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Mathematical modeling in black holes study.

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ABSTRACT
Most famously, black holes were predicted by Einstein's theory of general relativity, which showed that when a massive star dies, it leaves behind a small, dense remnant core. If the core's mass is more than about three times the mass of the Sun, the equations showed, the force of gravity overwhelms all other forces and produces a black hole. Scientists can't directly observe black holes with telescopes that detect x-rays, light, or other forms of electromagnetic radiation. In our work we try to find some mathematical used in blackhole research. Firstly we discuss about different essential branches which are effective in cosmic study and then finally by using MATLAB coding we try to find some solution.
KEYWORDS: dimension, space-time, galaxy, star, blackholes

I. INTRODUCTION

The idea of an object in space so massive and dense that light could not escape it has been around for centuries. Scientists can't directly observe black holes with telescopes that detect x-rays, light, or other forms of electromagnetic radiation.Black holes were predicted by Einstein's theory of general relativity, which showed that when a massive star dies, it leaves behind a small, dense remnant core. If the core's mass is more than about three times the mass of the Sun, the equations showed, the force of gravity overwhelms all other forces and produces a black hole. We can, however, infer the presence of black holes and study them by detecting their effect on other matter nearby. If a black hole passes through a cloud of interstellar matter, for example, it will draw matter inward in a process known as accretion. A similar process can occur if a normal star passes close to a black hole. In this case, the black hole can tear the star apart as it pulls it toward itself. As the attracted matter accelerates and heats up, it emits x-rays that radiate into space. Recent discoveries offer some tantalizing evidence that black holes have a dramatic influence on the neighborhoods around them - emitting powerful gamma ray bursts, devouring nearby stars, and spurring the growth of new stars in some areas while stalling it in others. Most black holes form from the remnants of a large star that dies in a supernova explosion. (Smaller stars become dense neutron stars, which are not massive enough to trap light.) If the total mass of the star is large enough (about three times the mass of the Sun), it can be proven theoretically that no force can keep the star from collapsing under the influence of gravity. However, as the star collapses, a strange thing occurs. As the surface of the star nears an imaginary surface called the "event horizon," time on the star slows relative to the time kept by observers far away. When the surface reaches the event horizon, time stands still, and the star can collapse no more - it is a frozen collapsing object. Black holes can come in any size, from microscopic to supermassive. In today's universe, massive stars detonate as supernovae and this can create stellar-mass black holes (1 solar mass = 1.9×1030 kg). When enough of these are present in a small volume of space, like the core of a globular cluster, black holes can absorb each other and in principle, can grow to several hundred times the mass of the sun. If there is enough matter (i.e., gas, dust, and stars) for a black hole to "eat," it can grow even larger. There is a black hole in the star-rich core of the Milky Way that has a mass equal to nearly 5 million suns. The cores of more massive and distant galaxies have supermassive black holes containing the equivalent of 100 million to as much as 10 billion suns. Astronomers are not entirely sure how these supermassive black holes evolved so quickly to their present masses given that the universe is only 14 billion years old.Currently, there are no known ways to create black holes with masses less than about 0.1 times the sun's mass, and through a speculative process called Hawking Radiation, black holes less than 1 trillion kg in mass would have evaporated by now if they had formed during the Big Bang.

II. OBSERVATION AND RESULT

In this 4-dimensional geometry, all points are called events, and all events have four coordinates (x, y, z, t). Draw a 2-D spacetime diagram with time increasing upwards along the vertical axis and 1-dimension of space increasing to the right along the horizontal axis. Label the x-axis with tick marks at 1 kilometer intervals. Label the t-axis with marks every hour. In the following problems, all coordinates have the form (x,t) where x is the space position and t is the time).A different Pythagorean Theorem has to be used to give the full 4-dimensional spacetime distance, S, between two events. An approximate form for this new distance formula is given by

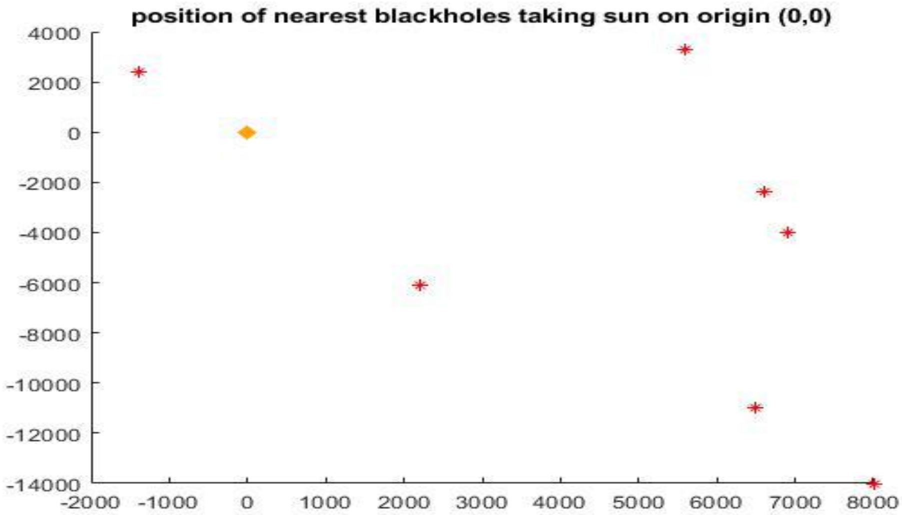
$$D = ((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - (t_2 - t_1)^2)^{\frac{1}{2}}$$

The Nearest Stellar Black Holes

How close is the nearest black hole to our own sun? Because our Milky Way galaxy is a very flat disk of stars, we can use Cartesian coordinates to map out where the nearest black holes are! Black holes are created when very massive stars explode as supernova. Fortunately, this does not happen very often in our corner of the Milky Way, so black holes are actually very far apart ! The table below gives the coordinates of the seven nearest black holes to our sun and solar system. The mass of each black hole is given in terms of solar mass units so that '16' means that the mass of the black hole is 16 times that of our sun. All distances (X, Y) are given in light years, where 1 light year = 9.6 trillion kilometers.

Name	Mass	X	Y
A) Cygnus X-1	16	6600	-2400
B) SS-433	11	8000	-14000
C) Nova Monocerotes 1975	11	-1400	2400
D) Nova Persi 1992	5	5600	3300
E) IL Lupi	9	6500	-11000
F) Nova Vulpeculi 1988	8	2200	-6100
G) V404 Cygni	12	6900	-4000

We can find the position of different nearby black holes by using MATLAB PROGRAM.



MATLAB PROGRAM:

```
x=[0 6600 8000 -1400 5600 6500 2200 6900];
y=[0 -2400 -14000 2400 3300 -11000 -6100 -4000 ];
hold on;
plot(x,y,'r*');
hold off
title('position of nearest blackholes taking sun on origin (0,0)');
```

Within the dense cores of most galaxies, lurk black holes that have grown over the eons into supermassive objects containing millions of times the mass of a stellar black hole. Some rare galaxies have two or three of these black holes, but far more have only one. Black holes may never lose mass. They steadily gain mass over the millennia by consuming interstellar gas, and even entire stars that are unfortunate enough to become trapped by their colossal gravity. The table below gives the distances and locations of the ten closest supermassive black holes to the Milky Way galaxy. The mass of each supermassive black hole is given in terms of solar mass units so that '90,000' means that the mass of the supermassive black hole is 90,000 times that of our sun. All distances (X, Y) are given in millions of light years, where 1 light year = 9.6 trillion kilometers.

Name	Mass	X	Y
NGC-205	90,000	1.1	2.0
Messier-33	50,000	0.5	2.6
Sagittarius A*	3 million	0	0
Messier-31	45 million	1.2	2.0
NGC-1023	44 million	-35.0	12.7
Messier-81	68 million	6.5	-11.3
NGC-3608	190 million	-70.5	-25.7
NGC-4261	520 million	-64.3	-76.7
Messier-87	3 billion	-33.4	-39.8

Using the 2- point distance formula to determine the distance, in millions of light years, between the Milky Way (0,0) and each of the nearby supermassive black holes by C/C^{++} PROGRAM :

```
#include<math.h>
#include<stdio.h>
int main()
{
float x1,y1,x2,y2,x3,y3,x4,y4,x5,y5,x6,x7,x8,x9,x10,y6,y7,y8,y9,y10,d1,d2,d3,d4,d5,d6,d7,d8,d9;
x1=0;y1=0;x2=1.1;
y2=2;x3=.5;x4=0;x5=1.2;x6=-35;x7=6.5;x8=-70.5;x9=-64.3;x10=-33.4;
y3=2.6;y4=0;y5=2;y6=12.7;y7=-11.3;y8=-25.7;y9=-76.7;y10=-39.8;
d1=sqrt((x2-x1)*(x2-x1)+(y2-y1)*(y2-y1));
d2=sqrt((x3-x1)*(x3-x1)+(y3-y1)*(y3-y1));
d3=sqrt((x4-x1)*(x4-x1)+(y4-y1)*(y4-y1));
d4=sqrt((x5-x1)*(x5-x1)+(y5-y1)*(y5-y1));
d5=sqrt((x6-x1)*(x6-x1)+(y6-y1)*(y6-y1));
d6=sqrt((x7-x1)*(x7-x1)+(y7-y1)*(y7-y1));
d7=sqrt((x8-x1)*(x8-x1)+(y8-y1)*(y8-y1));
d8=sqrt((x9-x1)*(x9-x1)+(y9-y1)*(y9-y1));
d9=sqrt((x10-x1)*(x10-x1)+(y10-y1)*(y10-y1));
printf("disances in light years of the closed supermassive blackholes to the Milky Way Galaxy: %.5f  %.5f  %.5f
%.5f  %.5f  %.5f%.5f%.5f%.5f ",d1,d2,d3,d4,d5,d6,d7,d8,d9);
}
```

Output:disances in light years of the closedsupermassiveblackholes to the Milky Way galaxy: 2.28254 2.64764 0.00000 2.33238 37.23291 13.03610 75.03825 100.08686 51.95768

Process returned 0 (0x0) execution time : 0.000 s

III. DISCUSSION

From our work we can find out the position of our sun or our galaxy in accordance with the position of the nearby black holes. If after some few time our galaxy or our sun come near of any nearby blackholes then what will be happened?Astronomers have suspected this for over a decade, and there have been many reports in the news media of 'proof' that such a thing existed. Now we have the most direct proof imaginable. The motion of the stars and gas near a region called SgrA* (pronounced 'sadge A star') in the constellation Sagittarius is faster than what you can account for if you just added up the mass of the gas and stars you see and worked out their 'gravitational speed'. This kind of motion has now been directly detected in a star called 'S2' located in orbit around the black hole. An international team of astronomers led by Rainer Schodel at the Max Planck Institute for Extraterrestrial Physics observed this star over the course of 10 years as it completed 2/3 of an orbit around a region centered on SgrA*. The star S2 approaches the central black hole to within three times the distance between the Sun and Pluto - while traveling at no less than 11 million miles per hour. The fast-moving star takes about 15 years to complete a single orbit. They used the Adaptive Optics Instrument on the 8.2-meter Very Large Telescope in Chile and captured a sequence of high-resolution images of this star as it orbited the black hole. The black hole, however, was not visible. Star velocities and variable X-ray emissions from the center of the Milky Way had indicated a compact source of radio waves, SgrA* can only be a black hole. SgrA* is the closest object to the actual Galactic Center. From the parameters of the elliptical orbit of S2 around the black hole, the investigators derived an enclosed mass between 2 to 5 million solar masses. This small volume of space, and large mass, completely excludes the possibility of a massive star cluster. Only a black hole fits the data, and at last settles this issue once and for all.Outside the black hole, it depends on what form the matter takes. If it happens to be in the form of gas that has been orbiting the black hole in a so-called accretion disk, the matter gets heated to very high temperatures as the individual atoms collide with higher and higher speed producing friction and heat. The closer the gas is to the black hole and its Event Horizon, the more of the gravitational energy of the gas gets converted to kinetic energy and heat. Eventually the atoms collide so violently that they get stripped of their electrons and you then have a plasma. All along, the gas emits light at higher and higher energies, first as optical radiation, then ultraviolet, then X-rays and finally, just before it passes across the Event Horizon, gamma rays.If the matter is inside a star that has been gravitationally captured by the black hole, the orbit of the star may decrease due to the emission of gravitational radiation over the course of billions of years. Eventually, the star will pass so close to the black hole that its fate is decided by the mass of the black hole. If it is a stellar-mass black hole, the tidal gravitational forces of the black hole will deform the star from a spherical ball, into a football-shaped object, and then eventually the difference in the gravitational force between the side nearest the black hole, and the back side of the star, will be so large that the star can no longer hold itself together. It will be gravitationally shredded by the black hole, with the bulk of the star's mass going into an accretion disk around the black hole. If the black hole has a mass of more than a billion times that of the sun, the tidal gravitational forces of the black hole are weak enough that the star may pass across the Event Horizon without being shredded. The star is, essentially, eaten whole and the matter in the star does not produce a dramatic increase in radiation before it enters the black hole.Once inside a black hole, beyond the Event Horizon, we can only speculate what the fate of captured matter is. General relativity tells us that there are two kinds of black holes; the kind that do not rotate, and the kind that do. Each of these kinds has a different anatomy inside the Event Horizon. For the non-rotating 'Schwarzschild black hole', there is no way for matter to avoid colliding with the Singularity. In terms of the time registered by a clock moving with this matter, it reaches the Singularity within a few micro seconds for a solar-massed black hole, and a few hours for a supermassive black hole. We can't predict what happens at the Singularity because the theory says we reach a condition of infinite gravitational force. For the rotating ' Kerr Black holes', the internal structure is more complex, and for some ingoing trajectories for matter, you could in principle avoid colliding with the Singularity and possibly reemerge from the black hole somewhere else, or at some very different future time thousands or billions of years after you entered.

IV. CONCLUSION

For black holes formed by collapsing stars, the body of the star itself exists in this zone so far as outside observers are concerned. In fact, as seen from a distance, the surface of the star is 'frozen' just outside the event horizon a few million million millionths of a centimeter from the horizon where the relativistic Doppler factor is enormous. Mathematically, for black holes old enough that the stellar material has collapsed all the way into the singularity, the region between the horizon and the singularity is occupied by a spacetime where the time and space coordinates are reversed from those of the outside world. What this means in terms of what you experience is unknown. Other more complex conditions can occur of the black hole is rotating. In that case the singularity becomes a ring around the center of the black hole. You can pass through the center, but the tidal gravitational field would be lethal in all likelihood. In nearly all cases there would be gravitational radiation rattling about, and this would cause distortions in spacetime that would probably lead to spectacular optical distortions. We are guided in our understanding of the interior of black holes by the theory of general relativity developed by Albert Einstein in 1915, and in particular, the mathematics of the complete, relativistic equation for gravity and space-time. This theory describes in considerable mathematical detail, both those regions of space-time that are accessible to humans, and those that are accessible only by individual observers but not distant observers. For black holes, distant observers will see only the outside of the event horizon, while individual observers falling into the black hole will experience quite another 'reality'. General relativity predicts that for distant observers outside the horizon, they will experience the 3 space-like coordinates and 1 time-like coordinate as they always have. For someone falling into a black hole and crossing the horizon, this crossing is mathematically predicted to involve the transformation of your single time-like coordinate into a space-like coordinate, and your three space-like coordinates into 3 time-like coordinates. Along any of these 3 former space-like coordinates, they now all terminate on the singularity, and you're experiencing them as time-like now means that you have no control over your destiny because all choices always terminate on the singularity...at least in the case of a non-rotating black hole. The coordinate which used to measure external time, now has a space- like character which affords you some wiggle room, but dynamically, in terms of these new reversed space and time coordinates, you find that no stable orbits about the singularity are possible no matter what you try to do. Without any stable orbits, and the inexorable free fall into the singularity, relativists often refer to this as the collapse of space-time geometry.

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