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Vertical Folding Assemblies for Multihull Boats

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Abstract: Catamarans and trimarans may have problems in road transportation or marina berthing because of their large width, therefore several folding mechanisms have been designed to reduce width. In this paper new vertical folding mechanisms are presented which are simpler and more stable than many existing. In one version the outriggers are fixed to the connecting beams of a trimaran and rotate during the folding process, in the other version the outriggers are hinged to the connecting beams and remain virtually upright. In one variant 2 struts are connected to 2 hinges on top of a trimaran main hull - in the other the struts are connected to one common hinge. The other ends of the struts are pivotally hinged to connecting beams, which rotate around a common central hinge. In all these assemblies, rotations of the central main hull of a trimaran during the folding process are prevented by a rope, making use of the elliptic course of points on the beams. Furthermore, a new vertical folding mechanism for catamarans is presented, which can be used on the water. Such mechanisms have not been used extensively so far. This paper focuses on the principle and mathematical background of the designs, while assemblies for practical application are given in a recent patent application.
Keywords: folding mechanism, catamaran, trimaran, multihull

I. INTRODUCTION

Multihull boats have excellent sailing properties, because they have slim hulls and don't need ballast. Their stability is achieved by having a wide beam. They consist mainly of two-hulled catamarans and three-hulled trimarans. Trimarans have a central main hull, connected to two outriggers on the sides. However, these boats have a disadvantage in that they are often too wide for transportation by road and also for berthing in marinas. To overcome this disadvantage, many proposals have been devised to enable the hulls or outriggers to be moved to reduce the overall width of the boat, preferably on the water without need to dismantle the boat.

A horizontal folding mechanism, known as the swing-wing configuration, has been used extensively. Connecting beams are hinged pivotally at one end to the outrigger and at the other end to the central hull of a trimaran or a central support of a catamaran, e. g. described in [1]. This mechanism often increases the length of the boat, hence requiring a longer marina berth. Furthermore, all horizontal folding mechanisms have a common disadvantage in that due to the limited length of the hinges an unfavorable leverage is a result, making laborious construction necessary.

Another horizontal mechanism consists of telescoping tubes, e. g. described in [2]. As the inner tube, connected to the outrigger, moves inside the outer tube at the main hull of a trimaran, the width reduction is limited and the movements of the crew may be hindered. Furthermore, slight damage of this assembly might cause a complete impasse.

Vertical folding mechanisms have the advantage, in that the connecting beams may be screwed on top of the (main) hull, thus relieving the strain on the hinges and stabilizing the assembly. A mechanism has been described in [3] and an improved version in [4]. This extensively used design consists of 2 struts on each side of a trimaran, which are both pivotally hinged at their ends to a connecting beam and to the main hull, one strut forming the lower side of a four-sided vertical figure and the other strut the upper side. When folded out, each connecting (half) beam on a side is screwed on top of the trimaran main hull with one screw, so stabilizing the assembly, but leaving some load on the hinges.

In this paper, new vertical folding mechanisms are presented that need far fewer hinges than those in [3] and [4], completely relieving the strain on the hinges of a trimaran when folded out and screwed on top of the main hull. In one version the outriggers are fixed to the connecting beams of a trimaran and rotate during the folding process, in the other version the outriggers are hinged to the connecting beams and remain virtually upright. One variant has 2 hinges on top of a trimaran main hull, the other using only one hinge. Furthermore, a new vertical folding mechanism for catamarans is presented which can be used on the water. Such mechanisms have not yet been used extensively.

II. METHODS

This paper focuses on the principle and mathematical background of the designs. Therefore, the figures are drawn schematically. Assemblies for practical application are presented in a recent patent application [5[.

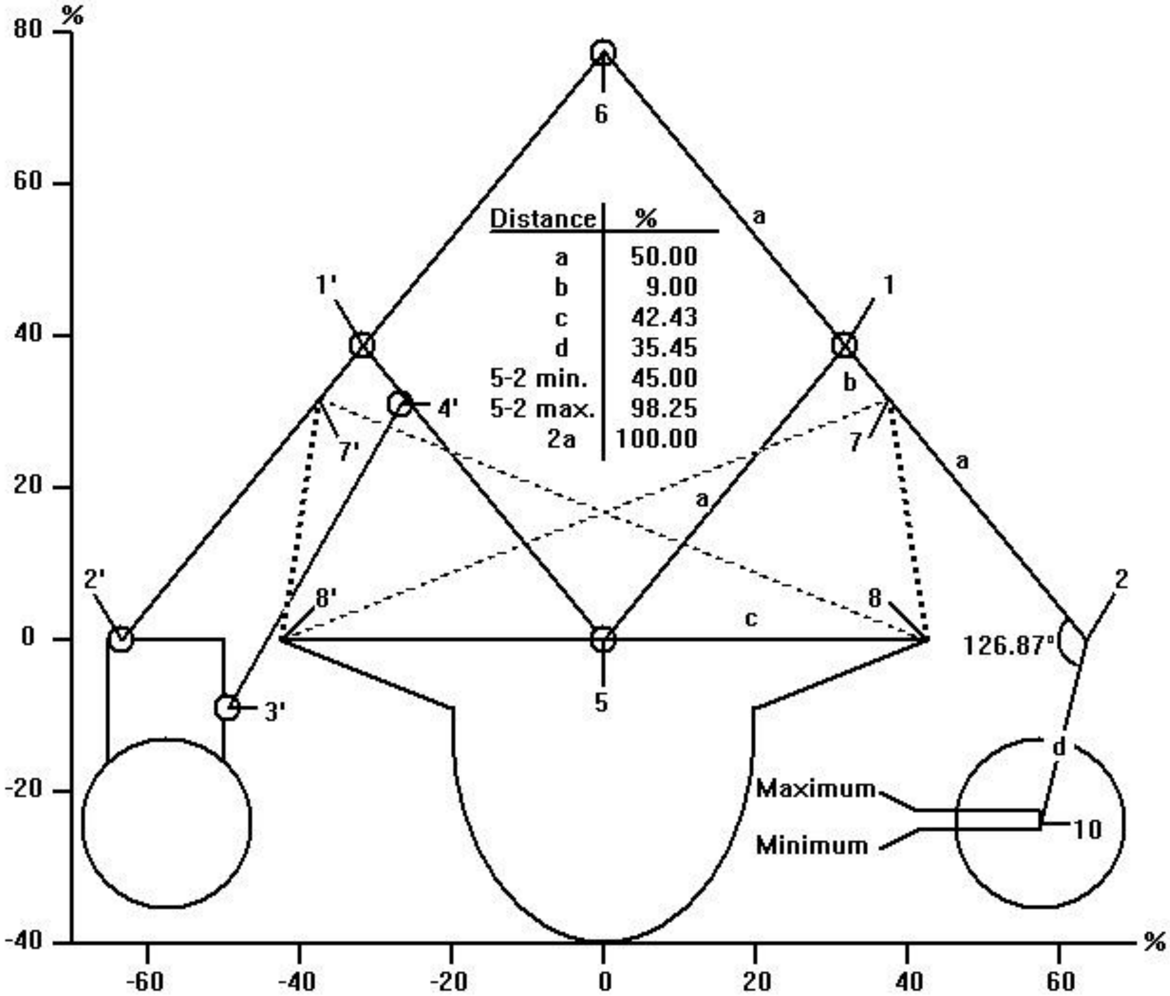

Fig. 1. Folding mechanism with 1 hinge on a trimaran main hull, right float rotating, left float fixed.(see text)

ll coordinates in the figures are given as a % of the half beam length in the structure. Characteristic points are numbered and pivot points additionally marked with small circles. Where points are symmetrical they have the same number, one with an apostrophe. The rigid half beams are pivotally connected to each other and to rigid struts, both represented by straight lines. Constant distances are represented as small letters and their values are shown in the integrated tables. Dotted lines in Fig. 1 and Fig. 2 represent ropes. Trimaran outriggers are shown as circles, their main hull as semicircle or semtellipse.

Fig. 1 shows an assembly where the symmetrical main structure forms a rhombus with side lengths a, and half beam length 2a. During the folding process pivot point 6 moves vertically and pivot points 2 and 2' move horizontally, while pivot points 1 and 1' form a circle with radius a, and center in pivot point 5. All other points on the half beams between point 6 and 2, as well as point 6 and 2', form ellipses, e. g. points 7 and 7' with the horizontal long half axis a+b and the vertical short half axis a-b. The reason for the elliptic course is the constant fraction of coordinates of e. g. points 1 and 7, horizontally (a+b)/a and vertically (a-b)/a, which can be simply seen from Fig. 1. On the right side, the outrigger is fixed to the half beam at a constant angle, rotates during the folding process and shows slight vertical movements; the maximum and minimum of the central point 10 are shown. On the left side, a strut and 3 more pivot points are added, keeping the outrigger practically upright and preventing vertical movements, as will be discussed later. The whole structure is not stable, because the main hull may rotate around pivot point 5. This can be avoided using the characteristics of an ellipse. The sum of the distances between any ellipse point and the ellipse foci is constant and equal to the whole long axis. A rope may be connected to the foci 8 and 8' and run through an eyelet at ellipse point 7, shown as a dotted line.

Turning the main hull in the direction of point 7 is not possible, because the said distance increases. Turning in the opposite direction is possible, because this distance decreases. Therefore, two such structures are necessary for a complete stabilization of the main hull. This is done by connecting point 7' to the foci as well. The mathematical relations are straightforward. In this and the other figures the coordinates of a point i are named Xi and Yi, as usual for the abscissa and the ordinate respectively. Applying the Pythagorean theorem, the following equations are obtained.

(1) $a^2 = X1^2 + Y1^2$

(2) $d^2 = Y10^2 + (X2 - X10)^2$

With X1 as independent variable Y1 may be obtained from equation (1). X2 = 2X1 and Y6 = 2Y1. The slope of line d between points 2 and 10 is -Y10/(X2 – X10). It may also be expressed by the known angle between line d and the half beam. If this angle is called ß, using the slope of the half beam -Y1/X1 and the tangent addition formula, the result is:

(3) $-Y10/(X2 - X10) = (\tan ß - Y1/X1)/(1 + \tan ß * Y1/X1)$

With equations (2) and (3) the remaining unknown variables X10 and Y10 may be calculated and the parameters may be optimized for minimum variation of Y10. If the assembly is folded out completely, an abrupt and strong vertical movement of point 10 occurs, which is normally unwanted and may cause strain in the structure. Therefore, the folding is limited and the minimum and maximum of the distances between points 2 and 5 are given in the table in FIG. 1.

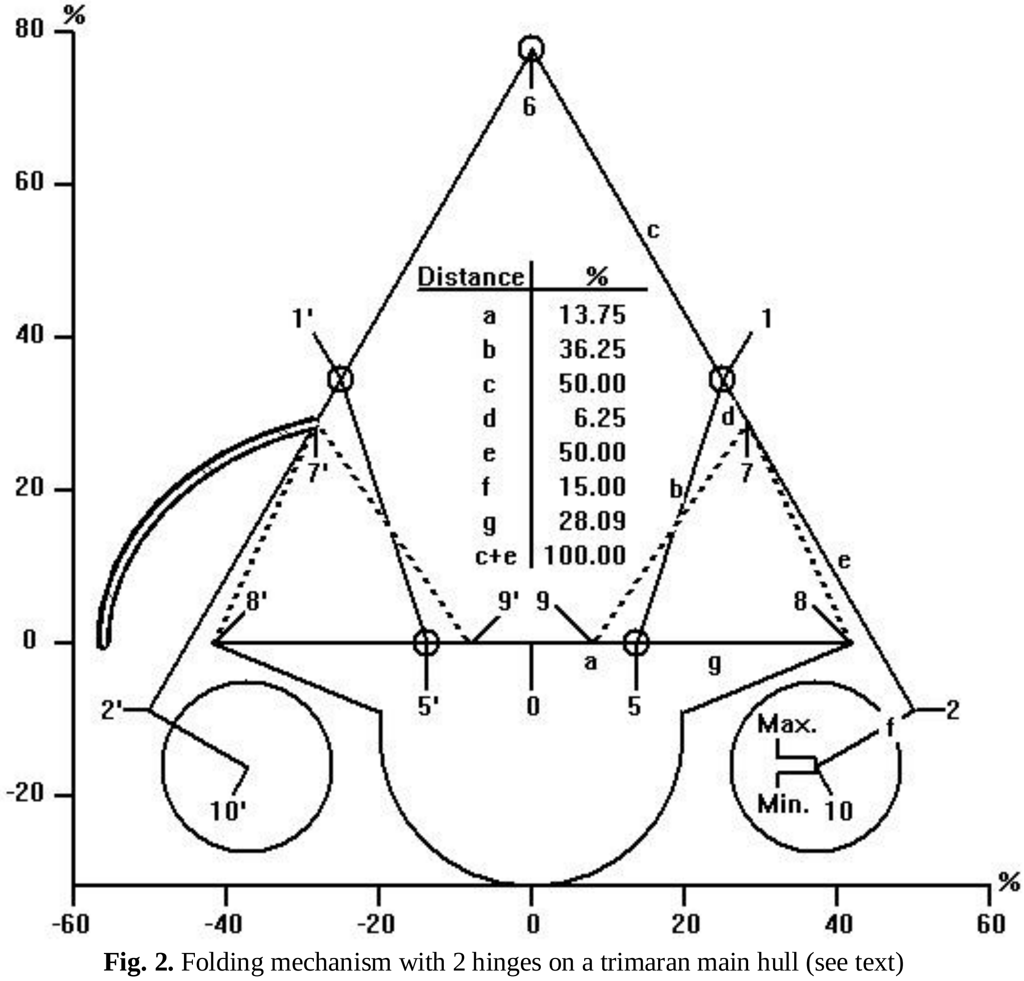

Fig. 2. Folding mechanism with 2 hinges on a trimaran main hull (see text)

Fig. 2 shows a mechanism with 2 pivot points 5 and 5' on the main hull of a trimaran. Other points correspond to those in Fig. 1 with a few exceptions. Points on the connecting half beams (a+e) between points 6 and 2 or 2' respectively don't produce exact ellipses during folding, and no point gives an exactly horizontal line. However, the course of points 7 and 7' may be approximated by ellipses, as shown for point 7', where an elliptic arc originates as the fat curve to the left. A white curve inside this elliptic arc shows the true course of point 7', which practically coincides with the ellipse. In this case 2 ellipses are obtained with foci 8 and 9 as well as 8' and 9'. Stabilization by ropes is necessary here as well, because the hull might otherwise swing and rotate. The central points 10 and 10' make slight vertical movements during folding and the maximum and minimum are shown. They correspond to a minimum distance between points 2 and 5 as shown in Fig. 2, to a maximum of 100 %, i.e. the beam is then in a horizontal position. The following equations are obtained applying the Pythagorean Theorem.

(4) $b^2 = Y1^2 + (X1-a)^2$
(5) $c^2 = X1^2 + (Y6-Y1)^2$
(6) $f^2 = (X2-X10)^2 + (Y2-Y10)^2$

With X1 as independent variable, Y1 and Y6 may be calculated solving for them and others using the relations:

(7) $c/(c+d) = X1/X7 = (Y6-Y1)/(Y6-Y7)$
(8) $c/(c+e) = X1/X2 = (Y6-Y1)/(Y6-Y2)$

For the variables X10 and Y10 a relationship may be obtained using perpendicular slopes.

(9) $(Y6-Y1)/X1 = (X2-X10)/(Y2-Y10)$

Using equation (6) these may be calculated and e. g. the parameters optimized for minimal variation of Y10.

In Fig. 3 two structures of the assembly on the left of Fig.1 are joined together, keeping the hulls of a catamaran practically upright. The data of this structure in Fig.1 corresponds to that in Fig.3, but rounded down to full %. The struts of length c in Fig.3 are pivotally connected to the half beam of length a+b on the opposite side at point 4 or 4', thus crossing each other. The other ends of both struts are pivotally connected to the catamaran hull at pivot point 3 or 3'. Both half beams have pivot points at their ends, point 1 connecting them with each other in the center of the assembly, and point 2 or 2' connecting them with a hull. Similar structures have been described before in [6], but in the presented assembly a significant further reduction of hull rotation has been achieved by shifting the pivot points 4 and 4' by a small amount e out of the straight line connecting the ends of the half beams, i.e. pivot points 1 and 2 or 2'. In [6] a different folding mechanism has been used for a trimaran. During the folding process points 2 and 2' move horizontally along the dotted line, while point 1 moves vertically in the center of the structure. Very small rotations occur around points 2 and 2' resulting in small vertical movements of points 3 and 3'. The curve passing through point 3' shows these movements. The curve has a minimum and a maximum: Fig. 3 shows the situation at the maximum, where the center line CL of the hull has the maximum deviation from the perpendicular line drawn next to it. At the minimum the hulls would rotate an equal amount in the opposite direction, while the hulls would stand upright at both endpoints of the folding process. The latter are given as the maximum and minimum distance between points 2 and 5 in the table integrated in Fig. 3. While folding out is limited in the structure shown in Fig. 1, because of abrupt vertical movements of the outriggers, here abrupt and relatively strong rotations of the hulls occur in this situation. Therefore, the folding out is limited here as well. Vertical movements of the hulls are negligible here. Applying the Pythagorean theorem the following equations are obtained.

(10) $(a+b)^2 = X2^2 + Y1^2$
(11) $c^2 = (X\,4 - X3)^2 + (Y4 - Y3)^2$
(12) $d^2 = (X2-X3)^2 + Y3^2$

If X2 is the independent variable; Y1 may be obtained with equation (10): For the calculation of other variables the knowledge of X4 and Y4 is necessary. If e=0, they can be calculated using the relationships $X4=X2*b/(a+b)$ and $Y4=Y1*a/(a+b)$. Otherwise, a small shift of their coordinates has to be evaluated. Let these shifts be dx and dy, then $dx^2+dy^2=e^2$. As line e is perpendicular to the beam a+b, their slopes have the relation $dy/dx=-X2/Y1$. The shifts may now be calculated and using equation (10) and the following equations are obtained.

(13) $X4 = -\,(X2*b - Y1*e)/(a+b)$
(14) $Y4 = (Y1*a - X2*e)/(a+b)$

Equation (12) may be solved for X3 and the solution used in equation (11), which leads to a quadratic equation

(15) $Y3 = 0.5*\{Y4*Z - [Y4^2*Z^2 - V*(Z^2 - W)]^{1/2}\}/V$,

where $W = 4*d^2*(X2 - X4)^2$, $V = (X2 - X4)^2 + Y3^2$ and $Z = V+d^2+c^2$.

Once Y3 is known, X3 can be calculated using equation (12). As the small hull rotation corresponds to the variation of Y3, the latter may be used for optimization of the parameters. For this purpose one parameter should be fixed as reference, e.g. the length of the half beam a+b. Furthermore a target average value of Y3 or the slope of line d should be fixed, because otherwise the optimization process might get out of control. The criteria could be the squared deviations

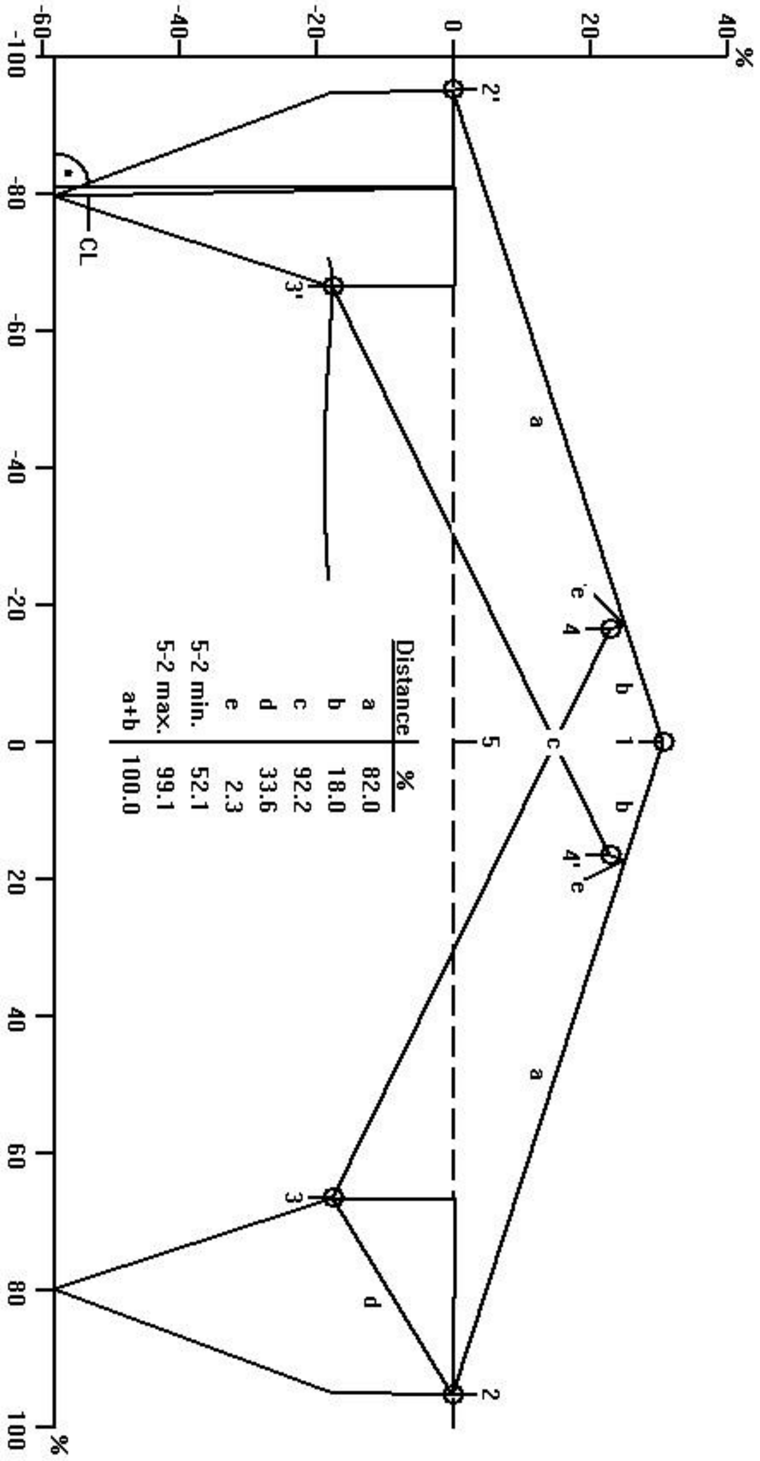

Fig. 3. Folding mechanism for a catamaran keeping the hulls virtually upright (see text)

of the extremes from the target value of Y3 and those at the desired endpoints of the folding process, normally (nearly) 100 % of the half beam and about 50 % when folded out and in respectively. According to the needed structure these squared deviations may be weighted. Preferably, Y3 at the endpoints of the folding process should be equal to the mean value of the extremes. If both endpoint values are equal, the hull may then be upright at both positions and the maximum rotation is the same in the opposite directions. Otherwise the rotation would be larger in one of the two directions. Therefore, for a certain value of Y3, the said mean value, the corresponding values of X3 have to be evaluated in order to compare it with the desired endpoint values. This means, that equation 15 has to be solved for X3. This can be done using Newton's method, but an explicit solution is possible, avoiding convergence problems during the approximation process. A 4^{th} order equation is obtained. For the evaluation of the extremes, equation (15) has to be differentiated with respect to X2 and equated to 0, which could only be done numerically because of resulting high order equations. An alternative is the calculation of c^2 with equation (11) using the target value of Y3 together with equation (12) or the target slope of line d. The value of c^2 shows now slight variations with extremes at X3 values very close to those at the extremes of Y3. The differentiation leads also to a 4^{th} order equation. The solution of both 4^{th} order equations is given as a C++ source code in the appendix.

III. DISCUSSION

Although this paper is focusing on the principles of the presented new vertical folding mechanisms, some remarks about their practical applicability will be included in the discussion. All assemblies have been tested in models, with a positive outcome..

in the mentioned cases, where the mechanism is not folded out to the maximum (Fig.1 and Fig. 3), the beams could be equipped with small extensions carrying the pivoting hinges, which could bring the beams to a horizontal position if required. Otherwise the beams could be left a bit inclined or given a curved structure. Such options are shown in [5].

The assemblies presented in Fig.1 and Fig. 2 may be screwed to the deck, or a support on the deck of a trimaran, by screws perforating the beams, when folded out. A good solution are extensions at the central ends of the half beams that overlap at their ends in such a way that the screws perforate both the extension end the other half beam, as shown in [5]. With this attachment 2 screws lock the whole assembly and relieve the loads on the hinges for the version with fixed outriggers at the beams. In the other version additional optional screws fixing the outriggers at the beams can take the loads from their additional hinges. The assembly presented in Fig. 2 may also be fitted with the mechanism keeping the outriggers upright, which

could be important in bigger ships, where outriggers carry passengers. In Fig. 1 the non rotating outrigger is equipped with a frame or support on top of the floats, which, together with the additional strut, could interfere with the central main hull when folded in. Therefore, one mechanism should be attached at the stern of the boat in such a way that these structures could slide behind it, while hinged to the beam, e. g. side to side. Normally a second mechanism is used at the front of the boat. This should be attached in front of the mast, or masts, where it cannot interfere much with the sail and the said problem with the interfering structures doesn't exist, because the main hull is slimmer there.

The ropes stabilizing the main hull in Fig. 1 and Fig. 2 may consist of natural material, plastic or steel and can be replaced by chains or (timing) belts. And the eyelet in point 7 or 7' may be replaced by a reel or a cogwheel. Small electric motors may be placed in that position for automatic folding. In case the ropes obstruct the passage of the crew, they could be removed after the folding process, if they are fixed with snap hooks at the foci of the ellipse.

In the assembly presented in Fig. 3 the half beams may also be extended at their central ends in a way that the beams can be screwed together at two positions, either side by side or overlapping. If a side by side position is chosen, where the (extended) strut is crossing that position, this strut could be fixed at the same time if perforated there. This produces a very stiff structure and is shown in [5[. As the hinges at the hulls are not relieved by this measure, the hulls could be screwed at the beams with additional optional screws here as well. In case the height of the assembly doesn't matter when folded in, e.g. if a marina berth is permanently available, the half beams could be equipped with (removable) long extensions at their central ends, which reach the hull at the opposite side and overlap there when folded out. They may then be screwed to the hull by one screw on each side, stabilizing the whole structure and relieving the strain from all hinges. Furthermore, the beams could then be made slimmer, because both take the load over their complete length.

hen folded in, trimaran outriggers could be screwed to the central main hull for stabilization, while in catamarans the ts could be fixed to the hulls. These attachments need not be very strong as they have not to resist rough waters.

The assembly in Fig. 1 may be changed in different ways. If the outriggers are fixed at the beams with their centers in position 2 and 2', no vertical movements would occur during the folding process, This is a difference to those widely used and described in [3] and [4], where such movements are unavoidable. The said variation is in principle possible, but the beams have then to be shifted and equipped with extensions for the attachment of the hinges, because otherwise the beam would dip into the water at the leeward side in strong winds. The assembly according to Fig. 1 was chosen because of greater flexibility. When folded in, the outriggers get under the main hull here, producing a further width reduction. The relative distances between the pivot points could be changed, for even greater flexibility, e. g. for further minimizing the said vertical movements. However, with this change of geometry the elliptic course of points like 7 and 7' would be lost and an approximated curve would have to be used. This could be done by introducing axis rotation as an additional parameter for the ellipse. Another option could be the use of a spanner, string or slightly elastic rope material to overcome the deviations from an exactly elliptic course.

The small shift of points 4 and 4' out of the linear connection between points 6 and 2 or 2' is an important difference compared to a similar system used for another folding mechanism in [6], because it leads to a further significant reduction of the hull rotation during the folding process. Another advantage is the fact, that the beam has not to be perforated for the attachment of the hinge; it may just be attached under the beam.

Folding mechanisms are normally attached outside the cabin areas, which is generally not a problem, because one mechanism is often attached to the stern and the other one to the front of the boat, where no cabins exist. In trimarans the cabin is often only in the central main hull. Catamarans that do not fold, normally have a wide cabin area not only in the hulls, but also filling the gap between them. Folding catamarans with cabins normally have slim cabins on either side extending a little over the hull width, and the gap between them is equipped with a trampoline consisting of a net or of water-proof material. The disadvantage of missing space could be overcome with the assembly in Fig. 3 using a hardtop. As the hull rotations during folding are minimal, a hardtop, covering both cabins when folded in, would glide over the hulls during the folding out process and would fill the gap between the cabins. The inner sides of the cabins could be left open, providing a substantial increase in available space. The trampoline should then consist of water-proof material. Fixing of the hardtop is not necessary, if the hardtop has lock seams at its border and center, which connect to corresponding lock seams at the hulls. The seams should be equipped with rubber material to avoid entry of water and wind. During folding the hardtop could be kept in place with rubber strips.

The assembly in Fig. 3 is symmetrical. There are asymmetrical multihulls, e.g. proas, consisting normally of a slim long main hull and one short outrigger. The latter serves only as ballast carrier and support. In the most common version, the pacific proa, the main hull is always directed to leeward. If the mechanism is made asymmetrical, the number of influencing parameters increases in such a way as to give a great variability to the system. It is possible to design an assembly, that pulls over the main hull and lowers there when folded in, thus avoiding the elevation of the symmetrical mechanism. Outrigger rotations aren't a problem here, because the outrigger doesn't serve as storage room nor for passenger transport. Asymmetrical assemblies for further reduction of the rotation in mechanisms like Fig. 3 may not be worthwhile.

IV. CONCLUSION

The new vertical folding mechanisms for multihull boats presented here have all a very simple yet stable structure. The fixing of this structure when folded out is simple and should withstand rough waters at high sea. Folding is quick and possible on the water, i. e. dismantling is not necessary.

Several versions for trimarans are described here, which have all the advantages compared to existing systems in terms of stability and simplicity. Improvements could be achieved, e. g. in reduction of outrigger rotation during folding. For the first time a stabilizing rope uses the elliptic geometry during folding.

For catamarans, vertical folding systems without need for dismantling, and that can be used on the water keeping the hulls upright, are not in common use. Here such a system is proposed with clues for practical application and the mathematical background for its establishment.

ACKNOWLEDGMENT

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Appendix

The following source code of a C++ program in compact form may be used after compilation for the optimization of the distances between pivot points shown in Fig. 3 in order to obtain a minimal rotation of the catamaran hulls during folding. Input is a number (1 – 4), space, value of the parameter belonging to the number (b, c, d or e) in % of half beam distance a+b and Enter. The parameters and the sum of squared deviations to be minimized are shown in the output.

```
// optimization of parameters b - e according to Fig. 3;
harald_breuer@hotmail.com
#include <iostream>
#include <iomanip>
#include <math>
using namespace std;

double cubic(double r, double s, double t) // cubic resolvent of quartic
wquation
{   double p, q, d, e, f, p3, pi, y[3], u[2], v;
    int i, j, l;
    p=s-r*r/3.0; q=2*r*r*r/27.0-r*s/3.0+t; d=p/3.0; p3=d*d*d;
d=0.25*q*q+p3; p3=r/3.0; if (d<0)
    { e=sqrt(-4*p/3.0); f=4*q/(e*e*e); f=asin(f)/3.0; pi=8*atan(1)/3.0;
for (i=0; i<3; i++)
        { y[i]=e*sin(f+i*pi)-p3; if (i==0) v=y[i]; else { if (v<y[i])
v=y[i]; }
         } // 3 real solutions
    } else // Cardano formula, one real solution
    { d=sqrt(d); y[0]=-0.5*q; y[2]=0; j=1; for (i=0;i<2;i++)
        { l=1; u[i]=y[0]+j*d; if (u[i]<0) l=-1; if (u[i]!=0)
u[i]=l*exp(log(fabs(u[i]))/3.0);
          y[2]+=u[i]; j=-j;
        } y[0]=-y[2]*0.5-p3; y[1]=sqrt(3.0)*(u[0]-u[1])*0.5; y[2]-=p3;
v=y[2];
    } return v;
}

double quartic(double aa[4]) // quartic equation with coefficients aa[i];
4 real solutions here
{ double a, b, c, d, x, y, z, w, r, s, t;
int i;
a=aa[3]; b=aa[2]; c=aa[1]; d=aa[0]; r=-b*0.5; s=a*c*0.25-d; t=((4*b-
a*a)*d-c*c)*0.125;
y=cubic(r,s,t); z=0.25*a*a-b+2*y; if (z>0)
    { z=sqrt(z); for (i=-1; i<2; i+=2)
        { x=-0.5*(a*0.5-i*z); w=y+0.5*i*(c-a*y)/z; w=x*x-w; if (w>=0)
            { aa[i+1]=x+sqrt(w);aa[i+2]=x-sqrt(w); // w is >= 0 here
            }
        }
    }  return z; // z has to be > 0
}

main()
{ int i,j;
double a=82.0, b=18.0, c=92.2, d=33.6, e=2.3, // optimized distances
```

```
between pivot points
f2,ff,g,g2,h0,h1,h2,h3,ab,a2,b2,c2,d2,e2,v,v1,s,t,t2,x,x2,y,y2,s1,s2,w,z,v
v,p[5],hh,f=0.615184;
// f = average slope of line d

cout << "      (a)    1: b    2: c    3: d    4: e   squares' sum   enter
no. 1-4 and parameter";
cout << " to minimize sum (end with no.=0)" << endl;
j=1; while (j!=0)
    { s1=s2=0; a=100-b; g=ab=a+b; hh=ab+b; g2=f2=ab*ab; e2=e*e; a2=a*a;
b2=b*b; c2=c*c; d2=d*d;
    // calculation of coefficients p[i] for the 4th order equation needed
to solve for the
    // independent variable, when the dependent is known:
       ff=sqrt(1+1/(f*f)); vv=-d/ff; v1=sqrt(d2-vv*vv); w=hh+a; z=hh-a;
p[4]=w*w*(z*z+4*e2);
        w=vv*ff; w=f2*(e2+a2-c2+w*w); z=2*f2*vv*(e/f+a); p[0]=w*w-z*z;
p[2]=2*w*(hh*hh-a2);
        p[1]=w*(hh/f+e); p[1]-=2*f2*e*(e/f+a)*(hh+a); p[1]*=4*ab*vv;
w=e*(hh+a); z=vv*(e/f+a);
        w=z*z-w*w; z=vv*(hh/f+e); w+=z*z; w*=4*f2; p[2]+=w;
        p[3]=4*ab*vv*(hh+a)*((hh-a)*(hh/f+e)+2*(e/f+a)*e); w=hh+a; z=hh-a;
p[4]=w*w*(z*z+4*e2);
        for (i=0;i<4;i++) p[i]=p[i]/p[4]; quartic(p); w=0; for
(i=0;i<3;i+=2) w+=(i-1)*p[i];
        w-=50; s1+=w*w; // deviation² from distance 50[%]




    // Calculation of coefficients p[i] for the 4th order wquation needed
to solve for the
    // independent variable at the extremes of c²:
        hh/=g; h0=2*(e*v1-a*vv)/g; h3=-2*e*(hh+a/g)/g; h1=2*(vv*e/g-
hh*v1); h2=hh*hh-a2/g2;
        p[4]=4*(h3*h3+h2*h2); p[3]=4*(h1*h2+h0*h3); p[2]=h1*h1+h0*h0-
4*g2*(h2*h2+h3*h3);
        p[1]=-2*g2*(h0*h3+2*h1*h2); p[0]=g2*(g2*h3*h3-h1*h1); for
(i=0;i<4;i++) p[i]/=p[4];
        quartic(p); for (i=2;i<4;i++) // calculation of dependent from
independent variable
        { x=p[i]; x2=x*x; y2=f2-x2; y=sqrt(y2); s=(b*x-e*y)/ab; t=(a*y-
e*x)/ab; t2=t*t; z=x+s;
            z*=z; w=4*z*d2; v=z+t2; z=v+d2-c2; v=0.5*(t*z-sqrt(t2*z*z-
v*(z*z-w)))/v; w=vv-v;
            s2+=(4-i)*(4-i)*w*w; // weighted deviations² from average
        }
        cout << setw(8) << a << setw(8) << b  << setw(8) << c  << setw(8)
<< d << setw(8) << e
        << setw(14) << (s1+s2)/6.0 << "      "; cin >> j; switch(j)
        { case 1: cin >> b; break; case 2: cin >> c; break; case 3: cin >>
d; break; case 4:
            cin >> e; break; default: break;
        }
    }
}
```