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Study of Waves in Piezoelectric Heterogeneous Poroelastic Layer over an Initially Stressed Heterogeneous Poroelastic Half Space

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Abstract: This paper is to study the wave propagation in piezoelectric heterogeneous isotropic poroelastic layer lying over an initially stressed heterogeneous poroelastic half space in the framework of Biot's theory. Heterogenity here is variation in densities of layer in a particular direction. Frequency equation is obtained for axially symmetric waves for two layered phenomena. Phase velocity is computed as a function of wavenumber for fixed piezoelectric, dielectric constants, and initial stress. For numerical purpose, two examples are considered, and the results are presented graphically.
Keywords: Piezoelectric, heterogeneous, initial stress, half-space, phase velocity, wavenumber.

I. INTRODUCTION

Piezoelectric materials are used in communication engineering, transport, healthcare, and manufacturing. Piezoelectricity is a phenomenon in which mechanical energy is converted into electrical energy and vice versa. A material possessing piezoelectricity will generate an electrical charge when a mechanical pressure is applied to it. The material geometry will change when an electrical charge is applied to it. Love wave propagation in a piezoelectric layer overlying in an inhomogeneous elastic half space is studied by Santanu *et.al* [1]. Danoyan and Piliposian investigated on surface electro-elastic Love waves in a layered structure with a piezoelectric substrate and a dielectric layer [2]. In the paper [2], the dispersion equation for the existence of Love surface wave is obtained. In the paper [3], the propagating nature of the elastic and electric wave in bone and porous PZT is investigated in detail. Dispersion relation for SH wave propagation in porous piezoelectric composite structure is investigated by Anushu [4]. From the paper [4], authors are concluded that phase velocity is significantly influenced by the porosity. Propagation of SH-wave in piezoelectric layer influenced by a point source is studied by Abhishek *et.al* [5]. Gupta and Vashishth [6] investigated the effects of piezoelectricity on the interaction of waves in fluid-loaded poroelastic half space. In the paper [6], the angle of refraction, amplitude ratios, displacements, electric potentials and vertical component of slowness are studied for particular case. Displacement distributions in a piezoelectric heterogeneous are investigated by Qiang *et.al* [7]. To the best of authors knowledge, the model consists of piezoelectric heterogeneous poroelastic layer lying over an initially stressed heterogeneous poroelastic half space is not yet investigated. Therefore, in this paper, the same is investigated in the framework of Biot's theory of poroelasticity [8].

This paper is organized as follows. In section 2, geometry and solution of the problem are discussed. Numerical results are given in section 3. Finally, conclusion is given in section 4.

II. GEOMETRY AND SOLUTION OF THE PROBLEM

Consider piezoelectric heterogeneous isotropic poroelastic layer over on initially stressed half space on cylindrical system. The origin is taken at the initial point of the interface between layer and half-space. Consider the axially symmetric waves, propagation direction is along the r axis. The z axis is taken as positive downwards as shown in figure 1.

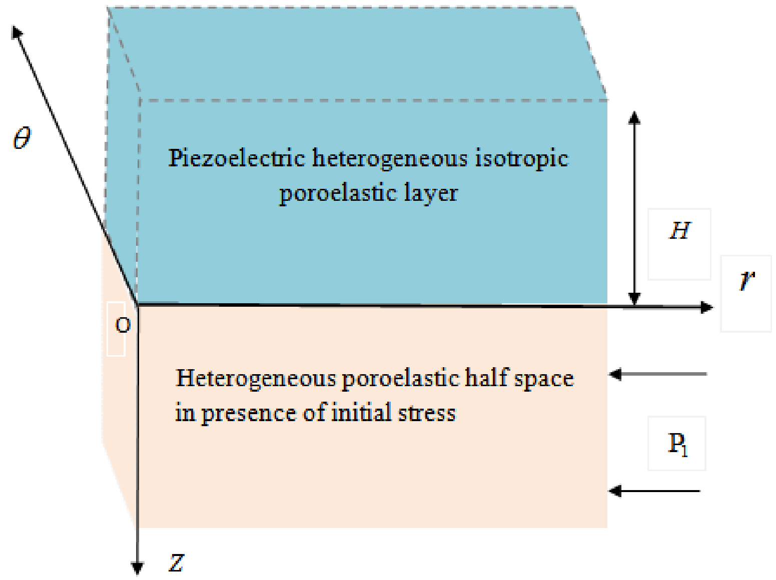

Figure 1: Geometry of the problem.

2.1 Upper piezoelectric heterogeneous isotropic poroelastic layer

For the upper piezoelectric heterogeneous poroelastic layer, $(u_1, 0, w_1)$ and $(U_1, 0, W_1)$ be the solid and fluid displacement components, respectively. For axially symmetric waves in the case of piezoelectric heterogeneous, the equations of motion is

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr}}{r} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}(1+\alpha z)u_1 + \rho_{12}(1+\alpha z)U_1\right),$$
$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rz}}{r} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}(1+\alpha z)w_1 + \rho_{12}(1+\alpha z)W_1\right),$$
$$Q\frac{\partial e}{\partial r} + R\frac{\partial \varepsilon}{\partial r} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}(1+\alpha z)u_1 + \rho_{22}(1+\alpha z)U_1\right),$$
$$Q\frac{\partial e}{\partial z} + R\frac{\partial \varepsilon}{\partial z} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}(1+\alpha z)w_1 + \rho_{22}(1+\alpha z)W_1\right),$$
$$\frac{\partial D_r}{\partial r} + \frac{\partial D_z}{\partial z} + \frac{D_r}{r} = 0. \tag{1}$$

In eq. (1), $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{rz}$ are the stress components, ρ_{ij} are mass coefficients, α is heterogeneous parameter. The solid stress σ_{ij}, fluid pressure s, and dielectric displacement D_i are given by

$$\sigma_{ij} = 2\mathrm{N}(1+\alpha z)e_{ij} + (\mathrm{A}e + Q\varepsilon)\delta_{ij} - \mathrm{E}_i \quad (i, j = 1,2,3),$$
$$s = Qe + R\varepsilon.$$
$$D_r = \mathrm{P}_{15}\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial r}\right) - \varepsilon_{11}\frac{\partial \phi}{\partial r},$$
$$D_z = \mathrm{P}_{31}\frac{\partial u_1}{\partial r} + \mathrm{P}_{31}\frac{u_1}{r} + \mathrm{P}_{33}\frac{\partial u_3}{\partial z} - \varepsilon_{33}\frac{\partial \phi}{\partial z}. \tag{2}$$

In eq. (2), $\mathrm{P}_{15}, \mathrm{P}_{31}, \mathrm{P}_{33}$ are piezoelectric constants, $\varepsilon_{11}, \varepsilon_{33}$ are dielectric constants. Strain components e_{ij} and electric field E_i are given by

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\quad (i,j=1,2,3),\ \ \mathrm{E}_i=-\phi_{,i}\quad (i=1,2,3). \qquad (3)$$

Suppose that solution of eq.(3) in the frequency domain are as follows:

$$u_1(r,z,t)=C_1e^{j\omega t-j(k_1r+k_3z)},\quad U_1(r,z,t)=C_3e^{j\omega t-j(k_1r+k_3z)},$$

$$w_1(r,z,t)=C_2e^{j\omega t-j(k_1r+k_3z)},\quad W_1(r,z,t)=C_4e^{j\omega t-j(k_1r+k_3z)},$$

$$\phi(r,z,t)=C_5e^{j\omega t-j(k_1r+k_3z)}. \qquad (4)$$

In eq. (4), $C_i\,(i=1,2...5)$ are arbitrary constants, j is the complex unity, and $k_i\,(i=1,3)$ is the wavenumber in the i^{th} direction such that the wavenumber $k=\sqrt{k_1^2+k_3^2}$. Using eqs. (2), (3), and (4) in eq. (1), the following systems of equations are obtained:

$$2\mathrm{N}(1+\alpha z)C_1(-jk_1)^2+\mathrm{A}\left(C_1(-jk_1)^2+\frac{C_1}{r^2}-\frac{C_1(-jk_1)}{r}+C_2(-jk_3)(-jk_1)\right)$$
$$+Q\left(C_3(-jk_1)^2+\frac{C_3}{r^2}-\frac{C_3(-jk_1)}{r}+C_4(-jk_3)(-jk_1)\right)+\mathrm{P}_{31}C_5(-jk_3)(-jk_1)$$
$$+\mathrm{N}(1+\alpha z)C_2(-jk_1)(-jk_3)+C_2(-jk_1)\mathrm{N}\alpha+\mathrm{N}(1+\alpha z)C_1(-jk_3)^2+C_1(-jk_3)\mathrm{N}\alpha$$
$$+\mathrm{P}_{15}C_5(-jk_1)(-jk_3)+\frac{2\mathrm{N}(1+\alpha z)C_1(-jk_1)}{r}+\frac{\mathrm{A}}{r}\left(C_1(-jk_1)+\frac{C_1}{r^2}-\frac{C_1(-jk_1)}{r}+C_2(-jk_3)\right)$$
$$+\frac{Q}{r}\left(C_3(-jk_1)+\frac{C_3}{r^2}-\frac{C_3(-jk_1)}{r}+C_4(-jk_3)\right)+\frac{\mathrm{P}_{31}C_5(-jk_3)}{r}+\omega^2\rho_{11}(1+\alpha z)C_1+\omega^2\rho_{12}(1+\alpha z)C_3=0,$$

$$\mathrm{N}(1+\alpha z)C_2(jk_1)^2+\mathrm{N}(1+\alpha z)C_1(-jk_3)(-jk_1)+\mathrm{P}_{15}C_5(-jk_1)^2+\mathrm{N}(1+\alpha z)C_2(-jk_1)(-jk_3)$$
$$+C_2(-jk_1)\mathrm{N}\alpha+\mathrm{N}(1+\alpha z)C_1(-jk_3)^2+C_1(-jk_3)\mathrm{N}\alpha+\mathrm{P}_{15}C_5(-jk_1)(-jk_3)+\frac{\mathrm{N}(1+\alpha z)C_2(-jk_1)}{r}$$
$$+\frac{\mathrm{N}(1+\alpha z)C_1(-jk_3)}{r}+\frac{\mathrm{P}_{15}C_5(-jk_1)}{r}+\omega^2\rho_{11}(1+\alpha z)C_2+\omega^2\rho_{12}(1+\alpha z)C_4=0,$$

$$Q\left(C_1(-jk_1)^2+\frac{C_1}{r^2}-\frac{C_1(-jk_1)}{r}+C_2(-jk_3)(-jk_1)\right)+R\left(C_3(-jk_1)^2+\frac{C_3}{r^2}-\frac{C_3(-jk_1)}{r}+C_4(-jk_3)(-jk_1)\right)$$
$$\omega^2\rho_{12}(1+\alpha z)C_1+\omega^2\rho_{22}(1+\alpha z)C_3=0,$$

$$Q\left(C_1(-jk_1)(-jk_3)+\frac{C_1(-jk_3)}{r}+C_2(-jk_3)^2\right)+R\left(C_3(-jk_1)(-jk_3)+\frac{C_3(-jk_3)}{r}+C_4(-jk_3)^2\right)$$
$$+\omega^2\rho_{12}(1+\alpha z)C_2+\omega^2\rho_{22}(1+\alpha z)C_4=0,$$

$$\mathrm{P}_{15}C_1(-jk_3)(-jk_1)+\mathrm{P}_{15}C_2(-jk_1)^2-\varepsilon_{11}C_5(-jk_1)^2+\mathrm{P}_{31}C_1(-jk_1)(-jk_3)$$
$$+\frac{\mathrm{P}_{31}C_1(-jk_3)}{r}+\mathrm{P}_{33}C_2(-jk_3)^2-\varepsilon_{33}C_5(-jk_3)^2+\frac{\mathrm{P}_{15}C_1(-jk_3)}{r}+\frac{\mathrm{P}_{15}C_2(-jk_1)}{r}-\frac{\mathrm{P}_{15}\varepsilon_{11}C_5(-jk_1)}{r}=0. \qquad (5)$$

2.2 The lower heterogeneous poroelastic layer in presence of initial stress

The lower half space is heterogeneous poroelastic half-space in the sense, densities vary with the $z-$coordinate. The equations of motion under initial compression stress $\sigma_{zz}=-\mathrm{P}_1$ are [9].

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{\partial \sigma_{rz}}{\partial z}+\frac{\sigma_{rr}}{r}-\mathrm{P}_1\frac{\partial \omega_\theta}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{110}(1+\alpha z)u_2+\rho_{120}(1+\alpha z)U_2\right),$$

$$\frac{\partial \sigma_{rz}}{\partial r}+\frac{\partial \sigma_{rz}}{\partial z}+\frac{\sigma_{rz}}{r}-\mathrm{P}_1\frac{\partial \omega_\theta}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{110}(1+\alpha z)w_2+\rho_{120}(1+\alpha z)W_2\right),$$

$$Q\frac{\partial e}{\partial r}+R\frac{\partial \varepsilon}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho_{120}(1+\alpha z)u_1+\rho_{220}(1+\alpha z)U_1\right),$$

$$Q\frac{\partial e}{\partial z}+R\frac{\partial \varepsilon}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{120}(1+\alpha z)w_2+\rho_{220}(1+\alpha z)W_2\right),$$

$$\varepsilon_{110}\frac{\partial^2 \phi_1}{\partial r^2}+\varepsilon_{110}\frac{1}{r}\frac{\partial \phi_1}{\partial r}+\varepsilon_{330}\frac{\partial^2 \phi_1}{\partial z^2}=0. \qquad (6)$$

In eq. (6), $(u_2, 0, w_2)$ and $(U_2, 0, W_2)$ are the displacement components of solid and fluid respectively, ε_{110} dielectric constant. The rotational components ω_θ is given by

$$\omega_\theta=\frac{1}{2}\left(\frac{\partial u_2}{\partial z}-\frac{\partial w_2}{\partial r}\right). \qquad (7)$$

Suppose that solution of eq. (6) in the frequency domain are as follows:

$$u_2(r,z,t)=C_6e^{j\omega t-j(k_1r+k_3z)}, \quad U_2(r,z,t)=C_8e^{j\omega t-j(k_1r+k_3z)},$$

$$w_2(r,z,t)=C_7e^{j\omega t-j(k_1r+k_3z)}, \quad W_2(r,z,t)=C_9e^{j\omega t-j(k_1r+k_3z)},$$

$$\phi_1(r,z,t)=C_{10}e^{j\omega t-j(k_1r+k_3z)}. \qquad (8)$$

In eq. (8), $C_i\,(i=6,7...10)$ are arbitrary constants, j is the complex unity, and $k_i\,(i=1,3)$ is the wavenumber in the i^{th} direction such that the wavenumber $k=\sqrt{k_1^2+k_3^2}$. Using eqs. (3), (7), (8) in eq. (6), the following systems of equations are obtained:

$$2\mathrm{N}(1+\alpha z)C_6(-jk_1)^2+\mathrm{A}\left(C_6(-jk_1)^2+\frac{C_6}{r^2}-\frac{C_6(-jk_1)}{r}+C_7(-jk_3)(-jk_1)\right)$$
$$+Q\left(C_8(-jk_1)^2+\frac{C_8}{r^2}-\frac{C_8(-jk_1)}{r}+C_9(-jk_3)(-jk_1)\right)+\mathrm{N}(1+\alpha z)C_7(-jk_1)(-jk_3)$$
$$+C_7(-jk_1)\mathrm{N}\alpha+\mathrm{N}(1+\alpha z)C_6(-jk_3)^2+C_6(-jk_3)\mathrm{N}\alpha+\frac{2\mathrm{N}(1+\alpha z)C_6(-jk_1)}{r}$$
$$+\frac{\mathrm{A}}{r}\left(C_6(-jk_1)+\frac{C_6}{r^2}-\frac{C_6(-jk_1)}{r}+C_7(-jk_3)\right)+\frac{Q}{r}\left(C_8(-jk_1)+\frac{C_8}{r^2}-\frac{C_8(-jk_1)}{r}+C_9(-jk_3)\right)$$
$$-\frac{\mathrm{P}_1}{2}\left(C_6(-jk_3)^2-C_7(-jk_1)(-jk_3)\right)+\omega^2\rho_{110}(1+\alpha z)C_6+\omega^2\rho_{120}(1+\alpha z)C_8=0,$$

$$\mathrm{N}(1+\alpha z)C_7(-jk_1)^2+\mathrm{N}(1+\alpha z)C_6(-jk_3)(-jk_1)+\mathrm{N}(1+\alpha z)C_7(-jk_1)(-jk_3)$$
$$+C_7(-jk_1)\mathrm{N}\alpha+\mathrm{N}(1+\alpha z)C_6(-jk_3)^2+C_6(-jk_3)\mathrm{N}\alpha+\frac{\mathrm{N}(1+\alpha z)C_7(-jk_1)}{r}$$
$$+\frac{\mathrm{N}(1+\alpha z)C_6(-jk_3)}{r}+\frac{\mathrm{P}_1 C_7(jk)^2}{2}+\omega^2\rho_{110}(1+\alpha z)C_7+\omega^2\rho_{120}(1+\alpha z)C_9=0,$$
$$Q\left(C_6(-jk_1)^2+\frac{C_6}{r^2}-\frac{C_6(-jk_1)}{r}+C_7(-jk_3)(-jk_1)\right)+R\left(C_8(-jk_1)^2+\frac{C_8}{r^2}-\frac{C_8(-jk_1)}{r}+C_9(-jk_3)(-jk_1)\right)$$
$$\omega^2\rho_{120}(1+\alpha z)C_1^{|}+\omega^2\rho_{220}(1+\alpha z)C_3^{|}=0,$$
$$Q\left(C_6(-jk_1)(-jk_3)+\frac{C_6(-jk_3)}{r}+C_7(-jk_3)^2\right)+R\left(C_8(-jk_1)(-jk_3)+\frac{C_8(-jk_3)}{r}+C_9(-jk_3)^2\right)$$
$$+\omega^2\rho_{120}(1+\alpha z)C_7+\omega^2\rho_{220}(1+\alpha z)C_9=0,$$
$$\varepsilon_{110}C_{10}(-jk_1)^2+\frac{\varepsilon_{110}C_{10}(-jk_1)}{r}+\varepsilon_{330}C_{10}(-jk_3)^2=0. \qquad (9)$$

III. NUMERICAL RESULTS

For the numerical results, the wave propagation along the r direction. In this case $k_3=0$, eqs. (5) and (9) reduces the following matrix form:

$$[\mathrm{A}_{lm}][C_l]=0,\ l,m=1,2,3,4,5,6,7,8,9. \qquad (10)$$

In order to obtain a non trivial solution of the system, determinant of coefficients must be zero. Accordingly, one obtains the following frequency equation.

$$|\mathrm{A}_{lm}|=0,\ l,m=1,2,3,4,5,6,7,8,9. \qquad (11)$$

The frequency eq. (11) gives implicit relation between frequency, and wavenumber. For numerical process, the following materials are used.

The material parameters for upper layer taken as Berea sandstone saturated with water [5, 10] (Say Mat-I),

$$\mathrm{A}=7.9709\ Gpa,\ \mathrm{N}=6.7\ Gpa,\ Q=1.1506\ Gpa,\ R=4.778\ Gpa,\ \rho_{11}=-0.7999\times10^3 kg/m^3,$$
$$\rho_{12}=-0.0833\times10^3 kg/m^3,\ \rho_{22}=2.1666\times10^3 kg/m^3,\ \mathrm{P}_{15}=17c/m^2,\ \varepsilon_{11}=2770\times10^{-10}\ f/m.$$

The material parameters for lower layer in presence of initial stress taken as Berea sandstone saturated with kerosene [5, 10] (Say Mat-II),

$$\mathrm{A}=6.9298\ Gpa,\ \mathrm{N}=6.7\ Gpa,\ Q=7.183\ Gpa,\ R=2.983\ Gpa,\ \rho_{110}=2055.8384,$$
$$\rho_{120}=-68.3348,\ \rho_{220}=273.3598,\ \varepsilon_{110}=128\times10^{-10}\ f/m.$$

Employing these values in frequency equation, the implicit relation between the phase velocity and wavenumber is obtained and the results are depicted in figure 2. From figure 2, phase velocity is computed as a function of wavenumber for various values of heterogeneous parameter i.e., 0.1, 0.2, 0.3, and for fixed dielectric constant, piezoelectric constant, and initial stress. It is observed that phase velocity decreases as wavenumber increases. Also it is clear that as heterogeneous parameter increases, phase velocity decrease. Moreover, in absence of heterogeneous parameter and initial stress, phase velocity decrease and phase velocity is same for all the values of piezoelectric and dielectric constants. From this it is clear that phase velocity is independent of heterogeneous parameter, piezoelectric, dielectric constants and initial stress.

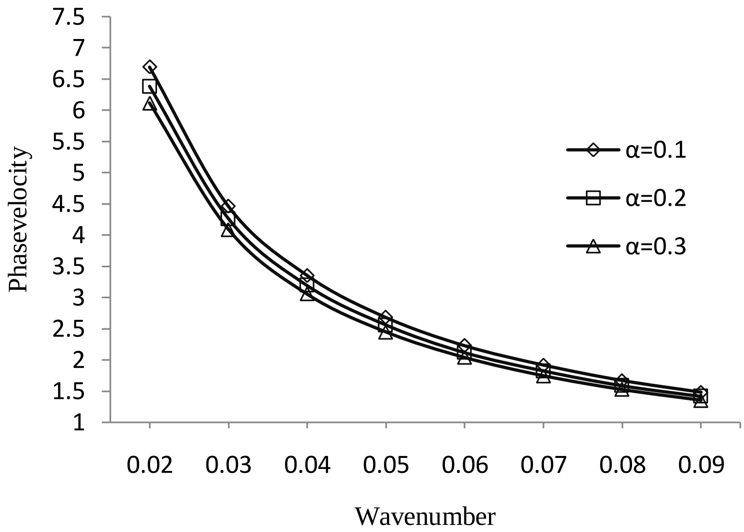

Figure2. Variation of phase velocity as a function of wavenumber

IV. CONCLUSION

Axially symmetric vibrations in piezoelectric heterogeneous poroelastic layer lying over an initially stressed heterogeneous poroelastic half space in the framework of Biot's theory is investigated. Phase velocity is computed as a function of wavenumber. From the numerical results, it is observed that as wavenumber increases, phase velocity decreases for both the solids. Both the solids are sandstone related and differ in only the fluid part. Hence, it can be inferred that the fluid part is causing the above discrepancy.

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