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Analysis of S-Metric in Electron Scatterings

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Abstract:- In quantum electrodynamics, s-metrics are the best way to describe the scattering of the particle and it gives more accurate information about what happening in the process of scattering. in this article, I would like to do an analysis and comparison of the main five scatterings of an electron. Frist is Crompton scattering, second is bremsstrahlung, a third is an electron-positron annihilation, fourth is electron-electron scattering and fifth is electron-positron scattering. In this article, we are also discussing hamiltonian of the system of each scattering.

Keywords: Scattering, Amplitude, Momentum

I. INTRODUCTION

The research work is introducing some basics of quantum electrodynamics and it's the lagrangian equation and hemeltonian operator.it can build strong base of the work. Scattering of the electron is regarded as the main topic of this work. there is an analisis of deferent s-metrics.

II. PHENOMENON OF QUANTUM ELECTRODYNAMICS

In classical theory, when two negative charges come near, it repels each other and going far with respect to each other. And the positive and negative charges attract each other and come toward each other. But why that's happening no one knows. So as quantum physics does progress, Richard Feynman comes with a tremendous theory of quantum electrodynamics. And it solves the mystery of electromagnetism by proposing a theory of transmitting and receiving virtual quantum.[1] Hamiltonian of this process for a non-relativistic particle is described by, $H=H_p+H_I+H_C+H_{tr}$ where $H_P+H_I=\sum_n \frac{1}{2} m_n^{-1}\left(p_n - p_n A^{tr}(x_n)\right)^2$ is Hamiltonian of the particle of mass m_n. charge e, and coordinate x_n. and momentum p_n.[1] and Hamiltonian of the transverse field is $H_{tr}=\frac{1}{2}\sum_k \sum_{r=1}^{4}\left(\left((P_k^{(r)})^2 + k^2(q_k^{(r)})^2\right)\right)$where $P_k^{(r)}$is momentum conjugate $q_k^{(r)}$. And the hamiltonian of Coulomb interaction $H_e=\frac{1}{2}\sum_n\sum_m \frac{e_n e_m}{r_{nm}}$where $r_{nm}{}^2=(x_n-x_m)^2$. so, let's talk about the Lagrange of quantum electrodynamics. as per gauge-invariant interacting theory, $\mathcal{L}(\mathrm{x})=\mathcal{L}_e(x)+\mathcal{L}_\gamma(x)$ wher$\mathcal{L}_\gamma=-\frac{1}{4}\left[\partial_\mu A_v(x)-\partial_v A_\mu(x)\right][\partial^\mu A^\nu(x)-\partial^\nu A^\mu(x)]=-\left(\frac{1}{2}\right)F_{\mu v}F^{\mu\nu}$ and $\mathcal{L}_e=\bar{\psi}(x)(i\gamma^\mu D-m)\psi(x)$. That's why the gauge field we can write $D\psi(x,t)=\left(\nabla-\frac{i}{c}eA\right)\psi(x,t)$. so we can write Lagrange of quantum electrodynamics as $\mathcal{L}(x)=\bar{\psi}(x)(i\gamma^\mu D-m)\psi(x)-\left(\frac{1}{2}\right)F_{\mu v}F^{\mu\nu}$,so this is how we can represent the theory of quantum electrodynamic.

For all scattering, there are main four parts first is the propagator of particle. The second is the Feynman diagram third is 4×4 metrics in spinor space and the fourth is its s-metrics three scarring. first lets talk about The free-particle propagator. The free-particle propagator can now be constructed from the field operators as the vacuum expectation value $G(x,t;x',t')=\langle 0|\hat{T}\hat{\psi}(x,t)\hat{\psi}^\dagger(x',t')|0\rangle$ The free-particle propagator coincides with the Green function of the Schrödinger differential operator. Recall that a Green function of a homogeneous differential equation is defined by being the solution of the inhomogeneous equation with a δ-function $G(x,t;x',t')\left(-i\hbar\partial_t+\frac{\hbar^2}{2M}\partial_x^2\right)=i\hbar\delta(t-t')\delta^3(x-x')$, The righthand side follows directly from the fact that the field $\hat{\Psi}(x,t)$ satisfies the Schrödinger equation and the obvious formula $\partial_t\theta(t-t')=\delta(t-t')$ the free field propagator is calculated as follows$G(x,t;x',t')=\Theta(t-t')\langle 0|\hat{\Psi}(x,t)\hat{\Psi}^\dagger(x',t')|0\rangle$.Inserting the expansion $\Psi(x,t)=\int -d^3p\psi_p(x,t)\hat{a}(p)$,with the wave functions$\psi p(x,t)=\langle x,t|p\rangle=\langle x,t|\hat{a}_p^\dagger\rangle=\frac{1}{\sqrt{V}}\exp\left\{\frac{i}{\hbar}\left(p^m x-\frac{p^{m^2}}{2M}t\right)\right\}$, and using$[\hat{a}(p),\hat{a}^\dagger(p')]_\mp=\delta^3(p-p')$, so the right-hand side becomes$\Theta(t-t')\int d^3p d^3p'$ e^i([(px − p′x′) − (p²t/2M − p′²t′/2M)]/ℏ) $\langle 0|\hat{a}(p)\hat{a}^\dagger(p')|0\rangle = \Theta(t-t')\int d^3p$ e^i([p(x − x′) − p²(t − t′0/2M]/ℏ) . the factor after Θ(t−t′) is simply the one-particle matrix element of the time evolution operator$\langle 0|\hat{\Psi}(x,t)\hat{\Psi}^\dagger(x',t')|0\rangle=\langle 0|\hat{\Psi}(x)e^{-iH(t-t')/\hbar}\hat{\Psi}^\dagger(x')|0\rangle=\langle X|\hat{U}(t-t')|x'\rangle$. It describes the probability amplitude that a single Free Particle Has Propagated from x to x′ in the time t − t′ > 0.

And For $t - t' < 0$, G vanishes. So The Fourier-transformed propagator is $G(p,E) = \int d^2x \int_{-\infty}^{\infty} dt\, e^{-(px-Et)/\hbar} G(x,t) = \frac{i\hbar}{(E-P^2)/m+i\eta}$
the second part is the Feynman diagram. It content most of the information of scattering. As per reputation theory, The propagator of the free photon depends on the gauge. It is most simple in the Gupta-Bleuler quantization form, where $G_0^{\mu\nu}(x,x') = -g^{\mu\nu} G_0^{\mu\nu}(x,x') = -g^{\mu\nu} \int \frac{d^4}{(2\pi)^4} \frac{i}{q^2+i\eta} e^{ik(x-x')}$
the free particle propagator of the electron was given as $S(x-x') = \int \frac{d^4p}{(2\pi)^4} \frac{i}{\gamma p - m + i\eta} e^{ik(x-x')}$
as per wick expansion of $e^{iA^{int}}$, each contraction is represented by one of these two propagators $S(x-x'$ *and* $G0\mu\nu x - x'$ in the Feynman diagram, they are pictured by the line the lines gμνi(q2+i∈), and iγp−m. The interaction Lagrangian is $\mathcal{L}(x) = -e\bar{\psi}\gamma^\mu A_\mu(x)$,so now we can consrect vertex as $(-e\gamma^\mu)$. the four-fermion Coulomb interactions derived from an auxiliary interaction $A^{int} = \int d^4x\, j^0 A_0$, The photon propagator is contracted with an electron current as follows $\bar{u}(p', s_3')\gamma_\mu u(P, s_3')P^{\mu\nu}_{Phys'eff}(q)$, $q = p' - p$ the spinners on the right and left-hand side satisfy the Dirac equation, the current is conserved and satisfies $\bar{u}(p', s_3')\gamma_\mu u(P, s_3')q^\mu = 0$. the magnetic field caused by the orbital motion of an electron leads to a coupling of the orbital angular momentum L = x × p with a g-factor g = 1. In order to see this relative factor 2 most clearly, consider the interaction Hamiltonian $H^{int} = \int d^3x A(x)\langle p', s_3'|j(x)|p, s_3\rangle$For slow electrons, we may neglect quantities of second order in the momenta, so that the normalization factors E/M are unity, and we obtain $H^{int} = \int d^3x A(x)\, \bar{u}(p', s_3')\gamma_\mu u(P, s_3')e^{(p'-p)x}$. At this place, we make use of the so-called Gordon decomposition formula $\bar{u}(p', s_3')\gamma^\mu u(P, s_3') = \bar{u}(p', s_3')\left[\frac{1}{2m}(p'^\mu + p^\mu) + \frac{i}{2m}\sigma^{\mu\nu}q_\nu\right]u(P, s_3')$ where q ≡ p ′− p is the momentum transfer. This formula follows directly from the anticommutation rules of the γ-matrices and the Dirac equation. An alternative decomposition is $\langle P'|j^\mu|P\rangle = e\bar{u}(p', s_3')\left[\frac{1}{2m}(p'^\mu + p^\mu)F_1(q^2) + \frac{i}{2m}\sigma^{\mu\nu}q_\nu F_2(q^2)\right]u(P, s_3')$. thus the hamiltonian became $H^{int} = \int d^3x A(x)\, \bar{u}(p', s_3')(p + q - iq \times S)u(P, s_3')e^{-iqx}$ .The Green functions carry all pieces of information contained in the theory. In particular, they can be used to extract scattering amplitudes. The scattering amplitude of free particle can be defined like this $S(q^4, q^3|q^1, q^2) \equiv \frac{S_N(q^4, q^3|q^1, q^2)}{Z[0]}$,

III. RUTHERFORD SCATTERING

The scattering of electrons on the Coulomb potential of nuclei at charge Ze,

$$V_c(r) = -\frac{ZE^2}{4\pi r} = -\frac{Z\alpha}{r} \tag{1}$$

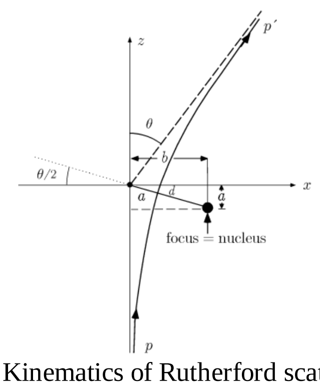


Fig 1: Kinematics of Rutherford scattering

The current density of a single randomly incoming electron is j = v/V . It would pass through the annular ring, with a probability per unit time

$$dp^{\cdot} = j2\pi b db \tag{2}$$

With this probability it winds up in the solid angle dΩ. , we find the differential cross section as

$$\frac{d\sigma}{d\Omega} = \frac{dP^{\cdot}}{d\Omega} = 2\pi b \frac{db}{d\Omega} = \frac{1}{4\sin 4\left(\frac{\theta}{2}\right)}\left(\frac{Z\alpha}{2E}\right)^2 \tag{3}$$

Let us now see how the above cross section formula is modified in a relativistic calculation involving Dirac electrons. The scattering amplitude is now became

$$S_{fi} = -ie\langle p', s_3' \left| \int d^4x \bar{\Psi}(x)(\gamma^\mu)\Psi(x) \right| p, s_3\rangle A_\mu(x) \quad (4)$$

where $A_\mu(x)$ has only the time-like component

$$A_0(x) = -\frac{Ze}{4\pi r} = -\int \frac{d^3q}{(2\pi)^3} e^{iqx} \frac{Ze}{|q|^2}. \quad (5)$$

The time-ordering operator has been dropped in (4) since there are no operators at different times to be ordered in first-order perturbation theory. By evaluating the matrix element of the current in (4), and performing the space-time integral we obtain

$$S_{fi} = i2\pi\delta(E' - E)\sqrt{\frac{M^2}{V^2E'E}}\,\bar{u}(p', s_3')\gamma^0 u(p, s_3)\,(Ze^2/|q|2\,)\,. \quad (6)$$

where E and E′ are the initial and final energies of the electron, which are in fact equal in this elastic scattering process

IV. COMPTON SCATTERING

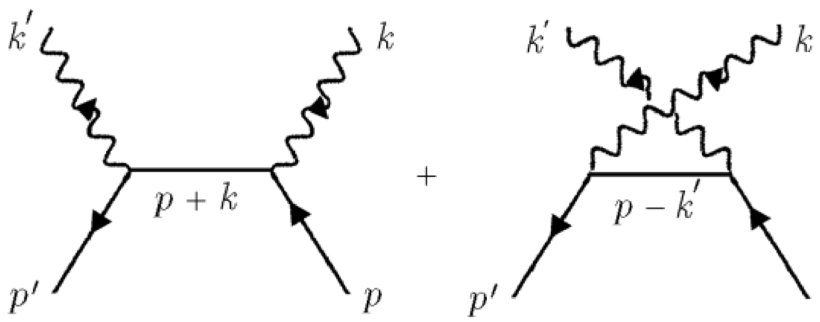


Fig 2 : Lowest-order Feynman diagrams contributing to Compton Scattering and giving rise to the Klein-Nishina formula.

A simple scattering process, whose cross section can be calculated to a good accuracy by means of the above diagrammatic rules, is photon-electron scattering, also referred to as Compton scattering. It gives an important contribution to the blue color of the sky. Consider now a beam of photons with four-momentum k_i and polarization $\lambda_i$impinging upon an electron target of four-momentum $p_i$and spin orientation σ_i. The two particles leave the scattering regime with four-momenta kf and pf , and spin indices λf , σf , respectively.scattering amplitude to this situation we have,

$$S_{fi} = S(p', s_3'; k', h'|k, h; p, s_3) \equiv \frac{\left\langle 0 \left| a(p', s_3')a(k'h')\hat{T}e^{-i\int_{-\infty}^{\infty} dt V_I(t)} a^+(k,h)a^+(P,s_3) \right| 0 \right\rangle}{\left\langle 0 \left| e^{-i\int_{-\infty}^{\infty} dt V_I(t)} \right| 0 \right\rangle} \quad (7)$$

$$e^{-i\int_{-\infty}^{\infty} dt V_I(t)} = e^{-ie\int d^4x\bar{\Psi}(x)(\gamma^\mu)\Psi(x)} \quad (8)$$

Expanding the exponential in powers of e, we see that the lowest-order contribution to the scattering amplitude comes from the second-order term which gives rise to the two Feynman diagrams shown in Fig. 2. In the first, the electron s1 of momentum p absorbs a photon of momentum k, and emits a second photon of momentum k ′ , to arrive in the final state of momentum p ′ . In the second diagram, the acts of emission and absorption have the reversed order. Before we calculate the scattering cross section associated with these Feynman diagrams.

Classically, the above process is described as follows. A target electron is shaken by an incoming electromagnetic field. The acceleration of the electron causes an emission of antenna radiation. For a weak and slowly oscillating electromagnetic field of amplitude, the electron is shaken non-relativistically and moves with an instantaneous acceleration

$$\ddot{X} = \frac{e}{M}E = \frac{e}{M} \in E_0 e^{-i\omega t + ik\cdot x}, \quad (9)$$

e direction of the emitted light. For a later comparison with quantum electrodynamic calculations we associate this emitted power with a differential cross section of the electron with respect to light. According to the definition in a cross section is obtained by dividing the radiated power per unit solid angle by the incident power flux density $cE_0^2/2$ This yields

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi Mc^2}\right)^2 \sin^2\beta = r_e^2 \sin^2\beta, \quad (10)$$

$r_e = \frac{e^2}{4\pi Mc^2} = \frac{h\alpha}{Mc} \approx 2.82 \times 10^{-13}$cm, is the classical electron radius.

The scattering amplitude corresponding to the two Feynman diagrams in Fig. 12.6 is obtained by expanding : $S\,(q^4,q^3|q^1,q^2) \equiv \frac{S_N(q^4,q^3|q^1,q^2)}{Z[0]}$ up to second order in e we get here we use $\mathbb{P} is\ \gamma^\mu P_\mu$ and K is $\gamma^\mu K$

$$S_{fi} = -e^2 \int d^4x d^4y \left\langle 0 \left| a_{p',s_3'} \hat{T}\, \bar{\Psi}(y) \notin' \Psi(y)\bar{\Psi}(x) \notin' \Psi(x) a^\dagger_{p,s_3} \right| 0 \right\rangle,$$

After Fourier-expanding the intermediate electron propagator, $G_0(y,x) = \int \frac{d^4p^i}{(2\pi)^4} e^{-ip^i(y-x)} \left(\frac{i}{\gamma p^i - M}\right)$,

$$S_{fi} = -e^2 \int\ d^4x d^4y\, e^{k'y-kx} \int \frac{d^4p^i}{(2\pi)^4} \left(\frac{M}{\sqrt{V2EE'2\omega 2\omega'}}\right)$$
$$\times\, [e^{i\left((p'-p^i)y-(p-p^i)x\right)}\, \bar{u}(p',s_3') \notin'^{*} \left(\frac{i}{\mathbb{P}^i - M}\right) \notin u(P,s_3')$$
$$e^{i\left((p'-p^i)x-(p-p^i)y\right)}\, \bar{u}(p',s_3') \notin \left(\frac{i}{\mathbb{P}^i - M}\right) \notin' u(P,s_3')]$$

$\mathbb{P} is\ \gamma^\mu P_\mu$ and K is $\gamma^\mu k_\mu$ spatial integrals fixes the intermediate momentum in accordance with energy-momentum conservation, the other yields a δ (4)-function for overall energymomentum conservation. The result is

$$S_{fi} = -i(2\pi)^4 \delta^4(p' + k' - p - k) e^2 \left(\frac{M}{\sqrt{V2EE'2\omega 2\omega'}}\right)\ \bar{u}(p',s_3') H \notin u(P,s_3')\,.$$

where H is the 4 × 4-matrix in spinor space

$$H \equiv \notin'^{*} \left(\frac{(\mathbb{P}+K)+M}{(p+k)^2 - M}\right) \notin\ + \notin \left(\frac{(\mathbb{P}-K)+M}{(p+k')^2 - M}\right) \notin'^{*}$$

We have written ∈(k,h), ωk as ∈, ω, and ∈ (k ′, h′), ωk ′ as ∈′, ω′ , respectively, with a similar simplification for E and E′ . The second term of the matrix H arises from the first by the crossing symmetry ∈↔ ∈'
$\in \leftrightarrow \in', k \leftrightarrow -k'$.
Simplifications arise from the properties (12.246). It can, moreover, be simplified by recalling that external electrons and photons are on their mass shell, sothat
$P^2 = p'^2 = M^2, k^2 = k'^2 = 0,\ k \in = k', \in' = 0.$
A further simplification arises by working in the laboratory frame in which the initial electron is at rest, p = (M, 0, 0, 0). Also, we choose a gauge in which the polarization vectors have only spatial components. Then $p \in = p \in' = 0$,
since p has only a temporal component and ∈ only space components. We also use the fact that H stands between spinors which satisfy the Dirac equation $(\mathbb{P} - M)u(p,s_3) = 0, \bar{u}(p,s3)(\mathbb{P} - M) = 0$.. Further we use the commutation rules (4.566) for the gamma matrices to write

$$\mathbb{P} \notin = -\mathbb{P} \notin + 2p \in$$

The second term vanishes by virtue of Eq. (12.248). Similarly, we see that $\mathbb{P}$ anticommutes with ∉ ′ . Using these results, we may eliminate the terms ℙ+M occurring in M. Finally, using Eq. (12.247), we obtain
$\bar{u}(p',s_3') H u(P,s_3') = \bar{u}(p',s_3') \left\{ \notin'^{*} \left(\frac{K}{2pk}\right) \notin + \notin \left(\frac{K'}{2pk'}\right) \notin'^{*} \right\} u(P,s_3')$
To obtain the transition probability, we must take the absolute square of this. If we do not observe initial and final spins, we may average over the initial spin and sum over the final spin directions. This produces a factor 1/2 times the sum over both spin directions, which is equal to $F = \sum_{s_3',s_3} \left|\bar{u}(p',s_3') H u(P,s_3')\right|^2 = \sum_{s_3',s_3} \bar{u}(p',s_3') H u(P,s_3') \bar{u}(p',s_3') H u(P,s_3')$

V. BREMSSTRAHLUNG

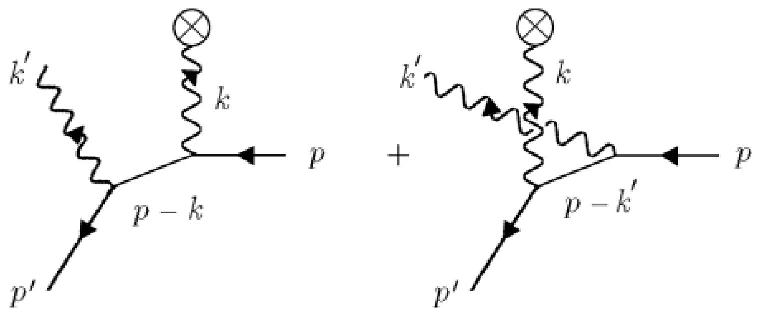


Fig :3 Trajectories in the simplest classical Bremsstrahlung process: An electron changing abruptly its momentum

Consider a trajectory in which a particle changes its momentum abruptly from p to p ′
The trajectory may be parameterized as:

$$x_\mu(\tau) = \{\tau p / M, \tau p' / M \text{ for } \tau\} < 0, \tau > 0,$$

Where τ is the proper time. The electromagnetic current associated with this trajectory is

$$j_\mu(x) = \frac{e \int d\tau \ dx^\mu(\tau)}{d\tau} \ \delta^{(4)}\big(x - x(\tau)\big)$$

After a Fourier decomposition of the δ-functions, this can be written as

$$j_\mu(x) - \int \frac{d^4k}{(2\pi)^4} e^{-ikx} j^\mu(k) \ ,$$

We now turn to the more realistic problem of an electron scattering on a nucleus. Here the electron changes its momentum within a finite period of time rather than abruptly. Still, the Bremsstrahlung will be very similar to the previous one. Let us consider immediately a Dirac electron, i.e., we study the Bremsstrahlung emitted

in Mott scattering. The lowest-order Feynman diagrams governing this process are shown in Fig. 12.15. The vertical photon line indicates the nuclear Coulomb potential

$$V_C(x) = -\frac{Z\alpha}{4\pi r}.$$

The scattering amplitude is found from the Compton amplitude by simply interchanging the incoming photon field

$$eA^\mu(x) = \epsilon^\mu_{k,,\lambda} \ e^{-ikx}\sqrt{2Vk_0}$$

The scattering amplitude is therefore

$$S_{fi} = i\frac{4\pi Z\alpha e}{|q|^2} 2\pi\delta(p'^0 + k' - p^0)\left(\frac{M}{\sqrt{V^2E'E}}\right)\left(\frac{1}{\sqrt{2Vk^{0'}}}\right)$$
$$\times [\notin'^* \left(\frac{1}{\mathbb{P}' + K' - M}\right)\gamma^0 + \notin \left(\frac{1}{\mathbb{P}' - K' - M}\right)\notin']$$

Where

$$q = p' + k' - p$$

is the spatial momentum transfer. The amplitude conserves only energy, not spatial momentum. The latter is transferred from the nucleus to the electron without any restriction. The unpolarized cross section following from Sfi is

$$d\sigma \ = M^2Z^2(4\pi\alpha)^3 \frac{1}{2k^{0'}E'E} \frac{1}{v} \int \frac{d^3p' d^3k'}{(2\pi)^6} 2\pi\delta(p'^0 + k'^0 - p^0)(\frac{F}{|q|^2}) \ ,$$

where we have used the incoming particle current density v/V = p/EMV and set

$$F \ \equiv \frac{1}{2}\sum_h tr\,[(\notin'^* \left(\frac{\mathbb{P}' + K' + M}{2\mathbb{P}'K'}\right)\gamma^0 + \gamma^0\left(\frac{\mathbb{P} \ - K' + M}{2pk}\right)\notin') \ \frac{\mathbb{P} + M}{2M}$$
$$(\gamma^0\left(\frac{\mathbb{P} \ + K + M}{2\mathbb{P}'K'}\right)\notin' + \notin'\left(\frac{\mathbb{P} \ - K \ + M}{2pk}\right)\gamma^0) \ \frac{\mathbb{P}' + M}{2M}]$$

VI. ELETRON-ELECTRON SCATTERING

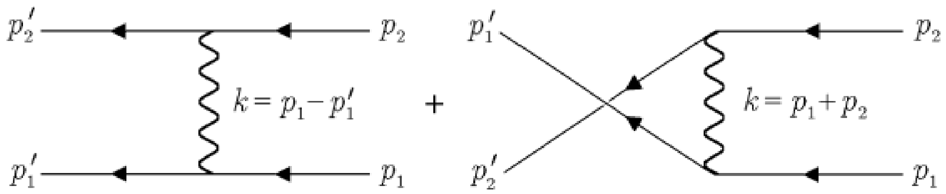


Fig:4 Lowest-order Feynman diagrams contributing to electron-electron scattering

The leading Feynman diagrams are shown in Fig. 12.17. The associated scattering amplitude is given by

$$S_{fi} = (2\omega)^4\delta^4(p_1' + p_2' - p_1 - p_2)(-ie)^2(12.352)$$
$$\times$$

$$\Big[-\bar{u}(p_1', \varepsilon_1')\gamma^\nu u(P_1, \varepsilon_1) - \frac{ig_{\nu\rho}}{(p_1 - p'_1)^\wedge 2}\bar{u}(p_2', \varepsilon_2')\gamma^\rho u(P_2, \varepsilon_2) + \quad \bar{u}(p_2', \varepsilon_2')\gamma^\nu u(p_1, \varepsilon_1) -$$

igνρ(p1−p′2)^2up1′, 𝜀1′γρuP2, 𝜀2

For the scattering amplitude s_{fi} defined by

$$S_{fi} = -ie^4(2\pi)^4\delta^4(p_1' + p_2' - p_1 - p_2)t_{fi}$$

We find

$$t_{fi} = \frac{\bar{u}(p_1',\varepsilon_1')u(P_1,\varepsilon_1)\bar{u}(p_2',\varepsilon_2')\gamma_\nu u(P_2,\varepsilon_2)}{(p_1-p_1')^2} - \frac{\bar{u}(p_2',\varepsilon_2')\gamma^\nu u(p_1,\varepsilon_1)\bar{u}(p_1',\varepsilon_1')\gamma_\nu u(P_2,\varepsilon_2)}{(p_1-p_2')^2}$$

There is a manifest antisymmetry of the initial or final states accounting for the Pauli principle. Due to the identity of the electrons, the total cross section is obtained by integrating over only half of the final phase space. Let us compute the differential cross section for unpolarized initial beams, when the final polarizations are not observed. The kinematics of the reaction in the center of mass frame is represented in Fig. 12.18, where θ is the scattering angle in this frame. The energy E is conserved, and we denote$|p| = |p'| = p = \sqrt{(E^2 - m^2)}$. Using the general formula (9.311) with the covariant fermion normalization V → 1/E, we obtain d

$$\frac{d\sigma}{d\Omega_{CM}} = \frac{M^2e^4}{4E^2(2\pi)^2}\ |t_{fi}|2\,.$$

The bar on the right-hand side indicates an average over the initial polarizations and a sum over the final polarizations. More explicitly, we must evaluate the traces1

$$|t_{fi}|^2 = \frac{1}{4}\sum_{\varepsilon1\varepsilon2\varepsilon'1\varepsilon'2}|t_{fi}|^2 \;= \frac{1}{4}\{\mathrm{tr}\left(\gamma_\nu\,\frac{\mathbb{P}_1+M}{2M}\,\gamma_\rho\,\frac{\mathbb{P}'_1+M}{2M}\right) + \mathrm{tr}\left(\gamma^\nu\,\frac{\mathbb{P}+M}{2M}\,\gamma^\rho\,\frac{\mathbb{P}'_2+M}{2M}\right)\frac{1}{\left[(p_1'-p_1)^2\right]^2}$$

$$-\mathrm{tr}\left(\gamma_\nu\,\frac{\mathbb{P}+M}{2M}\gamma_\rho\,\frac{\mathbb{P}_2+M}{2M}\right)\left(\gamma^\nu\,\frac{\mathbb{P}+M}{2M}\,\gamma^\rho\,\frac{\mathbb{P}'_1+M}{2M}\right)\frac{1}{(p_1'-p_1)^2(p_2'-p_1)^2} + (p_1' \leftrightarrow p_2')\}\ .$$

This can be expressed in terms of the Mandelstam variables s, t, u

$$|t_{fi}|^2 = \frac{1}{2M^4}\left\{\frac{1}{t^2}\left[\frac{s^2+u^2}{4} + 2m^2(t-m^2)\right] + \frac{1}{u^2}\left[\frac{s^2+t^2}{4} + 2m^2(u-m^2)\right] + \frac{1}{tu}\left[\left(\frac{s}{2} - m^2\right)\left(\frac{s}{2} - 3m^2\right)\right]\right\}.$$

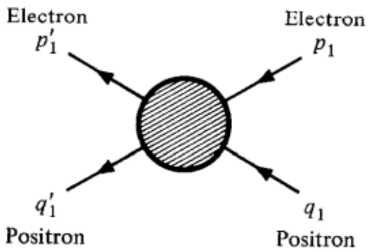
This is leads to the Møller formula which is describe here

$$\frac{d\sigma}{d\Omega_{CM}} = \alpha^2(2E_{CM}^2 - M^2)^2/4E_{CM}^2(E_{CM}^2 - M^2)^2\left[\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta} + \frac{(E_{CM}^2 - M^2)^2}{(2E_{CM}^2 - M^2)^2}\left(1 + \frac{3}{\sin^2\theta}\right)\right].$$

Comparing (12.364) with the classical Rutherford formula for Coulomb scattering in Eq. (12.194), we see that the forward peak is the same for both if we set Z = 1 and replace M by the reduced mass M/2. The particle identity yields, in addition, the backward peak.

VII. ELETRON-POSITRON SCATTERING

Let us now consider electron-positron scattering. The kinematics and lowest-order diagrams are depicted in Figs. 5 and 6Polarization indices are omitted



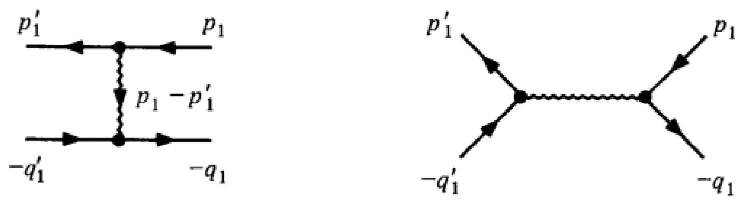


Fig 5: General form of diagrams contributing to electron-positron scattering.
Fig 6: Lowest-order contributions to electron-positron scattering

and in Fig. 6 four-momenta are oriented according to the charge flow. The scattering amplitude may then be obtained by substituting

$$p_2 \to q_1',\quad u(p_2) \to v(q_1'),$$
$$p_2' \to -q_1',\ u(p_2') \to v(q_1'),$$

and by changing the sign of the amplitude. The center of mass cross section is then given by the formula

dσ dΩ = M4 e 4 4E2 CM(2π) 2 |tf i| 2
with

$$|t_{fi}|^2 = \frac{1}{2M^4}\frac{\{[(p_1,q_1')^2 + (p_1,q_1)^2 - 2M^2(p_1q_1 - p_1q_1')]}{[(p_1' + p_1)^2]^2} + \frac{\{[(p_1,q_1')^2 + (p_1,p'_1)^2 + 2M^2(p_1p'_1 - p_1q_1')]}{[(p_1 + p_1)^2]^2}$$
$$+ 2\frac{\{[(p_1,q_1')^2 + 2M^2(p_1q_1')]}{(p_1 - p'_1)^2(p_1 + q_1)^2}.$$

It is then straightforward to derive the cross section formula first obtained by Bhabha (1936)
The results of Eqs. (12.362) and (12.368) may be compared with experimental data. At low energies we show in Fig. 12.21 some experimental data for electronelectron scattering at 90 degrees [7]. Møller's formula (12.362) is a good agreement with the data. The agreement confirms the fact that the spin of the electron is really 1/2. If it was zero, the agreement would have been bad (see the dashed curve in Fig. 12.21).
Electron-positron scattering data are fitted well by Bhabha's cross section, and the annihilation term is essential for the agreement. The energy of the incident particle in the laboratory frame plotted on the abscissa is chosen in the intermediate range where neither the nonrelativistic nor the ultrarelativistic approximation is valid. The numerical values show a significant departure from the ratio 2:1 between e −e − and e −e + cross sections, expected on the basis of a naive argument of indistinguishability of the two electrons.

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