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On the Flow of a Dusty Gas through a Channel over a Porous Layer Saturated with a Dusty Gas

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Abstract: Flow through a free-space channel underlain by a porous layer is considered. Both regions are saturated with particle-laden fluid driven by the same pressure gradient. Velocity distribution in both regions are derived and velocity and shear stress conditions at the interface are obtained. Effects of mixture and porous medium parameters on the flow characteristics are investigated in Cartesian and in curvilinear coordinates. Analysis shows that when the dust-phase velocity is the product of a position function and the fluid-phase velocity, the particle number density does not influence conditions at the interface between the two layers.
Key Word:Fluid-particle mixture; Porous layers; Slip flow; Interfacial conditions.

I. INTRODUCTION

Fluid-particle transport through porous media is rooted in the many applications in which this mixture flow occurs. These applications involve deep-bed filtration processes, industrial design of liquid-dust separators, slurry transport in porous media, water purification plants, and the natural solute transport and the transport of suspended contaminants in deep repositories (*cf*. [10-13], [23-31] and the references therein). These and many other applications mandate the need for flow modelling and the solution to initial and boundary value problems, [18-22].

An important aspect of modelling fluid-particle flow through porous structures is the continuum approach that involves intrinsic averaging of Saffman's dusty gas equations, [27], over a representative control volume, [14]. Various models have been introduced to describe flow situations in a variety of settings and involve assumptions on the porous media, on the flow, and on the empirical parameters of the flow and the medium, [1, 2, 3, 4, 7, 8, 16, 17, 20, 21, 22]. Appropriate boundary conditions on the flowing phases have been proposed for some models, and numerical simulation has been carried out, [7, 8, 18, 19], to better understand the behaviour of the flowing phases and the effects of the media parameters on the flow. Effects of the porous structure on the flowing phases takes into account the Darcy resistance, Forchheimer effects, or the porous microstructure that is based on granular or consolidated media considerations, [14].

A recent model developed by Roach *et.al.*, [26], accounts for pressure-dependent viscosity of the fluid. However, an early model that uses the Darcy resistance to account for the effects of the porous medium on flowing phases was developed by Hamdan and Barron, [17]. This model involves the viscous shear effects of Brinkman's equation and Lapwood's macroscopic inertial effects, and can be used to describe dusty gas flow with either constant or variable number density through a porous structure when a macroscopic boundary is present. The model is based on Saffman's dusty gas model [27] and has conveniently been labelled as the Darcy-Lapwood-Brinkman-Saffman (DLBS) model. It can be argued that the DLBS model is useful in describing flow through a mushy zone undergoing rapid freezing. The model has been applied to study flow into a two-dimensional sink [8].

Of interest to the current work is the flow of a dusty gas in free space over a porous layer containing a dusty gas. This problem is reminiscent of the Navier-Stokes flow over a Darcy porous layer that was introduced by Beavers and Joseph [9] and gave rise to the popular Beavers and Joseph slip condition. In the analysis to follow we consider flow through a free-space channel bounded on one side by a solid, impermeable wall and bounded by a porous layer on the other side. The porous layer is bounded by a solid, impermeable wall on one of its sides, as shown in **Fig. 1**, or could extend to infinity,

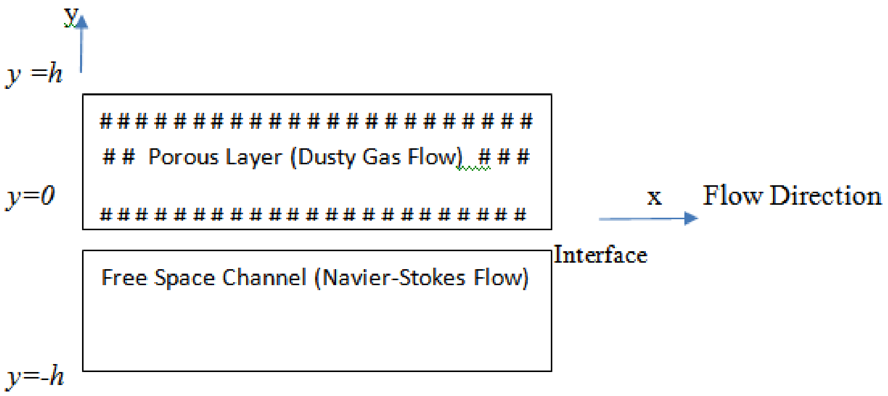

Fig. 1. Representative Sketch

Flow through the free-space channel is governed by Saffman's dusty gas model, [27], and through the porous layer by the DLBS model, [17]. While we take the number density to be variable, we introduce the simplifying assumption that the dust-phase velocity is the product of a position function and the fluid-phase velocity. At the interface between the two regions, we implement velocity and shear stress continuity conditions. Velocity profiles in both regions are determined, together with velocity and shear stress at the interface. We solve the stated problem in both rectangular and curvilinear coordinates in order to better understand the influence of the particle number density on the velocity profiles and shear stress within the assumptions made.

II. PROBLEM FORMULATION AND GOVERNING EQUATIONS

Amodeldescribingtheflowofadustygas,through an isotropicporousmedium, was developed by Hamdan and Barron, [17]. The model is based on Saffman's dusty gas model, [27], and has conveniently been labelled the Darcy-Lapwood-Brinkman-Saffman (DLBS) model so that it is not confused with the Saffman's dusty gas model that is only valid in free-space. DLBS assumes a small concentration of dust particles in the fluid mixture. It also assumes a two-way interaction between the flowing phases in that the force exerted by one phase on the other phase is proportional to the relative velocity of the phases involved. However, the porous microstructure exerts a damping force (Darcy resistance) expressed in terms of the fluid-phase velocity and not in terms of the relative velocity of the phases involved.

Equations governing the flow through a porous sediment are thus given by the following continuity and momentum equations, for each of the phases involved:

Fluid-phase:

$$\nabla \cdot \vec{u} = 0 \qquad \dots (1)$$

$$\rho(\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \mu_e \nabla^2 \vec{u} + KN(\vec{v} - \vec{u}) - \frac{\mu}{\eta}\vec{u} \qquad \dots (2)$$

Dust-phase:

$$\nabla \cdot N\vec{v} = 0 \dots (3)$$

$$mN(\vec{v} \cdot \nabla)\vec{v} = KN(\vec{u} - \vec{v}) \qquad \dots (4)$$

wherein ρ is the fluid density, m is the mass of a dust particle, N is the particle number density (number of particles per unit volume), k is the Stokes' coefficient of resistance, p is the pressure, μ is the fluid-phase base viscosity, η is the permeability, μ_e is the effective viscosity (that is, viscosity of fluid-phase saturating the porous medium), $\vec{u}, \vec{v}$ are the fluid- and dust-phase velocity fields, respectively.

In the absence of dust, $N = 0$, equations (3) and (4) are identically satisfied, and (2) reduces to the Darcy-Lapwood-Brinkman (DLB) momentum equation governing single phase flow through porous media. In

addition, if the flow is in free-space, that is if $\eta \to \infty$, and $\mu_e = \mu$, equations (1)-(4) reduce to Saffman's dusty gas model, [27].
System (1)-(4) represents a determinate system of 8 scalar equations in the 8 unknowns $\vec{u}$, $\vec{v}$, p and N as functions of position.
Assuming the flow to be in two space dimensions, and letting $\alpha(x, y)$ be a scalar function of position such that

$$\vec{v} = \frac{\alpha(x,y)}{N(x,y)}\vec{u} \quad \ldots (5)$$

then using (5) in (3) we obtain:

$$\alpha\nabla\cdot\vec{u} + \vec{u}\cdot\nabla\alpha = 0 \ldots (6)$$

whence using (1) in (6), we get

$$\nabla\alpha\cdot\vec{u} = 0 \quad \ldots (7)$$

For flow in two space dimensions, equation (7) signifies that α is constant along the streamlines.
Upon using (5) in (2) and (4) we obtain, respectively:

$$(\vec{u}\cdot\nabla)\vec{u} = -\frac{\nabla p}{\rho} + \frac{\mu_e}{\rho}\nabla^2\vec{u} + \frac{KN}{\rho}\left(\frac{\alpha}{N} - 1 - \frac{\mu}{\eta KN}\right)\vec{u} \quad \ldots (8)$$

$$(\vec{u}\cdot\nabla)\left(\frac{\alpha}{N}\right)\vec{u} = \frac{KN}{\alpha m}\left[1 - \frac{\alpha}{N}\right]\vec{u} \quad \ldots (9)$$

In what follows we will present solutions in two coordinate systems: ***a)*** In (*x*,*y*) coordinates and, ***b)*** in (ϕ, ψ) coordinates.

(a) Solution in (x, y) Coordinates

Letting $\vec{u} = (u, v)$ and $\beta(x, y) = \frac{\alpha(x,y)}{N(x,y)}$, and assuming $\mu = \mu_e$, equation (9) takes the following components' form

$$\left(u\frac{\partial\beta u}{\partial x} + v\frac{\partial\beta u}{\partial y}\right) = \frac{K}{\beta m}[1-\beta]u \ldots (10)$$

$$\left(u\frac{\partial\beta v}{\partial x} + v\frac{\partial\beta v}{\partial y}\right) = \frac{K}{\beta m}[1-\beta]v \quad \ldots (11)$$

For unidirectional flow, $v = 0, \frac{\partial u}{\partial x} = 0,$ and $u = u(y)$. Thus, (11) is automatically satisfied, and (10) reduces to

$$\left[u\frac{\partial\beta}{\partial x}\right] = \frac{K}{\beta m}[1-\beta] \quad \ldots (12)$$

Now, (7) reduces to

$$u\frac{\partial\beta N}{\partial x} = 0 \quad \ldots (13)$$

which implies that $\alpha(x, y) = \beta(x, y)N(x, y)$ is a function of y. Since $u = u(y)$ and $\alpha = \alpha(y)$, we can take $N = N(y)$ and $\beta = \beta(y)$. Equation (10) then implies that

$$\frac{K}{\beta m}[1-\beta] = 0 \ldots (14)$$

or $\beta = 1$, hence $\alpha = N$ and the dust-phase tangential velocity is the same as the fluid-phase tangential velocity. With the dust-phase equation of motion automatically satisfied, the fluid-phase momentum equation reduces to

$$u_{yy} - \frac{u}{\eta} = \frac{p_x}{\mu} \qquad \ldots (15)$$

Equation (15) is the same equation one would obtain in the unidirectional flow through a porous medium governed by Brinkman's equation. Clearly, (15) is independent of dust-phase effects, and its solution is independent of the presence of a dust-phase in the flow field.

Solution to (15) is given by

$$u = A\cosh\frac{y}{\sqrt{\eta}} + B\sinh\frac{y}{\sqrt{\eta}} - \frac{\eta p_x}{\mu} \qquad \ldots (16)$$

with

$$u_y = \frac{A}{\sqrt{\eta}}\sinh\frac{y}{\sqrt{\eta}} + \frac{B}{\sqrt{\eta}}\cosh\frac{y}{\sqrt{\eta}} \qquad \ldots (17)$$

As $\eta \to \infty$, equation (15) reduces to:

$$u_{yy} = \frac{p_x}{\mu} \qquad \ldots (18)$$

Equation (16) is the governing equation for unidirectional flow involving Saffman's dusty gas model in free space. Clearly, it is the same equation that Navier-Stokes equations would reduce to for flow in the same configuration, and it is free and independent of dust effects.

Solution to (16) takes the form:

$$u = \frac{p_x}{2\mu}y^2 + Cy + D \qquad \ldots (19)$$

with

$$u_y = \frac{p_x}{\mu}y + C \qquad \ldots (20)$$

Equations (16) and (19) must satisfy the no-slip at $y = h$ and $y = -h$, respectively, namely

$$u(h) = 0 \qquad \ldots (21)$$

$$u(-h) = 0 \qquad \ldots (22)$$

and the following velocity and shear stress continuity conditions at the interface $y = 0$:

$$u(0^+) = u(0^-) \qquad \ldots (23)$$

$$u_y(0^+) = u_y(0^-) \ldots (24)$$

From (16) and (17), we obtain, respectively,

$$u(0^+) = A - \frac{\eta p_x}{\mu} \qquad \ldots (25)$$

$$u_y(0^+) = \frac{B}{\sqrt{\eta}} \qquad \text{... (26)}$$

From (19) and (20), we obtain, respectively,

$$u(0^-) = D \qquad \text{... (27)}$$

$$u_y(0^-) = C \qquad \text{... (28)}$$

Conditions (21)-(28) yield the following values for $A, B, C,$ and D:

$$A = \left[\frac{\sigma + \left[1 - \frac{\sigma^2}{2}\right] \sinh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \frac{\eta p_x}{\mu} \qquad \text{... (29)}$$

$$B = \left[\frac{1 - \left[1 - \frac{\sigma^2}{2}\right] \cosh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \frac{\eta p_x}{\mu} \qquad \text{... (30)}$$

$$C = \left[\frac{1 - \left[1 - \frac{\sigma^2}{2}\right] \cosh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \frac{\sqrt{\eta} p_x}{\mu} \qquad \text{... (31)}$$

$$D = \left[\frac{\sigma + \left[1 - \frac{\sigma^2}{2}\right] \sinh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]} - 1\right] \frac{\eta p_x}{\mu} \qquad \text{... (32)}$$

where $\sigma = \frac{h}{\sqrt{\eta}}$.

Using (29)-(32) in (16) and (19) gives the following velocity profile in the porous layer and in the free-space channel, respectively:

$$u(y) = \left\{\left[\frac{\sigma + \left[1 - \frac{\sigma^2}{2}\right] \sinh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \cosh\frac{y}{\sqrt{\eta}} + \left[\frac{1 - \left[1 - \frac{\sigma^2}{2}\right] \cosh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \sinh\frac{y}{\sqrt{\eta}} - 1\right\} \frac{\eta p_x}{\mu} \qquad \text{... (33)}$$

$$u(y) = \frac{p_x}{2\mu} y^2 + \left[\frac{1 - \left[1 - \frac{\sigma^2}{2}\right] \cosh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \frac{\sqrt{\eta} p_x}{\mu} y + \left[\frac{\sigma + \left[1 - \frac{\sigma^2}{2}\right] \sinh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]} - 1\right] \frac{\eta p_x}{\mu} \qquad \text{... (34)}$$

Using either (33) or (34) we obtain the velocity and shear stress at the interface, respectively as:

$$u(0) = \left[\frac{\sigma + \left[1 - \frac{\sigma^2}{2}\right] \sinh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]} - 1\right] \frac{\eta p_x}{\mu} \qquad \text{... (35)}$$

$$u_y(0) = \left[\frac{1 - \left[1 - \frac{\sigma^2}{2}\right] \cosh\sigma}{[\sigma\cosh\sigma + \sinh\sigma]}\right] \frac{\sqrt{\eta} p_x}{\mu} \qquad \text{... (36)}$$

Equations (33)-(36) suggest the following observation.

Observation 1: The velocity profiles in the porous layer and in the free-space channel, and the velocity and shear stress at the interface between layers, are independent of the dust particle number density, N(x,y). Rather, they depend on layers' widths, driving pressure gradient, permeability and viscosity.

(a) Solution in (ϕ, ψ) Coordinates

In their analysis of admissible geometries in the flow of a dusty gas in a porous sediment, Awartani and Hamdan [5, 6] transformed the flow equations into the (ϕ, ψ) net in which $\psi = constant$ represents the streamlines of the flow and $\phi = constant$ represents their orthogonal trajectories. The velocity vector is tangent to the curves $\psi = constant$, and makes an angle $\vartheta(\phi, \psi)$ with the x-direction. The squared element of arc length along any curve is given by

$$dS^2 = E_1^2(\phi, \psi)d\phi^2 + E_2^2(\phi, \psi)d\psi^2 \quad \ldots (37)$$

where E_1andE_2 are metric coefficients that are related to $\vartheta(\phi, \psi)$ by

$$\vartheta_\phi = -\frac{(E_1)_\psi}{E_2} \quad \ldots (38)$$

$$\vartheta_\psi = \frac{(E_2)_\phi}{E_1} \quad \ldots (39)$$

Assuming that $\vartheta(\phi, \psi)$ is continuous then $\vartheta_{\phi\psi} = \vartheta_{\psi\phi}$, which implies that E_1andE_2 satisfy Gauss' equation, namely

$$(\frac{(E_2)_\phi}{E_1})_\phi + (\frac{(E_1)_\psi}{E_2})_\psi = 0 \quad \ldots (40)$$

Equation (7) implies that α is constant along streamlines. It reduces to $\alpha_\phi = 0$ or

$$\alpha = \alpha(\psi) \quad \ldots (41)$$

Awartani and Hamdan, [5], designated $U = |\vec{u}|$ to be the speed of the fluid-phase, and $\vec{\omega}$ the vorticity vector of the fluid-phase, and chose the orthornomal set of vectors $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}\}$ to represent a right-handed coordinate system, with $\overrightarrow{e_1}$ tangent to the streamlines, $\overrightarrow{e_2}$ is normal to $\overrightarrow{e_1}$, and $\overrightarrow{e_3}$ normal to the plane of the flow, and expressed velocity and vorticity vectors as

$$\vec{u} = U\overrightarrow{e_1} \quad \ldots (42)$$

$$\vec{\omega} = (0,0,\omega) = -\frac{(UE_1)_\psi}{E_1E_2}\overrightarrow{e_3} \quad \ldots (43)$$

With the above notation, they [5] expressed equation (9) in the following components' forms in the ϕ and ψ directions, respectively:

$$U_\phi + NU(\frac{1}{N})_\phi = \frac{k}{m}\left\{\frac{N}{\alpha}\right\}^2\left(1 - \frac{\alpha}{N}\right)E_1 \quad \ldots (44)$$

$$-\frac{U^2}{E_1}\left\{\frac{\alpha}{N}\right\}^2 (E_1)_\psi = 0 \quad \ldots (45)$$

Equation (45) implies $(E_1)_\psi = 0$ or

$$E_1 = E_1(\phi) \quad \ldots (46)$$

and, consequently, (38) and (39) imply that

$$\vartheta = \vartheta(\psi) \quad \ldots (47)$$

Equation (47) implies that every streamline $\psi = \text{constant}$ makes a constant angle with the x-axis, and the flow is either radial or in parallel straight lines.

With $(E_1)_\psi = 0$, Gauss' equation reduces to

$$(\frac{(E_2)_\phi}{E_1})_\phi = 0 \quad \ldots (48)$$

and vorticity surviving component becomes

$$\omega = -\frac{U_\psi}{E_2} \quad \ldots (49)$$

Awartani and Hamdan [5] showed that the fluid-phase momentum equation (8) takes the following components' form in the the ϕ and ψ directions, respectively:

$$UU_\phi + \frac{1}{\rho}p_\phi = -\frac{\mu}{\rho}\frac{E_1}{E_2}\omega_\psi + \frac{kN}{\rho}\left[\frac{\alpha}{N} - 1 - \frac{\mu}{\eta kN}\right]E_1U \quad \ldots (50)$$

$$\frac{1}{\rho}p_\psi = \frac{\mu}{\rho}\frac{E_2}{E_1}\omega_\phi \quad \ldots (51)$$

Fluid-phase continuity equation becomes:

$$(E_2U)_\phi = 0 \quad \ldots (52)$$

Governing equations have thus been reduced to (41), (46), (44), (49), (50), (51), and (52) in the 7 unknowns $U, \omega, p, \alpha, N, E_1, E_2$, with E_1, E_2 satisfying Gauss' equation (48)

III. Flow in parallel straight lines

Equation (48) implies that flow is either in parallel straight lines or is radial flow. Awartani & Hamdan [5] showed that there does not exist a steady plane radial flow of a dusty gas through a porous medium when the particle number density is either constant throughout the flow field, or is constant along streamlines.

For flow in parallel straight lines, we can choose $E_1, = E_2 = 1$. This choice satisfies Gauss' equation (48) automatically. Equation (52) reduces to:

$$U_\phi = 0 \quad \text{... (53)}$$

which implies that

$$U = U(\psi) \quad \text{... (54)}$$

and (49) becomes:

$$\omega = -U_\psi = -U'(\psi) \quad \text{... (55)}$$

whence

$$\omega_\psi = -U''(\psi) \quad \text{... (56)}$$

where prime notation denotes ordinary differentiation.

Equation (51) reduces to:

$$p_\psi = 0 \quad \text{... (57)}$$

which implies that

$$p = p(\phi) \quad \text{... (58)}$$

Equation (50) thus becomes:

$$\mu U''(\psi) + kN\left[\frac{\alpha}{N} - 1 - \frac{\mu}{\eta k N}\right]U(\psi) = p'(\phi) \quad \text{... (59)}$$

Equation (44) reduces to

$$U\left(\frac{1}{N}\right)_\phi = \frac{k}{m}\left\{\frac{N}{\alpha}\right\}^2\left(1 - \frac{\alpha}{N}\right) \quad \text{... (60)}$$

which gives rise to two cases:

Case 1:$N = constant$ or $N = N(\psi)$.

In this case, equation (60) reduces to

$$\frac{k}{m}\left\{\frac{N}{\alpha}\right\}^2\left(1 - \frac{\alpha}{N}\right) = 0 \quad \text{... (61)}$$

which implies that $\alpha = N$, or $\vec{v} = \vec{u}$, and (59) reduces to

$$U''(\psi) - \frac{1}{\eta}U(\psi) = p'(\phi) \quad \text{... (62)}$$

where $p'(\phi) = constant$. Solution to (62) is given by

$$U(\psi) = Ae^{\psi/\sqrt{\eta}} + Be^{-\psi/\sqrt{\eta}} - \frac{\eta}{\mu}p'(\phi) \quad \text{... (63)}$$

where A and B are arbitrary constants. Equation (63) illustrates how the permeability η influences the velocity distribution. It should be noted that we have chosen here to express the velocity in exponential form as opposed to the equivalent hyperbolic form due to ease of analyzing flow through an infinite porous layer. For example, if $\psi \to \infty$, then to keep $U(\psi)$ finite we choose $A = 0$ in (63).

For flow through free-space, velocity variations are given by

$$U(\psi) = \frac{p'(\phi)}{2\mu}\psi^2 + C\psi + D \quad \text{... (64)}$$

where C and D are arbitrary constants.

Observation 2:The velocity profiles in the porous layer and in the free-space channel, and the velocity and shear stress at the interface between layers, are independent of the dust particle number density,$N = N(\psi)$.

Case 2:$N = N(\phi, \psi)$

In this case equation (60) reduces to

$$\frac{1}{N^2(\alpha - N)}N_\phi = \frac{k}{m\alpha^2}\frac{1}{U} \quad \text{... (65)}$$

with solution given by

$$U = \frac{-\frac{\phi k}{m}}{\frac{\alpha}{N} + \ln\left|\frac{\alpha}{N} - 1\right| - h(\psi)} \quad \text{... (66)}$$

where $h(\psi)$ is an arbitrary function of ψ.

For the case of dusty gas flow through free space, velocity distribution is the same as that given by (66). This leads to the following observation.

Observation 3:Permeability of the porous layer does not influence the velocity distribution in the porous layer when the particle number density $N = N(\phi, \psi)$ and the flow is in parallel straight lines.

In light of this conclusion, the problem at hand involves parallel straight lines with (64) representing velocity distribution in the free-space layer and (63) representing velocity distribution in the porous layer. In the chosen flow configuration, if we choose $y = -h$ to correspond to $\psi = \psi_{min}$ and $y = h$ to correspond to $\psi = \psi_{max}$, with $\psi = 0$ to represent the interface, then the following matching conditions of velocity and shear stress are valid at $\psi = 0$:

$$U(0^+) = U(0^-) \qquad \dots (67)$$

$$U_\psi(0^+) = U_\psi(0^-) \qquad \dots (68)$$

Conditions at ψ_{min} and ψ_{max} are the no-slip conditions,

$$U(\psi_{min}) = 0 \qquad \dots (69)$$

$$U(\psi_{max}) = 0 \qquad \dots (70)$$

Using these conditions in (63) and (64), and solving for the arbitrary constants A, B, C and D yields:

$$A = \frac{\frac{p'(\phi)}{\mu}\left\{\eta - \frac{(\psi_{min})^2}{2} - \eta e^{\frac{\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}}{\left\{\left[1 + \frac{\psi_{min}}{\sqrt{\eta}}\right] - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}} \qquad \dots (71)$$

$$B = \frac{\eta}{\mu}p'(\phi)e^{\psi_{max}/\sqrt{\eta}} - \frac{\frac{p'(\phi)}{\mu}e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left\{\eta - \frac{(\psi_{min})^2}{2} - \eta e^{\frac{\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}}{\left\{\left[1 + \frac{\psi_{min}}{\sqrt{\eta}}\right] - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}} \qquad \dots (72)$$

$$C = \left[1 + e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\right]\frac{\frac{p'(\phi)}{\mu\sqrt{\eta}}\left\{\eta - \frac{(\psi_{min})^2}{2} - \eta e^{\frac{\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}}{\left\{\left[1 + \frac{\psi_{min}}{\sqrt{\eta}}\right] - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}} - \frac{\sqrt{\eta}}{\mu}p'(\phi)e^{\frac{\psi_{max}}{\sqrt{\eta}}} \qquad \dots (73)$$

$$D = \frac{\eta}{\mu}p'(\phi)\left[e^{\frac{\psi_{max}}{\sqrt{\eta}}} - 1\right] + \left[1 - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\right]\frac{\frac{p'(\phi)}{\mu}\left\{\eta - \frac{(\psi_{min})^2}{2} - \eta e^{\frac{\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}}{\left\{\left[1 + \frac{\psi_{min}}{\sqrt{\eta}}\right] - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}} \qquad \dots (74)$$

Using these values of arbitrary constants in (63) and (64) gives the complete velocity distribution in each of the layers. Furthermore, velocity at the interface $U(\psi = 0) = D$, is given by:

$$U(0) = \frac{\eta}{\mu}p'(\phi)\left[e^{\frac{\psi_{max}}{\sqrt{\eta}}} - 1\right] + \left[1 - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\right]\frac{\frac{p'(\phi)}{\mu}\left\{\eta - \frac{(\psi_{min})^2}{2} - \eta e^{\frac{\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}}{\left\{\left[1 + \frac{\psi_{min}}{\sqrt{\eta}}\right] - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}} \qquad \dots (75)$$

and shear stress at the interface is given by

$$U_\psi(0) = \left[1 + e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\right]\frac{\frac{p'(\phi)}{\mu\sqrt{\eta}}\left\{\eta - \frac{(\psi_{min})^2}{2} - \eta e^{\frac{\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}}{\left\{\left[1 + \frac{\psi_{min}}{\sqrt{\eta}}\right] - e^{\frac{2\psi_{max}}{\sqrt{\eta}}}\left[1 - \frac{\psi_{min}}{\sqrt{\eta}}\right]\right\}} - \frac{\sqrt{\eta}}{\mu}p'(\phi)e^{\frac{\psi_{max}}{\sqrt{\eta}}} \qquad \dots (76)$$

IV. CONCLUSION

In this work we considered the flow of a dusty gas through a free-space channel underlain by a porous layer. It was assumed that the dust-phase velocity is a product of a position function and the fluid-phase velocity. Solution to the governing equations was obtained in two coordinate systems: the Cartesian (x,y) and the curvilinear (ϕ, ψ) net in which $\psi = constant$ represents the streamlines of the flow and $\phi = constant$ represents their orthogonal trajectories. Conditions of velocity and shear stress continuity at the interface were employed in obtaining the velocity and shear stress profiles. Analyses support the following conclusions:

1) The velocity profiles in the porous layer and in the free-space channel, and the velocity and shear stress at the interface between layers, are independent of the dust particle number density, N(x,y). Rather, they depend on layers' widths, driving pressure gradient, permeability and viscosity.

2) The velocity profiles in the porous layer and in the free-space channel, and the velocity and shear stress at the interface between layers, are independent of the dust particle number density, $N = N(\psi)$.
3) Permeability of the porous layer does not influence the velocity distribution in the porous layer when the particle number density $N = N(\phi, \psi)$ *and the flow is in parallel straight lines.*
Dusty gas flow in the presence of an interface is usually accompanied by momentum and mass transfer across the interface between layers. This might require flow modelling with blowing or suction of dust particles. By assuming that the dust-phase velocity is proportional to the fluid-phase velocity, we have created a form of a tangency condition at the interface in that the interface between the layers is a dividing streamline and that the dust particles cannot cross this streamline. This assumption resulted in simplifying the presented analyses in which we arrived at the stated conclusions. Further consideration and analysis need to be carried out by relaxing the assumption of phase velocity proportionality in order to allow for dust particle transfer across the interface and to allow for reflection of dust particles back into the flow field.

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