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Supply Chain Inventory Model for Deteriorating Items with Multiple Buyers Single Vendor under Time and Price Dependent Demand

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ABSTRACT:- For deteriorating items following price and time price dependent demand, for vendor and buyer an optimal policy is developed. For obtaining system's optimal cycle time, multiple buyers single vendor model is formulated as profit maximization. Joint profits for buyers and vendor have also computed. To demonstrate the utility of the model, numerical example is presented with its sensitivity analysis.
Key Words:- Different Deterioration, Multiple Buyers, Optimal strategy, Price dependent demand, Single Vendor, Supply chain, Time dependent demand

I. INTRODUCTION

Collaboration between suppliers, vendors, distributers, buyers, etc. is supply chain. To satisfy the buyers' demands in current competitive business considerable information requires to be shared through the supply chain. Collaboration between vendors and buyers execution is necessary for current business scenario. Buyer purchases products from vendor in a supply chain inventory system. On long term agreements, buyer may be a regular purchaser of goods from vendor. A greater commitment to quality exists due to this association and over the period of time, buyer and vendor trusts among themselves. For buyers and vendor supply chain stock model under various assumptions for demand pattern like price-dependent, time dependent demand, etc. have been considered by various researchers in past.

Under the assumptions that all buyers have an equal cycle time, many buyers one vendor coordinated inventory replenishment model was proposed by Banerjee and Burton [1]. A buyer and a supplier integrated mathematical model in inventory cost of vendor is derived, was obtained by Ha and Kim [10]. Hill [11] derived supply chain model for global most favorable batching and delivery plan under manufacturing inventory strategy when single vendor and single buyer are considered in integrated system. Goyal [9] proposed and designed integrated inventory model that the consignment bulk would be estimated by first delivery amount. Deterioration of units start after receiving units in stock by buyers, under this supposition, a joint inventory model for single vendor and multiple buyers was derived by Yang and Wee [26]. For one wholesaler and one or more retailers a supply chain stock model to get maximum joint profit under customers price-sensitive demand was obtained by Boyaci and Gallego [2]. For decaying items, a multiple buyers single vendor production stock model was derived by Yang and Wee [27]. A quantity discount pricing offered by vendor to buyer, the integrating decaying stock model was obtained by Yang [28]. Wee and Jong [25] formulated one buyer one producer collaborative inventory model where the influences of unit price, deterioration factor, producer's and buyer's unit cost was taken into consideration. Zavanella and Zanoni [29] introduced collaborative inventory model where single vendor and multiple buyers are considered the consignment stock case. When shortages are permitted for buyers, one vendor and many buyers' production inventory model was considered by Singh and Chandramouli [23]. Supply chain inventory model derived by Shah et al. [21] considered multiple buyers and single vendor, demand is a function of increasing and quadratic time dependent with invariable deterioration unit. Using algebraic method, a single buyer single vendor stock model was formulated by Sarkar [19] under different probabilistic deterioration rate for items. Under uncertain lot receiving items and backorder price discount policy, a one buyer one vendor integrated model with trade credit was described by Priyan and Uthayakumar [17]. Supply chain inventory model was derived by Glock and Kim [8] when single supplier and multiple retailers where supplier transport complete manufactured products to the retailers while the supplier waits until the complete produced lot has been ended and consignments can be manufactured by batch. Under partial backlogging policy and stock dependent demand situation, a two echelon supply chain production stock model for decaying items was obtained by Khurana et al. [14] under inflationary setting. Giri and Roy [7] derived two levels supply chain by assuming the collaboration between multiple buyers and single manufacturer when lead-time demand was normally distributed under price dependent demand. Ghiami and Williams [6] delivered two levels production inventory models with multiple buyers and one manufacturer when deteriorating items has fixed production rate and the order quantities are dispatched by the manufacturer to the consumers for definite period and the surplus inventory supplies for successive deliveries. Under stochastic lead time situation, an integrated buyer vendor stock model was obtained by Jauhari [12]. A stock model having imperfect production system under inspection errors and warranty cost was formulated by Sarkar and Saren [20]. Kaya and Polat [13] discussed a decaying item stock model for obtaining jointly optimal pricing and inventory replenishment policy. A supply chain inventory model with two level trade credits under time and credit-sensitive demand involving default risk for obtaining optimal replenishment and credit policy was developed by Mahata et al. [15]. Under partial trade credits offer, Tiwari et al. [24] developed a decaying item supply chain stock model and derived optimum profit by setting optimal selling price. Many buyers one vendor collaborative inventory model are considered by assuming lead-time to reduce cost of supply chain system was developed by Ritha and Poongodisathiya [18]. For multiple products when single supplier and multiple buyers consist in inventory model and derived integrated model by Powar and Nandurkar [16] under supply chain policy determined the optimal joint reorder point, shipments and order quantity for each buyer subject to decrease the coordinated cost of buyers and vendor. A three-echelon supply chain model with carbon emissions from transportation, warehousing, and disposal of deteriorating items was developed by Daryanto et al. [5]. Chen et al. [4] studied a pricing and inventory replenishment problem in the presence of the uncertain demand distribution. Under discounted cash-flow analysis, Chang et al. [3] studied manufacturer's pricing and lot sizing decisions under various payment terms. Shah et al. [22] developed an inventory model with price sensitive and time dependent demand to obtain joint inventory policies for a manufacturer retailer supply chain. Demand for retailer's side is considered to be price sensitive as well as time dependent quadratic in nature. Manufacturer adopts a lot-for-lot policy for delivering retailer's demand and offers the retailer payment time dependent price for the product.

A one vendor multiple buyers combined inventory models for varying deterioration for buyers and time varying holding cost for vendor buyer both under time and price dependent demand is considered in this paper. We assume that vendor has better preservation technology, so preservation technology cost is included for vendor and therefore there is no deterioration cost for vendor.

II. ASSUMPTIONS AND NOTATIONS:

NOTATIONS:

The first objective of this section is to list all the used notations in the subsequent sections for easy reference.

$D(t, p)$: $a_i+b_it-\rho_ip_i$, where $a_i>0$, $0<b_i<1$, $p_i>0$, $\rho_i>0$

$I_{bi}(t)$: i^{th} buyer's inventory size at time t

$I_v(t)$: Vendor's inventory size at time t

A_{bi} : i^{th} buyer's ordering cost

A_v : Vendor's ordering cost

c_b : Unit cost of purchasing of buyer

θ_i : ith buyer's deterioration rate during $t_1 \le t \le t_2$, $0< \theta_i<1$

$\theta_i t$: ith buyer's deterioration rate during , $t_2 \le t \le \frac{T}{n_i}$, $0< \theta_i<1$

x_{bi} : ith Buyer's fixed holding cost

y_{bi} : ith Buyer's varying holding cost

x_v : Fixed holding cost of vendor

y_v : Varying holding cost of vendor

p_i : Selling price of ith buyer's per unit (decision variables)

m : Preservation technology cost for vendor (fixed)

n_i : Number of time orders placed by ith buyer during cycle time.

TP_{bi} : Total profit of i^{th} buyer

TP_v : Vendor's total profit

TP : Integrated total profit for both vendor and buyers per time unit.

$t_1=v_1*\frac{T}{n_i}$, $t_2=v_2*\frac{T}{n_i}$, where $T_b = T/n_i$

T = Cycle time of vendor (a decision variable).

ASSUMPTIONS:

Further we present the assumptions related to the work.

- Item's demand depends on time and price.
- Multiple buyers and one vendor are considered.

- Stock out is not permitted.
- Lead time is zero.
- During the cycle time, no repairing or replacement of deteriorated units and deterioration is dependent on time for buyer's inventory.
- For buyer and vendor both, time varying holding cost is considered.

III. THE MATHEMATICAL MODEL AND ANALYSIS:

Level of inventory of i^{th} buyer's at time t be given by $I_{bi}(t)$ $(0 \le t \le T/n_i)$ is shown below:
Buyers' Inventory

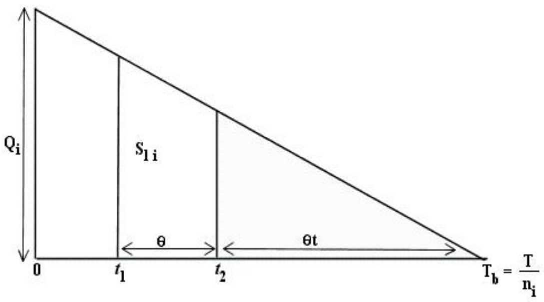


Figure 1

Two situations are discussed. In the first situation there is no collaboration between vendor and buyers, while in the second situation there is collaboration of buyers and vendor. Considering time and price dependent demand, inventory size is given for buyers and vendor.
Change in inventory sizes are given by following differential equations for buyers and vendor:

$$\frac{dI_{bi}(t)}{dt} = -(a_i + b_i t - \rho_i p_i), \qquad 0 \le t \le t_1 \quad (1)$$

$$\frac{dI_{bi}(t)}{dt} + \theta_i I_{bi}(t) = -(a_i + b_i t - \rho_i p_i), \qquad t_1 \le t \le t_2 \quad (2)$$

$$\frac{dI_{bi}(t)}{dt} + \theta_i t I_{bi}(t) = -(a_i + b_i t - \rho_i p_i), \qquad t_2 \le t \le \frac{T}{n_i} \quad (3)$$

$$\frac{dI_v(t)}{dt} = -\sum_{i=1}^{N}(a_i + b_i t - \rho_i p_i), \qquad 0 \le t \le T \quad (4)$$

under initial conditions $I_{bi}(0) = Q_i$, $I_{bi}(t_1) = S_{1i}$, $I_{bi}\left(\frac{T}{n_i}\right) = 0$ and $I_v(T)=0$.

These equations have solutions:

$$I_{bi}(t) = Q_i - \left(a_i t - \rho_i p_i t + \frac{1}{2} b_i t^2\right), \quad (5)$$

$$I_{bi}(t) = \begin{bmatrix} a_i(t_1-t) - \rho_i p_i(t_1-t) + \frac{1}{2} a_i\theta_i(t_1^2-t^2) - \frac{1}{2}\rho_i p_i \theta_i(t_1^2-t^2) + \frac{1}{2} b_i(t_1^2-t^2) \\ + \frac{1}{3} b_i\theta_i(t_1^3-t^3) - a_i\theta_i t(t_1-t) + \rho_i p_i\theta_i t(t_1-t) - \frac{1}{2} b_i\theta_i t(t_1^2-t^2) \end{bmatrix} + S_{1i}\left[1+\theta_i(t_1-t)\right] \quad (6)$$

$$I_{bi}(t) = \begin{bmatrix} a_i\left(\frac{T}{n_i}-t\right) - \rho_i p_i\left(\frac{T}{n_i}-t\right) + \frac{1}{2} b_i\left(\frac{T^2}{n_i^2}-t^2\right) + \frac{1}{6} a_i\theta_i\left(\frac{T^3}{n_i^3}-t^3\right) - \frac{1}{6}\rho_i p_i\theta_i\left(\frac{T^3}{n_i^3}-t^3\right) \\ + \frac{1}{8} b_i\theta_i\left(\frac{T^4}{n_i^4}-t^4\right) - \frac{1}{2} a_i\theta_i t^2\left(\frac{T}{n_i}-t\right) + \frac{1}{2}\rho_i p_i\theta_i t^2\left(\frac{T}{n_i}-t\right) - \frac{1}{4} b_i\theta_i t^2\left(\frac{T^2}{n_i^2}-t^2\right) \end{bmatrix}. \quad (7)$$

$$I_v(t) = \sum_{i=1}^{N}\left[a_i(T-t) - \rho_i p_i(T-t) + \frac{1}{2} b_i(T^2-t^2)\right]. \quad (8)$$

(by not considering higher powers of θ)

Substituting t = t_1, in (5) gives

$$Q_i = S_{1i} + \left(a_i t_1 - \rho_i p_i t_1 + \frac{1}{2} b_i t_1^2 \right). \qquad (9)$$

From equations (6) and (7), putting t = t_2, we have

$$I_{bi}(t_2) = \left[\begin{array}{l} a_i(t_1-t_2) - \rho_i p_i (t_1-t_2) + \frac{1}{2} a_i \theta_i (t_1^2-t_2^2) - \frac{1}{2} \rho_i p_i \theta_i (t_1^2-t_2^2) + \frac{1}{2} b_i (t_1^2-t_2^2) \\ + \frac{1}{3} b_i \theta_i (t_1^3-t_2^3) - a_i \theta_i t_2 (t_1-t_2) + \rho_i p_i \theta_i t_2 (t_1-t_2) - \frac{1}{2} b_i \theta_i t_2 (t_1^2-t_2^2) \end{array} \right] + S_{1i}\left[1+\theta_i(t_1-t_2)\right] \qquad (10)$$

$$I_{bi}(t_2) = \left[\begin{array}{l} a_i\left(\frac{T}{n_i}-t_2\right) - \rho_i p_i \left(\frac{T}{n_i}-t_2\right) + \frac{1}{2} b_i \left(\frac{T^2}{n_i^2} - t_2^2\right) + \frac{1}{6} a_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) - \frac{1}{6} \rho_i p_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) \\ + \frac{1}{8} b_i \theta_i \left(\frac{T^4}{n_i^4} - t_2^4\right) - \frac{1}{2} a_i \theta_i t_2^2 \left(\frac{T}{n_i} - t_2\right) + \frac{1}{2} \rho_i p_i \theta_i t_2^2 \left(\frac{T}{n_i} - t_2\right) - \frac{1}{4} b_i \theta_i t_2^2 \left(\frac{T^2}{n_i^2} - t_2^2\right) \end{array} \right]. \qquad (11)$$

So from equations (10) and (11), we get

$$S_{1i} = \frac{1}{\left[1+\theta_i(t_1-t_2)\right]} \left[\begin{array}{l} a_i\left(\frac{T}{n_i}-t_2\right) - \rho_i p_i \left(\frac{T}{n_i}-t_2\right) + \frac{1}{2} b_i \left(\frac{T^2}{n_i^2} - t_2^2\right) + \frac{1}{6} a_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) - \frac{1}{6} \rho_i p_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) \\ + \frac{1}{8} b_i \theta_i \left(\frac{T^4}{n_i^4} - t_2^4\right) - \frac{1}{2} a_i \theta_i \mu_2^2 \left(\frac{T}{n_i} - t_2\right) + \frac{1}{2} \rho_i p_i \theta_i t_2^2 \left(\frac{T}{n_i} - t_2\right) - \frac{1}{4} b_i \theta_i t_2^2 \left(\frac{T^2}{n_i^2} - t_2^2\right) \\ - a_i(t_1-t_2) + \rho_i p_i (t_1-t_2) - \frac{1}{2} a_i \theta_i (t_1^2-t_2^2) + \frac{1}{2} \rho_i p_i \theta_i (t_1^2-t_2^2) - \frac{1}{2} b_i (t_1^2-t_2^2) \\ - \frac{1}{3} b_i \theta_i (t_1^3-t_2^3) + a_i \theta_i t_2 (t_1-t_2) - \rho_i p_i \theta_i t_2 (t_1-t_2) + \frac{1}{2} b_i \theta_i t_2 (t_1^2-t_2^2) \end{array} \right]. \qquad (12)$$

Substituting from (12), value of S_1 in (6) gives

$$I_{bi}(t) = \frac{\left[1+\theta_i(t_1-t)\right]}{\left[1+\theta_i(t_1-t_2)\right]} \left[\begin{array}{l} a_i\left(\frac{T}{n_i}-t_2\right) - \rho_i p_i \left(\frac{T}{n_i}-t_2\right) + \frac{1}{2} b_i \left(\frac{T^2}{n_i^2} - t_2^2\right) + \frac{1}{6} a_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) - \frac{1}{6} \rho_i p_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) \\ + \frac{1}{8} b_i \theta_i \left(\frac{T^4}{n_i^4} - t_2^4\right) - \frac{1}{2} a_i \theta_i t_2^2 \left(\frac{T}{n_i} - t_2\right) + \frac{1}{2} \rho_i p_i \theta_i t_2^2 \left(\frac{T}{n_i} - t_2\right) - \frac{1}{4} b_i \theta_i t_2^2 \left(\frac{T^2}{n_i^2} - t_2^2\right) \\ - a_i(t_1-t_2) + \rho_i p_i (t_1-t_2) - \frac{1}{2} a_i \theta_i (t_1^2-t_2^2) + \frac{1}{2} \rho_i p_i \theta_i (t_1^2-t_2^2) - \frac{1}{2} b_i (t_1^2-t_2^2) \\ - \frac{1}{3} b_i \theta_i (t_1^3-t_2^3) + a_i \theta_i t_2 (t_1-t_2) - \rho_i p_i \theta_i t_2 (t_1-t_2) + \frac{1}{2} b_i \theta_i t_2 (t_1^2-t_2^2) \end{array} \right].$$

$$+ \left[\begin{array}{l} a_i(t_1-t) - \rho_i p_i (t_1-t) + \frac{1}{2} a_i \theta_i (t_1^2-t^2) - \frac{1}{2} \rho_i p_i \theta_i (t_1^2-t^2) + \frac{1}{2} b_i (t_1^2-t^2) \\ + \frac{1}{3} b_i \theta_i (t_1^3-t^3) - a_i \theta_i t (t_1-t) + \rho_i p_i \theta_i t (t_1-t) - \frac{1}{2} b_i \theta_i t (t_1^2-t^2) \end{array} \right]. \qquad (13)$$

Using (12) in (9), we have

$$Q_i = \frac{1}{\left[1+\theta_i(t_1-t_2)\right]} \left[\begin{array}{l} a_i\left(\frac{T}{n_i}-t_2\right) - \rho_i p_i \left(\frac{T}{n_i}-t_2\right) + \frac{1}{2} b_i \left(\frac{T^4}{n_i^4} - t_2^2\right) + \frac{1}{6} a_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) \\ - \frac{1}{6} \rho_i p_i \theta_i \left(\frac{T^3}{n_i^3} - t_2^3\right) + \frac{1}{8} b_i \theta_i \left(\frac{T^4}{n_i^4} - t_2^4\right) - \frac{1}{2} a_i \theta_i t^2 \left(\frac{T}{n_i} - t_2\right) + \frac{1}{2} \rho_i p_i \theta_i t_2^2 \left(\frac{T}{n_i} - t_2\right) \\ - \frac{1}{4} b_i \theta_i t_2^2 \left(\frac{T^4}{n_i^4} - t_2^2\right) - a_i(t_1-t_2) + \rho_i p_i (t_1-t_2) - \frac{1}{2} a_i \theta_i (t_1^2-t_2^2) + \frac{1}{2} \rho_i p_i \theta_i (t_1^2-t_2^2) \\ - \frac{1}{2} b_i (t_1^2-t_2^2) - \frac{1}{3} b_i \theta_i (t_1^3-t_2^3) + a_i \theta_i t_2 (t_1-t_2) - \rho_i p_i \theta_i t_2 (t_1-t_2) + \frac{1}{2} b_i \theta_i t_2 (t_1^2-t_2^2) \end{array} \right] \qquad (14)$$

$$+ \left(a_i t_1 - \rho_i p_i t_1 + \frac{1}{2} b_i t_1^2 \right)$$

Using value of Q in (5), we have

$$I_{bi}(t) = \frac{1}{\left[1+\theta_i\left(t_1-t_2\right)\right]}\left[\begin{array}{l} a_i\left(\frac{T}{n_i}-t_2\right)-\rho_i p_i\left(\frac{T}{n_i}-t_2\right)+\frac{1}{2}b_i\left(\frac{T^2}{n_i^2}-t_2^2\right)+\frac{1}{6}a_i\theta_i\left(\frac{T^3}{n_i^3}-t_2^3\right) \\ -\frac{1}{6}\rho_i p_i\theta_i\left(\frac{T^3}{n_i^3}-t_2^3\right)+\frac{1}{8}b_i\theta_i\left(\frac{T^4}{n_i^4}-t_2^4\right)-\frac{1}{2}a_i\theta_i t_2^2\left(\frac{T}{n_i}-t_2\right) \\ +\frac{1}{2}\rho_i p_i\theta_i t_2^2\left(\frac{T}{n_i}-t_2\right)-\frac{1}{4}b_i\theta_i t_2^2\left(\frac{T^2}{n_i^2}-t_2^2\right)-a_i\left(t_1-t_2\right) \\ +\rho_i p_i\left(t_1-t_2\right)-\frac{1}{2}a_i\theta_i\left(t_1^2-t_2^2\right)+\frac{1}{2}\rho_i p_i\theta_i\left(t_1^2-t_2^2\right)-\frac{1}{2}b_i\left(t_1^2-t_2^2\right) \\ -\frac{1}{3}b_i\theta_i\left(t_1^3-t_2^3\right)+a_i\theta_i t_2\left(t_1-t_2\right)-\rho_i p_i\theta_i\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b_i\theta_i\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] \quad (15)$$

$$+\left(a_i(t_1-t)-\rho_i p_i(t_1-t)+\frac{1}{2}b_i(t_1^2-t^2)\right).$$

Buyers relevant costs:

(i) Ordering cost (OC_b) = $\sum_{i=1}^{N} n_i A_{bi}$ (16)

(ii) Holding Cost:

$$(HC_b) = \sum_{i=1}^{N} n_i\left[x_{bi}\left\{\int_0^{\frac{T}{n_i}} I_{bi}(t)\,dt\right\}+y_{bi}\left\{\int_0^{\frac{T}{n_i}} tI_{bi}(t)\,dt\right\}\right]$$

$$= \sum_{i=1}^{N} n_i\left[\begin{array}{l} x_{bi}\left\{\int_0^{t_1} I_{bi}(t)\,dt+\int_{t_1}^{t_2} I_{bi}(t)\,dt+\int_{t_2}^{\frac{T}{n_i}} I_{bi}(t)\,dt\right\} \\ +y_{bi}\left\{\int_{t_1}^{\frac{T}{n_i}} tI_{bi}(t)\,dt+\int_{t_1}^{t_2} tI_{bi}(t)\,dt+\int_{t_2}^{\frac{T}{n_i}} tI_{bi}(t)\,dt\right\} \end{array}\right] \quad (17)$$

(iii) Deterioration Cost:

$$(DC_b)=\sum_{i=1}^{N} n_i\; c_b\left[\int_{\mu_1}^{t_2}\theta_i I_{bi}(t)\,dt+\int_{t_2}^{\frac{T}{n_i}}\theta_i tI_{bi}(t)\,dt\right]. \quad (18)$$

(iv) Sales Revenue:

$$(SR_b)=\sum_{i=1}^{N} n_i p_i\left[\int_0^{\frac{T}{n_i}}(a_i+b_i t-\rho_i p_i)dt\right] \quad (19)$$

(by not considering higher powers of θ)

(v) Total Profit (buyers):

$$TP_{bi}=\frac{1}{T}\left[SR_b-OC_b-HC_b-DC_b\right] \quad (20)$$

Vendor's Relevant costs:

(i) Ordering Cost (OC_v) = A_v (21)

(ii) Holding Cost:

$$(HC_v)=x_v\left[\int_0^T I_v(t)\,dt - \sum_{i=1}^{N} n_i\left[\left\{\int_0^{t_1} I_{bi}(t)\,dt + \int_{t_1}^{t_2} I_{bi}(t)\,dt + \int_{t_2}^{\frac{T}{n_i}} I_{bi}(t)\,dt\right\}\right]\right]$$

$$+\,y_v\left[\int_0^T tI_v(t)\,dt - \sum_{i=1}^{N} n_i\left[\left\{\int_{t_1}^{\frac{T}{n_i}} tI_{bi}(t)\,dt + \int_{t_1}^{t_2} tI_{bi}(t)\,dt + \int_{t_2}^{\frac{T}{n_i}} tI_{bi}(t)\,dt\right\}\right]\right]. \qquad (22)$$

(iii) Preservation Technology Cost (PTC_v) = m (23)

(iv) Sales Revenue:

$$(SR_v)=c_b\left(\sum_{i=1}^{N}\left[\int_0^T (a_i+b_i t-\rho_i p_i)dt\right]\right) \qquad (24)$$

(v) Total Profit (vendor):

$$TP_v=\frac{1}{T}\left[SR_v-OC_v-HC_v-PTC_v\right] \qquad (25)$$

Situation I: Buyer and vendor take independent decision:

Here the buyers and vendor make decision independently. For given value of n, TP_b can be maximized by solving

$$\frac{\partial TP_b(T_b,p_i)}{\partial T_b}=0,\ \frac{\partial TP_b(T_b,p_i)}{\partial p_i}=0,\ \text{where } T_b=\frac{T}{n_i}, \qquad (26)$$

provided it satisfies the second order condition

$$\begin{bmatrix}\frac{\partial^2 TP_b(T_b,p_i)}{\partial T_b^{\,2}} & \frac{\partial^2 TP_b(T_b,p_i)}{\partial p_i \partial T_b}\\ \frac{\partial^2 TP_b(T_b,p_i)}{\partial T_b \partial p_i} & \frac{\partial^2 TP_b(T_b,p_i)}{\partial p_i^2}\end{bmatrix} > 0. \qquad (27)$$

This solution (n,T,p_i) maximizes TP_v.

Then the total profit without collaboration is given by:

$TP = \max(TP_b + TP_v)$.

Situation-II: Joint decision of buyer and vendor:

Here joint decision is taken by buyers and vendor:

The optimum values of T and p_i must satisfy the following conditions which maximize total profit (TP) when buyers and vendor take joint decision.

$$\frac{\partial TP_v(T,p_i)}{\partial T}=0,\ \frac{\partial TP_v(T,p_i)}{\partial p_i}=0,\ \text{where } T_b=\frac{T}{n_i}. \qquad (28)$$

provided it satisfies the second order condition

$$\begin{bmatrix}\frac{\partial^2 TP_v(T,p_i)}{\partial T^2} & \frac{\partial^2 TP_v(T,p_i)}{\partial p_i \partial T}\\ \frac{\partial^2 TP_v(T,p_i)}{\partial T \partial p_i} & \frac{\partial^2 TP_v(T,p_i)}{\partial p_i^2}\end{bmatrix} > 0 \qquad (29)$$

where total profit (TP) with collaboration is given by:

$TP = TP_b + TP_v$ (30)

IV. NUMERICAL EXAMPLE

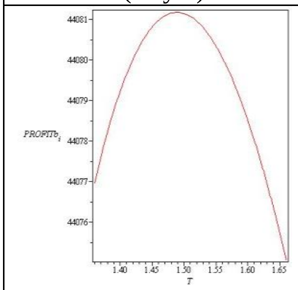
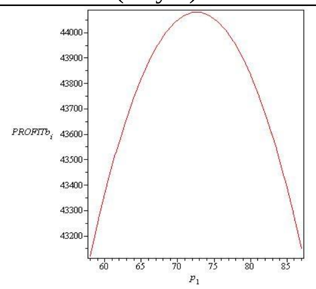
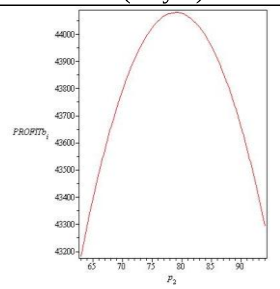
Various parameter values in appropriate units are taken for numerical illustration, A_{b1} = 85, A_{b2} = 65, a_1 = 650, a_2 = 550, b_1=0.05, b_2 =0.05, θ_1=0.06, θ_2=0.04, c_b=40, ρ_1=4.5, ρ_2=3.5, x_{b1} =4.5 , x_{b2} =5.5, y_{b1}=0.04, y_{b2}=0.06, m = 5, A_v=2000, x_v=3, y_v=0.03, v_1=0.3, v_2=0.50, N=2 in appropriate units.

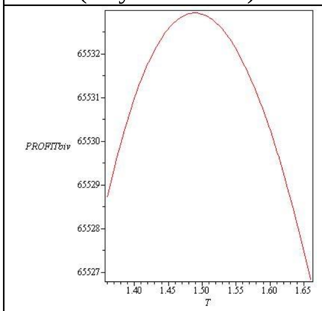
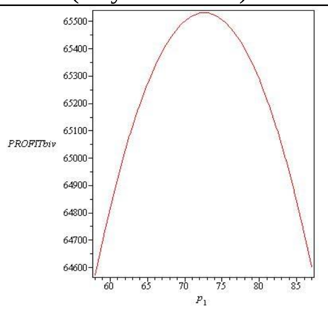
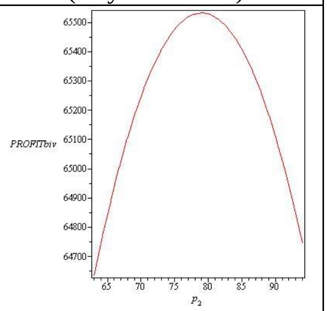
Table provides optimum independent and joint values of T, p_1, p_2 and profits for buyers and vendor. The second order conditions given in equation (27) and equation (29) are also satisfied.

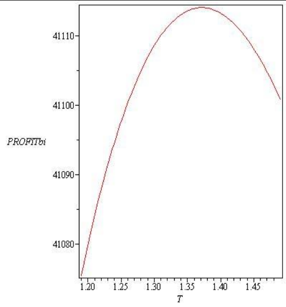
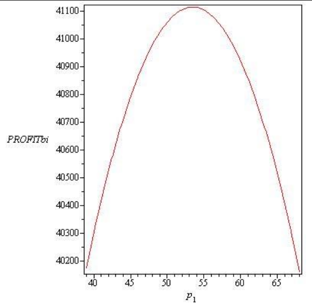
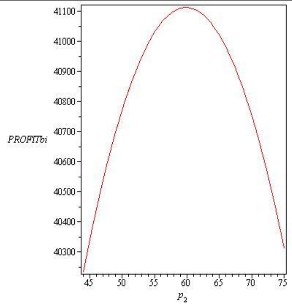
Table 1
Without collaboration and with collaboration optimum solution

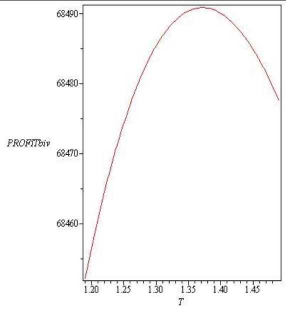
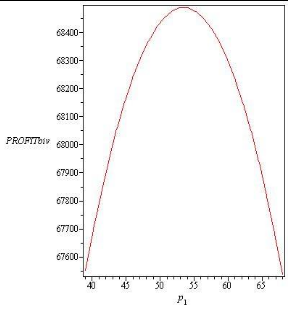
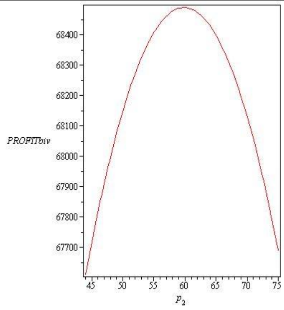
	Independent Decision	Joint Decision
n_1, n_2	n_1=5, n_2=5	n_1 = 4, n_2 = 4
T	1.4889	1.3713
p_1	72.6110	53.4468
p_2	79.0176	59.8614
Buyers' Profit	44081.1785	41114.1696
Vendor's Profit	21451.7681	27376.7288
Total Profit	65532.9466	68490.8984

Concavity of profit functions are shown in graph 1 to graph 12.

Graph Independent		
Graph of T and Profit (Buyer)	Graph of p_1 and Profit (Buyer)	Graph of p_2 and Profit (Buyer)
		
Graph 1	Graph 2	Graph 3

Graph Independent		
Graph of T and Profit (Buyer+Vendor)	Graph of p_1 and Profit (Buyer+Vendor)	Graph of p_2 and Profit (Buyer+Vendor)
		
Graph 4	Graph 5	Graph 6

Graph Jointly		
Graph of T and Profit (Buyer)	Graph of p_1 and Profit (Buyer)	Graph of p_2 and Profit (Buyer)
		
Graph 7	Graph 8	Graph 9

Graph Jointly		
Graph of T and Profit (Buyer+Vendor)	Graph of p_1 and Profit (Buyer+Vendor)	Graph of p_2 and Profit (Buyer+Vendor)
		
Graph 10	Graph 11	Graph 12

V. SENSITIVITY ANALYSIS

Study of one parameter at a time, table below gives post-optimality computations.

Table 2
Sensitivity Analysis
Independent Decision

Parameter	%	n_1	n_2	Profit(b)	Profit(v)	Profit(bv)
a_1,a_2	+20%	5	5	63821.4829	26031.6626	89853.1455
	+10%	5	5	53499.4525	23739.4408	77238.8933
	-10%	5	5	35566.9993	19169.3055	54736.3048
	-20%	5	5	27957.3521	16892.7962	44850.1483
A_{b1},A_{b2}	+20%	5	5	43984.9392	21454.3013	65439.2405
	+10%	5	5	44031.9847	21455.6417	65487.6264
	-10%	5	5	44132.8522	21440.9972	65573.8494
	-20%	6	6	44187.4327	21430.3585	65617.7912
x_{b1},x_{b2}	+20%	5	5	43996.4282	21411.5969	65408.0251
	+10%	5	5	44037.9575	21432.9498	65470.9073
	-10%	5	5	44126.3115	21467.3550	65593.6665
	-20%	5	5	44173.6280	21478.5207	65652.1487
θ_1, θ_2	+20%	5	5	44070.5788	21447.2533	65517.8321
	+10%	5	5	44075.8603	21449.5339	65525.3942
	-10%	5	5	44086.5341	21453.9643	65540.4984
	-20%	5	5	44091.9281	21456.0565	65547.9846
ρ_1, ρ_2	+20%	5	5	36568.0381	21426.6016	57994.6397
	+10%	5	5	39983.0894	21439.1878	61422.2772
	-10%	5	5	49089.9845	21464.3749	70554.3594
	-20%	5	5	55351.0263	21476.9838	76828.0101
A_v	+20%	5	5	44081.1785	21183.1134	65264.2919
	+10%	5	5	44081.1785	21317.4408	65398.6193
	-10%	4	4	44081.1785	21653.2281	65734.4066
	-20%	4	4	44081.1785	21787.5554	65868.7339
x_v	+20%	4	4	44081.1785	21319.1406	65400.3191
	+10%	4	4	44081.1785	21419.0207	65500.1992
	-10%	5	5	44081.1785	21558.3465	65639.5250
	-20%	5	5	44081.1785	21664.9249	65746.1034

Table 3
Sensitivity Analysis
Joint Decision

Parameter	%	n_1	n_2	Profit(b)	Profit(v)	Profit(bv)
	+20%	3	3	60726.8690	32110.0881	92836.9571
a_1,a_2	+10%	3	3	50418.2166	29792.1258	80210.3424

	-10%	4	4	32612.0550	25067.2095	57679.2645
	-20%	4	4	25015.8998	22760.2139	47776.1137
A_{b1}, A_{b2}	+20%	3	3	40942.4388	27479.0775	68421.5163
	+10%	3	3	40978.1812	27477.7730	68455.9542
	-10%	4	4	41157.2815	27377.6254	68534.9069
	-20%	4	4	41201.3302	27378.1040	68579.4342
x_{b1}, x_{b2}	+20%	4	4	41012.4432	27352.8667	68365.3099
	+10%	4	4	41062.5900	27365.0031	68427.5931
	-10%	3	3	41075.2958	27496.3518	68571.6476
	-20%	3	3	41138.3835	27516.0039	68654.3874
θ_1, θ_2	+20%	4	4	41101.0855	27374.1101	68475.1956
	+10%	4	4	41107.6194	27375.4177	68483.0371
	-10%	3	3	41022.2682	27478.8367	68501.1049
	-20%	3	3	41030.0997	27481.4476	68511.5473
ρ_1, ρ_2	+20%	4	4	33006.7870	28533.7905	61540.5775
	+10%	4	4	36719.2213	27954.8117	64674.0329
	-10%	4	4	46419.2856	26799.5797	73218.8653
	-20%	4	4	52976.0785	26223.3874	79199.4659
A_v	+20%	4	4	41118.7724	27090.7795	68209.5519
	+10%	4	4	41117.0724	27230.6576	68347.7300
	-10%	3	3	41021.6283	27627.4575	68649.0858
	-20%	3	3	41027.5601	27787.0327	68814.5928
x_v	+20%	3	3	41060.3514	27241.8401	68302.1915
	+10%	3	3	41038.2552	27357.0658	68395.3210
	-10%	4	4	41092.1999	27516.1234	68608.3233
	-20%	4	4	41067.8782	27661.7030	68729.5812

Based on the results of Table 2 and Table 3, we can observe about the optimal length of order cycle T*, prices, p_1*, p_2* and maximum total profits for independent as well as joint decisions.
There will be increase or decrease in value of profits when parameters 'a_1, a_2' increase/ decrease independent or as well as jointly, however, when A_{b1}, A_{b2}, x_{b1}, x_{b2}, x_v, A_v, θ_1, θ_2 and ρ_1, ρ_2 increase/decrease then total profit decrease/increase in independent and joint decision case.

VI. CONCLUSION

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyers and vendor take joint decision as compared to independent decision taken by buyers and vendor.

We can also observe that the vendors' profit is increased and number of times order placed by buyers during cycle time is decreased when buyers and vendor take joint decision.

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