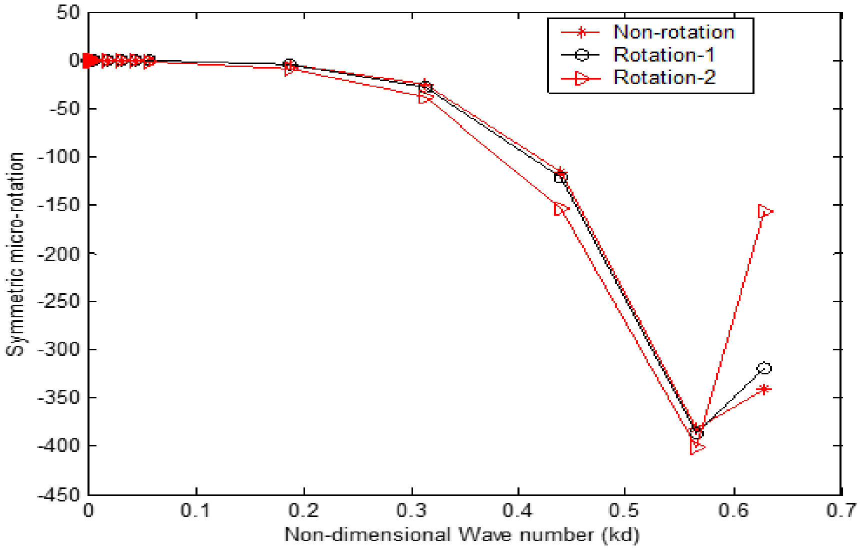

Figure 3: Wave number versus Symmetric micro-rotation

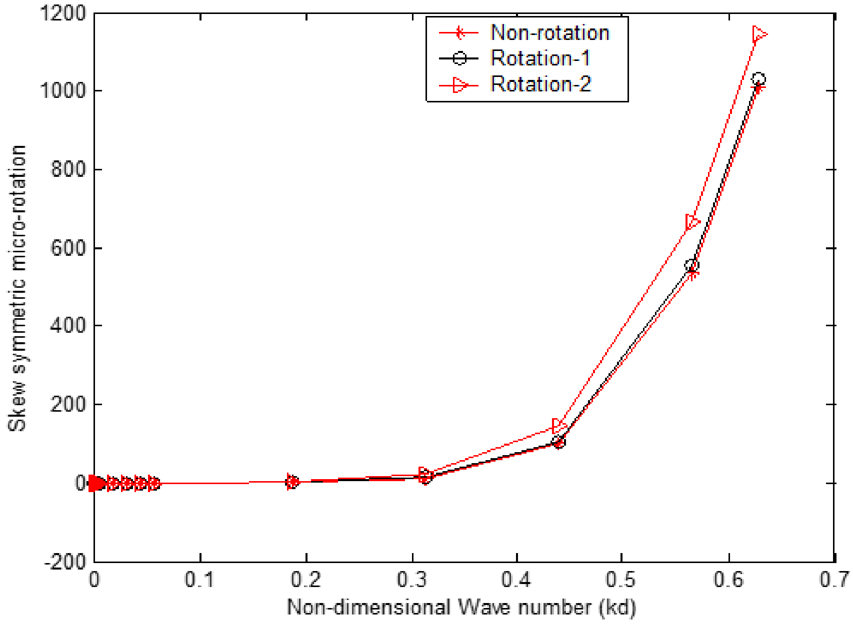

Figure 4.Wave number versus Skew symmetric micro-rotation

Angular rotation effect on axial displacements (AD), axial symmetric displacements (ASD) and axial skew-symmetric displacements (ASSD) are shown in figures 14 to 19. We observed that all types of axial displacements are vanishes in rotating and non-rotating solids and these displacements attains positive and negative values at zero wave number. Here we noticed that these theoretical results are stable for rotating and non-rotating solids, but numerical results may be vary in either different materials or in the range of different wave number.

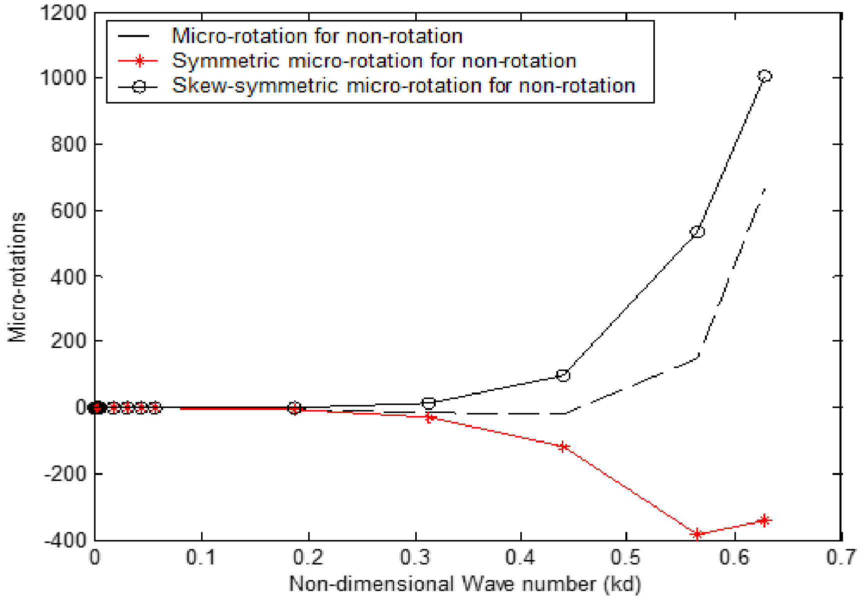

Figure5: Wave number versus Micro-rotations for non- rotation

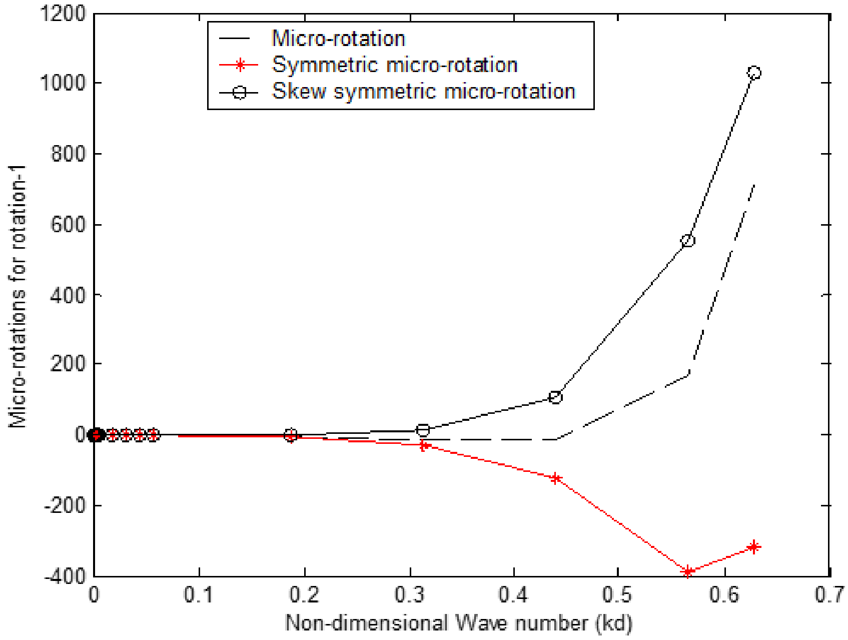

Figure 6*: Wave number versus Micro-rotations for roation-1*

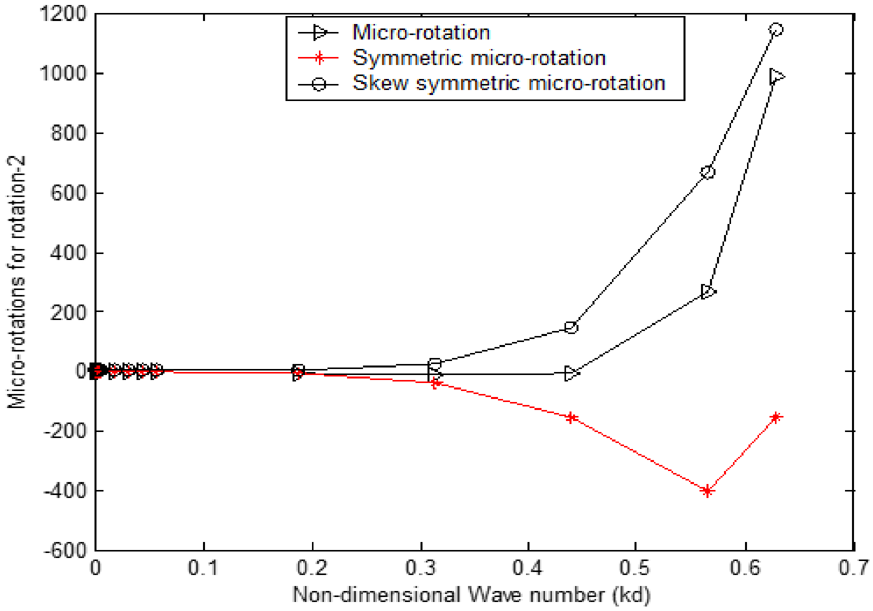

Figure 7: Wave number versus Micro-rotations for roation-2

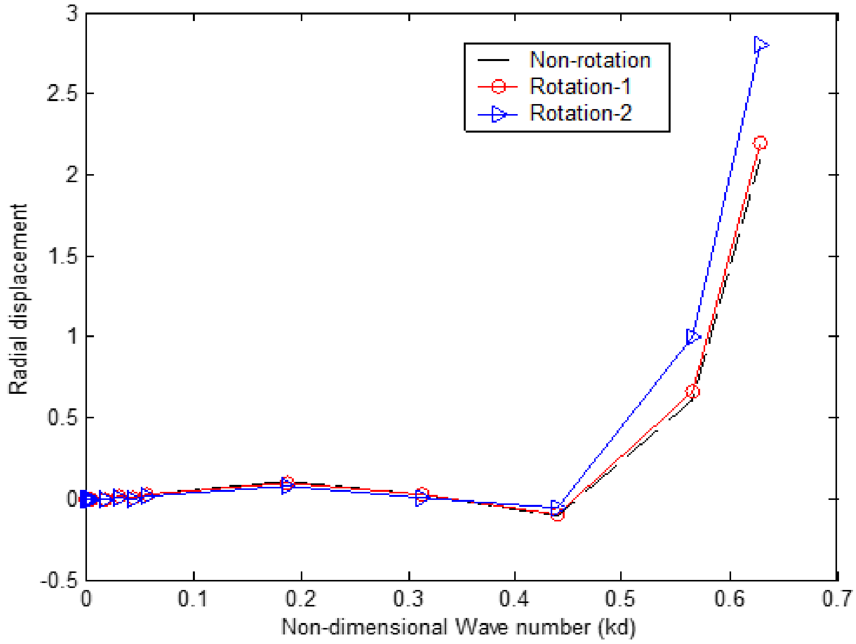

Figure 8: Wave number versus Radial displacement

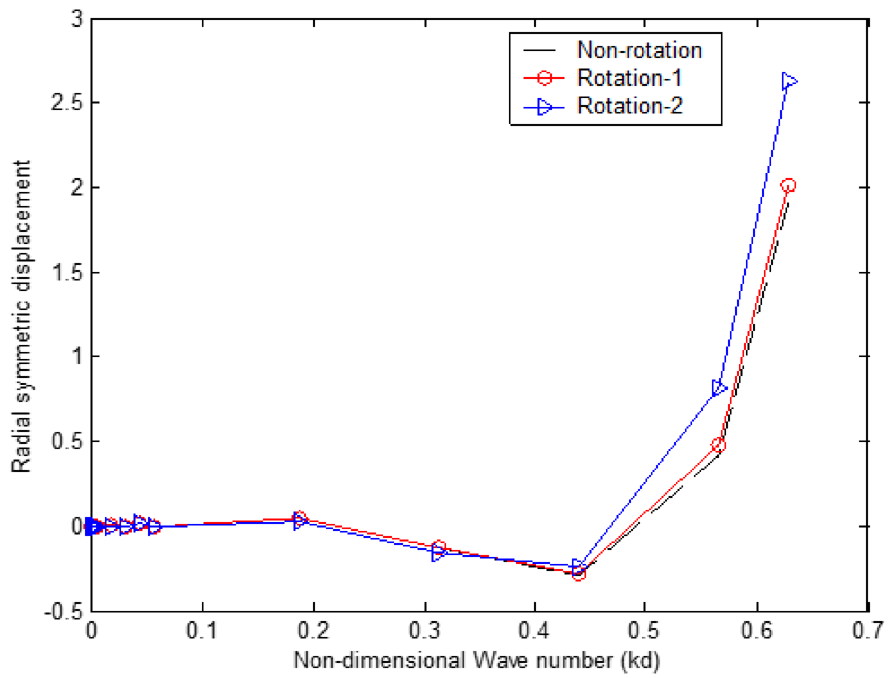

Figure 9: Wave number versus Radial symmetric Displacement

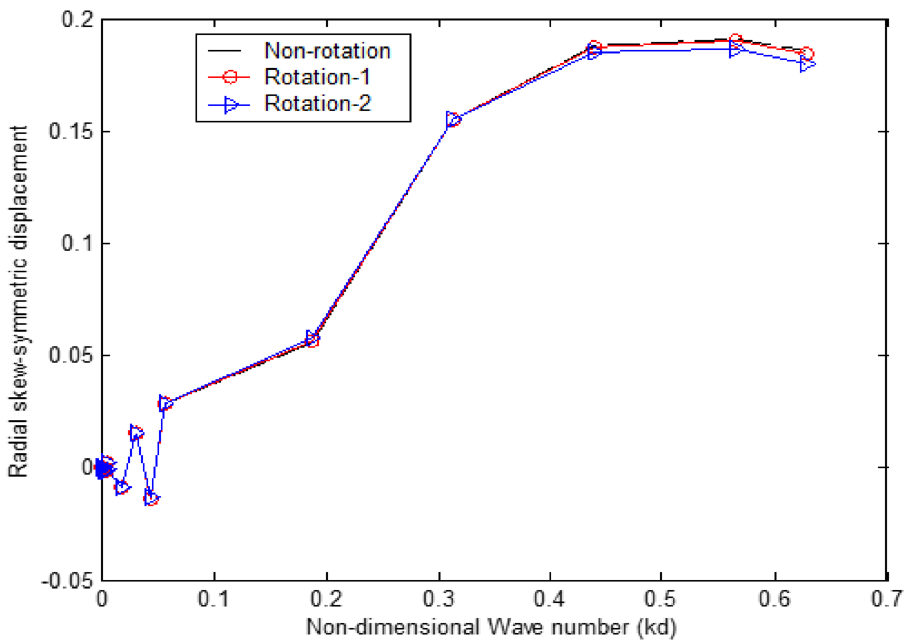

Figure 10: Wave number versus Radial skew symmetric displacement

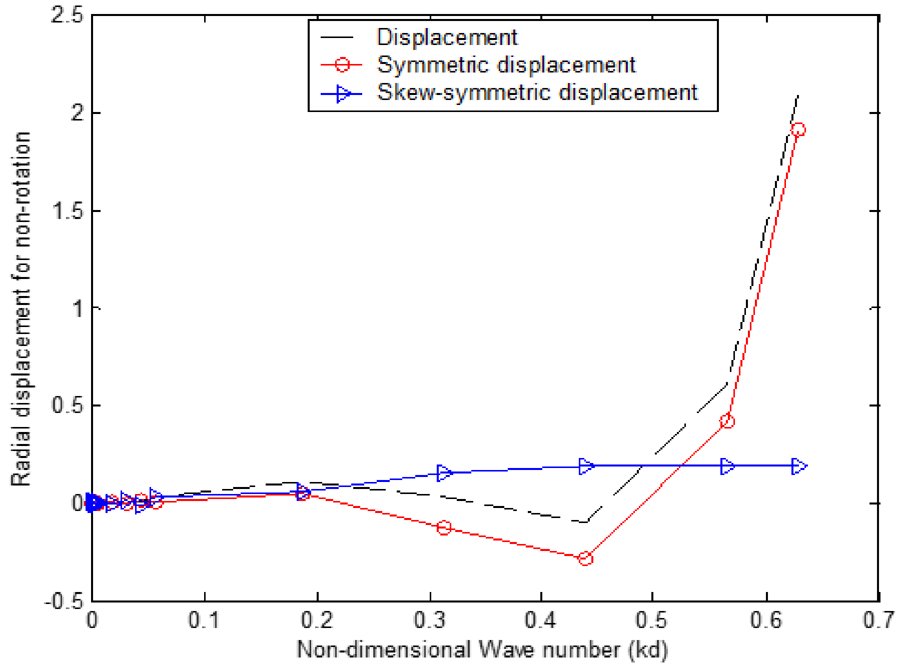

Figure 11: Wave number versus Radial displacement for non- rotation

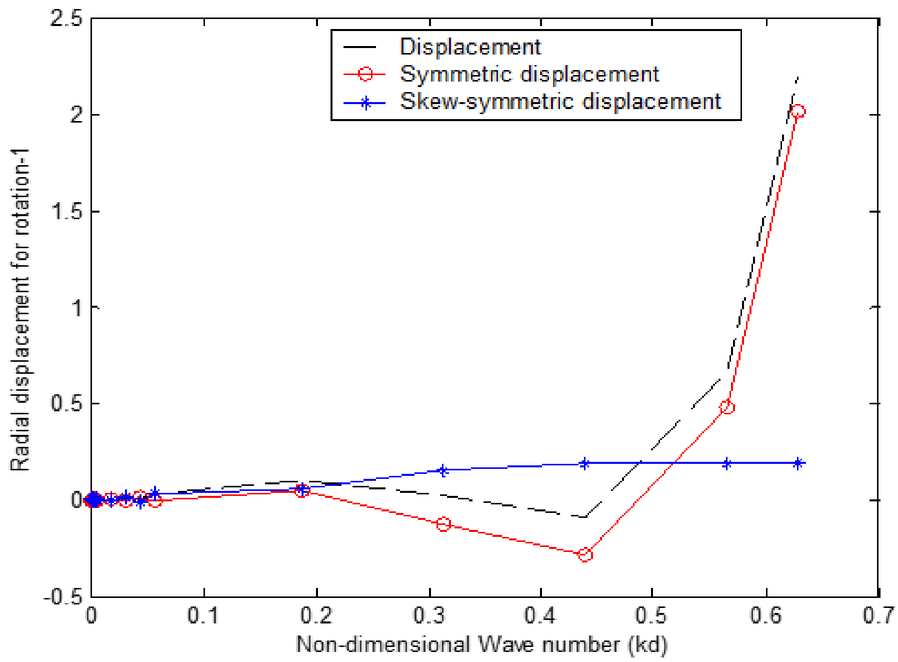

Figure 12: Wave number versus Radial displacement for rotation-1

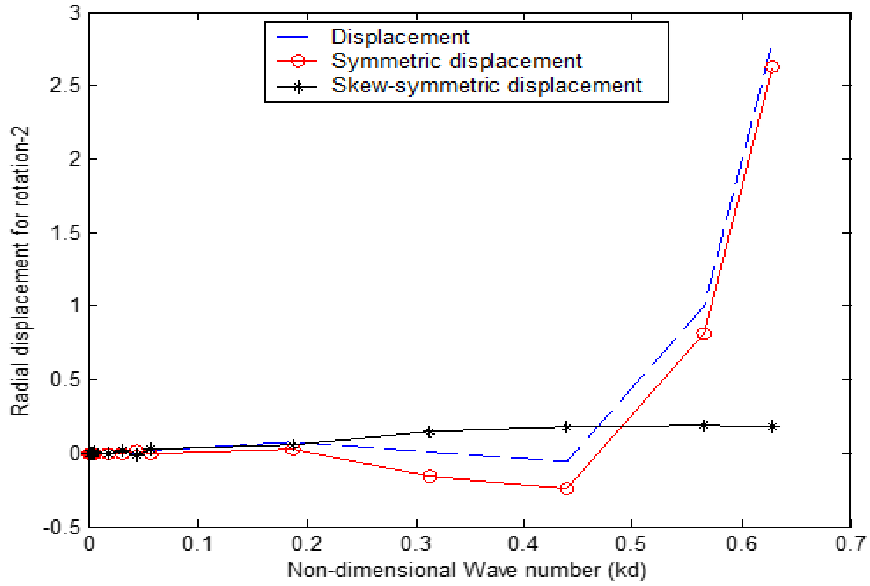

Figure 13: Wave number versus Radial displacement for rotation-2

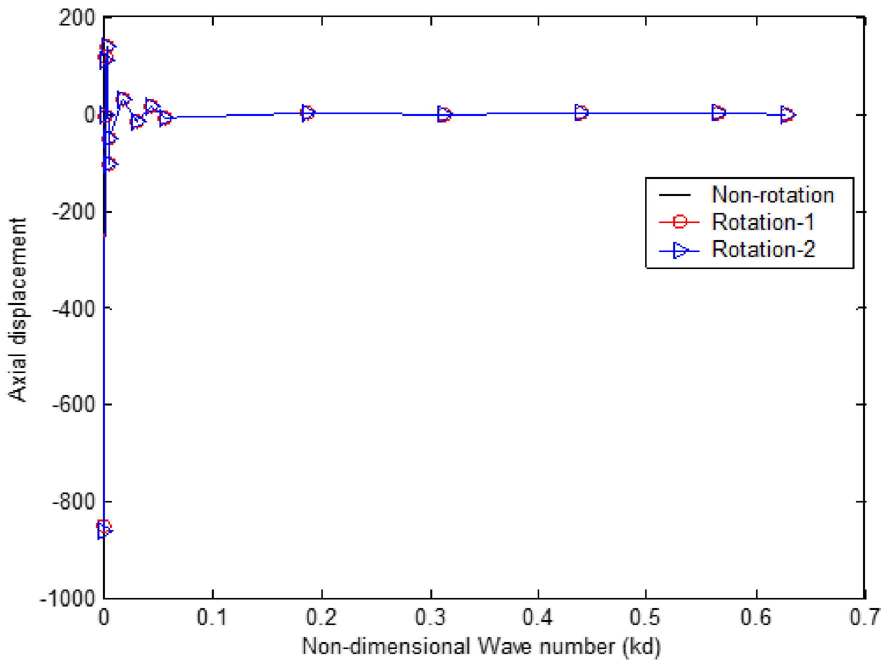

Figure 14: Wave number versus Axial displacement

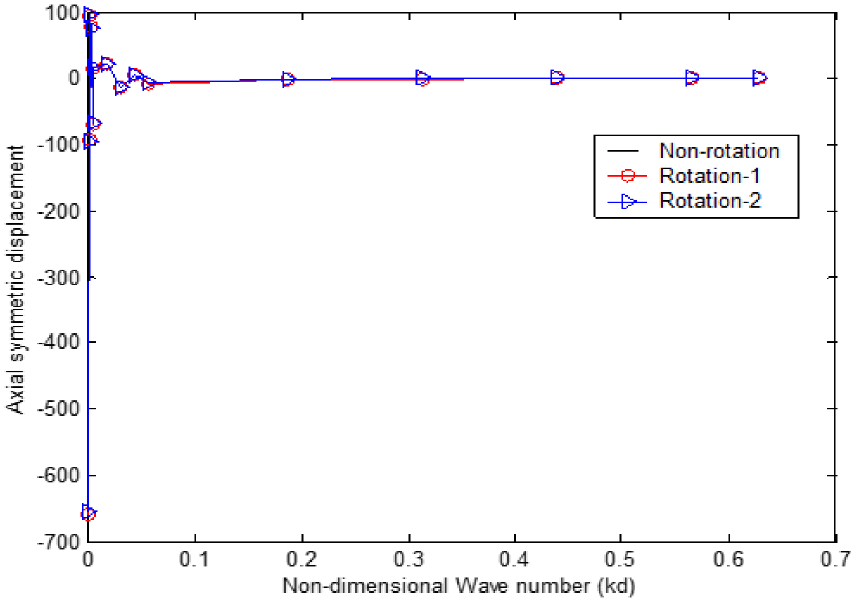

Figure 15: Wave number versus Axial symmetric displacement

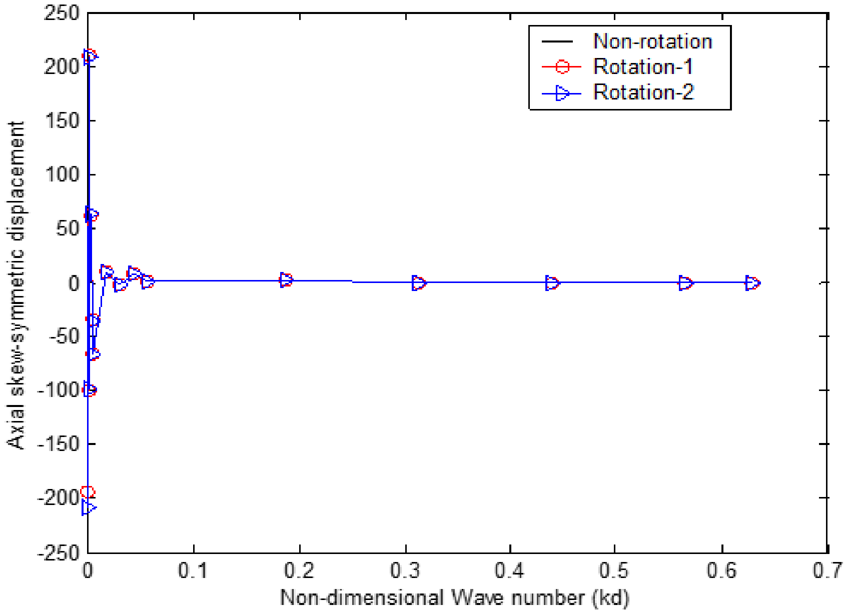

Figure 16: Wave number versus Axial skew-symmetric displacement

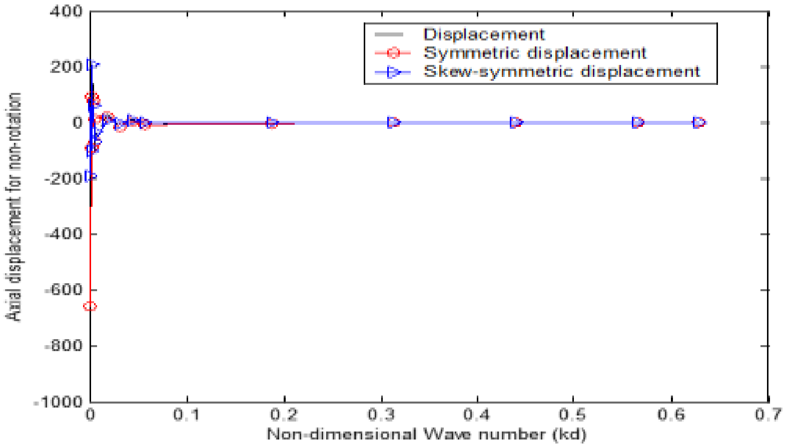

Figure 17: Wave number versus Axial displacement for non- rotation

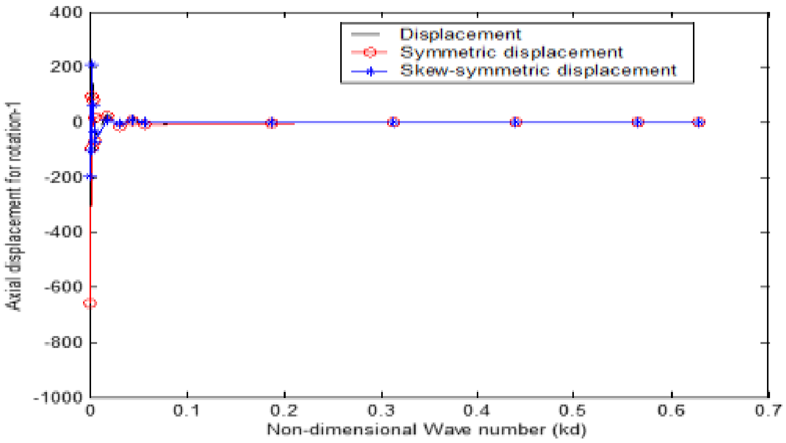

Figure 18: Wave number versus Axial displacement for rotation-1

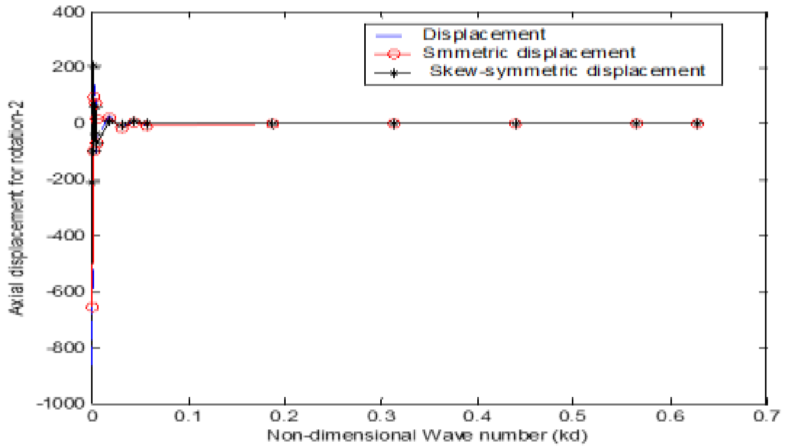

Figure 19: Wave number versus Axial displacement for rotation

VI. CONCLUSIONS

In the study of longitudinal vibrations in a micro polar elastic solid with rotation, the dispersion relation and micro rotations and radial, axial displacements connecting angular rotations are derived. Under theoretical illustrations and a particular numerical example we conclude that:

1. Micro rotations are lies between symmetric and skew-symmetric micro rotations in a rotating Solid.
2. Radial skew-symmetric displacements are coincides for all angular rotations.
3. Radial displacements are more than to radial symmetric displacements.
4. Axial displacements are vanishes in rotating and non-rotating solids, while these displacements attains positive and negative values at zero wave number.

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