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Scattered Destruction of a Twisted Shaft

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Abstract. A problem of initiation and propagation of a scattered destruction zone in a twisted isotropic solid shaft is considered. An incubation period formula is derived. A destruction front motion equation is obtained and solved.
Keywords: scattered destruction, damage, destruction front, incubation period.

I. INTRODUCTION

Shafts are one the most common structural elements of machine and mechanisms. The main form of the loads taken by them is torque. This is due precisely to their direct design purpose of transferring torque from one link of the machine to another one. In this connection analysis of life cycle of twisted shafts are still practically important and in demand. Their urgency is due to the need to take into account new factors one of which is material damage. In the presence of this phenomenon, destruction is delayed and consisting of latent destruction period called incubation period and apparent destruction stage accompanied by initiation and extension of the destructed part of the material. Within this conception we consider a problem of delayed destruction of a twisted solid isotropic shaft based on hereditary damage theory.

II. PROBLEM STATEMENT

We consider a problem of scattered destruction of a circular shaft of radius *R* and twisted by a constant moment *M*. Distribution of tangential stress τ within hereditary damage theory will be as for an elastic shaft:

$$\tau_0 = \frac{M}{J_p}\rho \tag{1}$$

where ρ is a current radius, $J_p = \frac{\pi R^4}{2}$ is a polar inertia moment of shaft's cross – section.

The greatest stress for $\rho = R$ will be

$$\tau_{\max} = \frac{M}{W_k}; \quad W_k = \frac{\pi R^3}{2} \tag{2}$$

where W_k is a torsional resistance moment.

Thereby, primary destruction will occur just at this place. The time $t = t_1$ when the external layer is destructed, is determined from the destruction criteria:

$$\tau + L^*\tau = \tau_0 \tag{3}$$

where L^* is an integral damage operator; τ_0 is an instantaneous shear strength.

III. PROBLEM SOLUTION

Taking into account (2) in (3), we get:

$$\tau_{\max} \cdot \left(1 + L^* \cdot 1\right) = \tau_0 \tag{4}$$

Or in the expanded form:

$$\tau_{\max}\left(1 + \int_0^{t_1} L(\xi)d\xi\right) = \tau_0 \tag{5}$$

Hence we find the time of latent period of propagation of destruction t_1, incubation period of destruction propagation.

$$\int_0^{t_1} L(\xi)d\xi = \frac{\tau_0}{\tau_{max}} - 1 \tag{6}$$

For the existence of incubation period of destruction propagation it is necessary that the right hand side of formula (6) be positive, i.e. the following condition be fulfilled:

$$\tau_{max} < \tau_0$$

Or

$$\frac{2M}{\pi R^3} < \tau_0 \tag{7}$$

This imposes restrictions on the external load and sizes of the shaft under which we realize the scattered destruction process. Subject to the condition (7), from the destruction criterion (6) we find the incubation period t_1.

For the kernel $L(\xi) = L_0 = const$ from (6) we find:

$$t_1 = \frac{1}{L_0}\left(\frac{\tau_0}{\tau_{max}} - 1\right) = \frac{1}{L_0}\left(\frac{\pi R^3}{2M}\tau_0 - 1\right) \tag{8}$$

For the kernel $L(\xi) = L_0 e^{-\mu\xi}$

$$t_1 = \frac{1}{\mu}\ln\left[1 - \frac{L_0}{\mu}\left(\frac{\pi R^3}{2M}\tau_0 - 1\right)\right]^{-1} \tag{9}$$

For the kernel $L(\xi) = L_0 \xi^{-\alpha}$ $0 < \alpha < 1$

$$t_1 = \left[\frac{1-2}{L_0}\left(\frac{\pi R^3}{2M}\tau_0 - 1\right)\right]^{\frac{1}{1-2}} \tag{10}$$

After destruction of the surface layer over time an extending ring – shaped destruction zone adjacent to the external boundary of the cross – section of the cylindrical shaft is formed. Assuming total loss of load – bearing ability of the shaft material behind the destruction front we get the equation of motion of interface of destructed and destructed parts of the shaft called a destruction front from the destruction criterion (3) allowing for tangential stress formula (1) in which preliminarily we make the following substitutions:

$$\rho = \beta(t); \quad R = \beta(\xi) \tag{11}$$

Here $\beta(t)$ is a radial coordinate of the destruction front for the current moment of time t; while $\beta(\xi)$ is a coordinate of the destruction front in the previous moment of time $\xi < t$.

By means of the radial coordinate of the destruction front, the tangential stress formula takes the form:

$$\tau(\beta(t); \beta(\xi)) = \frac{2M}{\pi}\frac{\beta(t)}{\beta^4(\xi)} \tag{12}$$

The destruction criterion (3) is written in the form:

$$\tau(\beta(t); \beta(t)) + \int_0^t L(t-\xi)\tau(\beta(t); \beta(\xi))d\xi = \tau_0 \tag{13}$$

Substituting (12) in (13), we get an equation of destruction front motion. We introduce the following dimensionless quantity

$$g = \frac{\pi R_0^3 \tau_0}{2M} \tag{14}$$

where R_0 is an initial radius of shaft's cross – section. We will assume the function $\beta(t)$ of the destruction front coordinate as dimensionless referring it to the same initial radius R_0 of the shaft's cross – section. Then the equation of the destruction front motion will be:

$$\frac{1}{\beta^4(t)} + \int_0^t L(t-\xi)\frac{1}{\beta^4(t)}d\xi = \frac{g}{\beta(t)}; \; t \ge t_1 \tag{15}$$

This time the desired function $\beta(t)$ has the following structure:

$$\beta(t) = \begin{cases} \beta_0 = 1 & for \quad t \le t_1 \\ \beta(t) < 1 & for \quad t > t_1 \end{cases} \tag{16}$$

Time in the equation (15) is nondimensionalized along the parameters of the kernel $L(t-\xi)$, that is possible when it is specified.

The equation of destruction front motion (15) is a nonlinear integral, second kind Volterra equation. For the general case of the kernel of the integral term its solution is a difficult mathematical problem. However, for simple special forms of the kernel of the integral term it becomes possible to obtain analytic solutions. In particular, such a possibility holds for the constant kernel $L(t-\xi) = L_0 = const$ . . In this case, we will assume the time t non – dimensionalized with respect to the parameter L_0. We will take this factor formally into account having put $L(t-\xi) = L_0 = 1$ in equation (15). Then we get the following destruction front equation:

$$\frac{1}{\beta^4(t)} + \int_0^t \frac{d\xi}{\beta^3(\xi)} = \frac{g}{\beta(t)} \tag{17}$$

Differentiating this equation with respect to time, we find an expression for the destruction front motion speed:

$$\frac{d\beta}{dt} = \frac{\beta}{4 - g\beta^3} \tag{18}$$

with the initial condition:

$$\beta = \beta_0 = 1 \quad \text{for} \quad t = t_1 \tag{19}$$

where t_1 is determined by formula (8) having in dimensionless quantities the form:

$$t_1 = g - 1 \tag{20}$$

The function $\beta(t)$ should be decreasing, then $\frac{d\beta}{dt} < 0$. This imposes restriction on the right hand side of the equation (18):

$$4 - g\beta^3 < 0 \tag{21}$$

Hence, in order the process of destruction zone extension hold, initial fulfilment of the condition $g > 4$ is necessary, or in dimensional quantities:

$$M < M_{kp} = \frac{\pi R_0^3}{8}\tau_0 \tag{22}$$

For the values of the torque greater or equal to M_{kp} the destruction will happen instantly.

Subject to the condition $g > 4$ there may be a situation when the denominator of the right hand side of equations (18) tends to infinity. The destruction front position corresponding to this moment of time will be determined by the formula:

$$\beta_{kp} = \sqrt{\frac{4}{g}} \tag{23}$$

Just this moment of time should be considered the total destruction time.
Integrating equation (18) for the initial condition (18) we get:

$$t = t_1 + 4\ln\beta + \frac{g}{3}\left(1 - \beta^3\right) \tag{24}$$

This formula determines the destruction zone size for any moment of time $t_1 < t < t_{kp}$.

The total destruction time is determined from (24) with regard to representation (23):

$$t_{kp} = t_1 + \frac{4}{3}\ln\left(\frac{4}{g}\right) + \frac{1}{3}(g-4) \tag{25}$$

From formulas (20) and (25) one can find relative ratio of latent destruction period t_1 and of apparent destruction period $t_{kp} - t_1$. For the values of the parameter $g = 32$ it will be: $t_1 = 31$; $t_{kp} - t_5 = 6.5$. According to these values, the apparent destruction time is significantly less than the latent destruction time and is 20% of this time.

IV. CONCLUSION

The equation of motion of scattered destruction front of a solid cylindrical shaft twisted by a constant moment was obtained and integrated. We carry out qualitative analysis of the solution of this equation. The estimations of incubation and apparent destruction periods are given.

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