

## Estimation of Angle of Attack on a Fighter Aircraft During Its Avionics Upgrade

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**Abstract:** This paper presents a scheme for estimation of angle of attack (AOA) and angle of side slip (AOS) on a fighter aircraft during its avionics upgrade. The scheme was implemented and validated on a fighter aircraft designed and developed in early 1970 which was required to undergo major avionics upgrade to enhance its operational capabilities with modern sensors and avionics systems. The original version of the aircraft had mechanical AOA which was not reliable and accurate to meet the combat performance (ground attack role) expected from the upgraded version of the aircraft. A scheme was developed for inflight estimation of AOA using inertial sensor data from the Inertial Navigation System having Laser Gyros. Algorithm was implemented in onboard flight computer developed for the upgrade task. The input to the algorithm was from air data computer and inertial navigation system. A Kalman filter was designed to recover AOA under gust/turbulence (Dryden gust model) conditions. The scheme was implemented in MATLAB/SIMULINK. Estimated AOA was used to estimate wind. Results of the flight test are presented. Estimated angle of attack matched well with mechanical AOA sensor and theoretically computed data.

**Keywords** – Inertial navigation system (INS), Kalman filter, Angle of attack, Stability matrix, Control matrix, Euler angle

### I. Introduction

The angle of attack (AOA) and angle of side slip (AOS) provide valuable indication of flow conditions around the aircraft. They provide flight safety critical warning on stall conditions and aircraft departure characteristics. AOA parameter is also used for stability and control of unstable fighter aircraft. Normally these angles are measured by vanes or pressure probes. They are placed outside the fuselage and at a proper location outside the flow field. These probes are susceptible to damage and are not accurate during high g- maneuvers. In the past many fatal accidents have occurred due to failure of these mechanical vanes. Also, AOA data plays an important role in estimation of wind conditions which directly affects the weapon accuracy in a combat aircraft. Further there is a requirement to have multiple redundancy because they are essential element of automatic flight control system. The test aircraft had vane type of AOA sensor which was placed outside the fuselage. The analog output was fed to flight computer through analog to digital interface.

Though there are several approaches to determine AOA from Inertial navigation system [1-2], in this paper two approaches are presented which were validated on the test aircraft. First approach is based on fusing of data from INS and Air Data Computer. Second approach is based on designing a state observer (Kalman filter) and estimating the AOA from measurement of aircraft acceleration, aircraft body rates, Euler angles (from INS).

### II. Methodology Of Estimation

**First Methodology (Direct Computation).** AOA of an aircraft is normally a function of lift coefficient ( $C_{LWB}$ ), Mach number (M) and altitude. This relationship is derived and discussed in various papers [3-5]. The scheme presented here is based on the fusion of INS and Air Data Computer output along with aircraft physical parameters. Computational steps consist of computing body-wing lift coefficient based on aircraft weight, moment of inertia, TAS, air density and body angle rates at different Mach number and altitude [5]. Regression analysis on post flight data is then carried out to establish relationship between AOA and lift coefficient ( $C_{LWB}$ ) and Mach number. This relationship is used to estimate AOA. Computational steps are outlined as follows:

**Step 1.** Compute approximate angle of attack assuming zero wind velocity and zero side slip as:

$\tan(\alpha_G) = \frac{V_{BZ}}{V_{BX}}$  where  $V_{BZ}$  and  $V_{BX}$  are velocity along aircraft body z-axis and x-axis with respect to earth coordinate system and is derived from the INS.

**Step 2.** Compute load factor n by the following equation:

$$n = \frac{(\sin \alpha_G) a_{BX} - (\cos \alpha_G) a_{BZ} + (\cos \varphi \cos \theta)}{g} \quad (1)$$

where  $a_{BX}$  and  $a_{BZ}$  are aircraft acceleration in body x and z-axis,  $\Phi$  and  $\theta$  are bank and pitch angle, which are available from INS.  $g$  is acceleration due to gravity.

**Step 3.** Compute coefficient of lift due to wing-body at different Mach number and altitude by the following equation

$$C_{LWB} = \frac{X_T n W + \dot{q} I_Y + p r (I_X - I_Z) - C_{m_0} \left(\frac{1}{2} \rho v^2 S\right) \bar{c}}{\left(\frac{1}{2} \rho v^2 S\right) X_T \left[1 + \frac{X_{WB}}{X_T}\right]} \quad (2)$$

Where

$C_{m_0}$  is coefficient of zero-lift pitching moment,  $X_T$  is tail distance from CG,  $X_{WB}$  is wing body aero dynamic centre from CG.  $p, q,$  and  $r$  are body angular rates (available from INS).  $I_X, I_Y, I_Z, I_{XZ}$  are mass moment of inertia.  $S$  and  $c$  are wing area and mean chamber.  $n$  is the load factor computed in step 1.  $W$  is the weight of the aircraft which is estimated by the on-board fuel sensor. The ratio  $\frac{X_{WB}}{X_T}$  is very small and can be neglected. Dynamic pressure  $\left(\frac{1}{2} \rho v^2\right)$  is available from air data computer.

**Step 4.** Carry out regression analysis to find the AOA as a function of Mach number and  $C_{LWB}$  from the post flight analysis of data. This could be written in the form  $AOA = a + b * C_{LWB} + c * M$  Where  $a, b$  and  $c$  are constants derived from the regression analysis.

**Second Methodology (State Observer/Kalman filter).** This approach is based on state observer and requires linear aircraft longitudinal and lateral model in terms of state equations. In body axis system, the AOA can be related to the ratio of the vertical velocity,  $w$ , to longitudinal velocity  $u$  as

$$\alpha = \tan^{-1} \left[ \frac{w}{u} \right] \quad (3)$$

these velocities during manoeuvres can be determined by perturbation equations discussed in Reference [4]. The longitudinal state equations can be written as

$$\dot{x} = A x + B \eta + G \xi \quad (4)$$

Where  $x, \eta,$  and  $\xi$  are the state, control, and gust disturbance vectors. Longitudinal equations are.

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{w} \\ \Delta \dot{q} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + Z_u M_w & M_w + Z_w M_w & M_q + M_w u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} X_\delta & X_{\delta_T} \\ Z_\delta & Z_{\delta_T} \\ M_\delta + Z_\delta M_w & X_{\delta_T} + M_w Z_{\delta_T} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \delta_T \end{bmatrix} + \begin{bmatrix} -X_u & -X_w & 0 \\ -Z_u & -Z_w & 0 \\ -M_u & -M_w & -M_q \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_g \\ w_g \\ q_g \end{bmatrix} \quad (5)$$

State equation can be further simplified if we take short period mode of longitudinal motion which consists of only two state variables ( $\alpha$  and  $q$ ). Stability and Control derivatives in  $A$  and  $B$  matrices are computed using MATLAB System Identification Tool Box. Since the stability derivatives will not be readily available if the original equipment manufacturer (OEM) is not the upgrade agency. For short period longitudinal motion,  $A$  matrix reduces to  $2 \times 2$  matrix,  $B$  &  $G$  are  $2 \times 1$  matrix. To design the Kalman filter to recover AOA in vertical gust, we need to develop state equations for the vertical gust as gust noise is not white, but according to [Mil.spec.1797,1987] has spectral density as given below

$$\Phi_w(\omega) = 2 L \sigma^2 \frac{1 + 3L^2 \omega^2}{(1 + L^2 \omega^2)^2} \quad (6)$$

Here  $\omega$  is the frequency in radian,  $\sigma$  the turbulence intensity, and  $L$  the turbulence scale length divided by true air speed. A noise shaping filter was determined with state equation as given below (detailed methodology is explained in [3]).

$$\dot{z} = A_w z + B_w w \quad (7)$$

$$w_g = C_w z \quad (8)$$

The shaping filter given by equation (7) & (8) is driven by the white noise  $w(t) \sim (0,1)$  that will generate vertical gust with spectral density given by (4). Next step is to combine aircraft state equation with vertical gust shaping filter to derive an augmented state equation

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{z1} \\ \dot{z2} \\ \dot{\delta_e} \end{bmatrix} = A \begin{bmatrix} \alpha \\ q \\ z1 \\ z2 \\ \delta_e \end{bmatrix} + B u + G w \quad (9)$$

To design Kalman filter we need to consider the measurement covariance matrix R which depends on the INS sensor accuracy and jitter. This matrix R for the given INS was determined as follows:

$$R = \begin{bmatrix} 1/200 & 0 \\ 0 & 1/600 \end{bmatrix} \quad (10)$$

The process noise covariance matrix was taken as Q=1. Having determined plant state model, measurement noise co-variance, output matrix y was computed as

$$y = \begin{bmatrix} nz \\ q \end{bmatrix} = \begin{bmatrix} a & b & 00 & 0 \\ 0 & 1 & 00 & 0 \end{bmatrix} x + v \quad (11)$$

where, x is state vector  $\begin{bmatrix} \alpha \\ q \end{bmatrix}$  and v is the measurement noise which is represented by the co-variance matrix R in equation (8). Constants a, b was derived from the aircraft equation of motion. Input to the Kalman filter is plant matrix A, B, G, Q and R. This was implemented in MATLAB. Output of the Kalman filter is estimated state variables  $\alpha, n_z, q$  and vertical gust  $w_g$ .

**Instrumentation.** A solid-state data recorder was used to capture INS and air data computer output. This recorder related to Mil-Std-1553 B data bus. Sampling rate could be programmed as per requirement.

### III. Flight Test Results & Discussion

**First Methodology (Direct Computation).** Two sorties of 45 minutes duration were flown towards calibration of mechanical vane type AOA sensors originally installed on the aircraft. Calibration consisted of comparing pitch angle derived from the INS in straight and level flight. Air data system was also calibrated for position error correction of pitot-static system. In level flight angle of attack is equal to the pitch angle. To derive the relation between angle of attack and lift coefficient/Mach number five inflight test points were selected. This consisted of flying the aircraft at five different altitudes and at different Mach number. Aircraft was flown in clean configuration. Results are tabulated in Table-1 below. AOA estimation was also validated through flying the aircraft in known wind condition and wind speed estimated matched well with actual prevailing wind.

**Table 1: Inflight Test Point**

Test point	1	2	3	4	5
$\frac{1}{2} \rho V^2$ (millibar)	189	118	149	123	244
Mach Number	.500	.400	.600	.550	.750
Altitude(m)	0	0	5000	5000	5000
$C_{LWB}$	.25	.089	.95	.280	.150
$\alpha_{true}$ (degree)	.9	-.9	10	.8	-.285

The regression analysis for 2 independent variables was used to establish relationship between  $\alpha$  as a function of  $C_{LWB}$  and Mach number. This regression for the subject aircraft provided the formula for estimating AOA as a function of  $C_{LWB}$  and Mach Number in the subsequent validation sorties. Functional relation was found linear. For each of the five test points,  $C_{LWB}$  required for the flight was found through the MATLAB code listed in Appendix. In straight and level flight for each test points, load factor n was 1 and body rates p, q, r was zero. This was verified after analysing the post flight data. Mach and TAS were derived from Air Data Computer. A comparison of estimated & true AOA is presented in Table-2. Results are shown in Fig 1. The average error over the flight range was found to be satisfactory.

**Table 2: Inflight Validation**

Data Points	1	2	3	4	5
$\alpha_{true}$ (degree)	.9	-.9	9.4	.80	-.3
$\alpha_{est}$ (degree)	1.08	-1.33	9.12	1.1	-.7
$\Delta$ error (degree)	.18	-.43	-.2	.2	.4

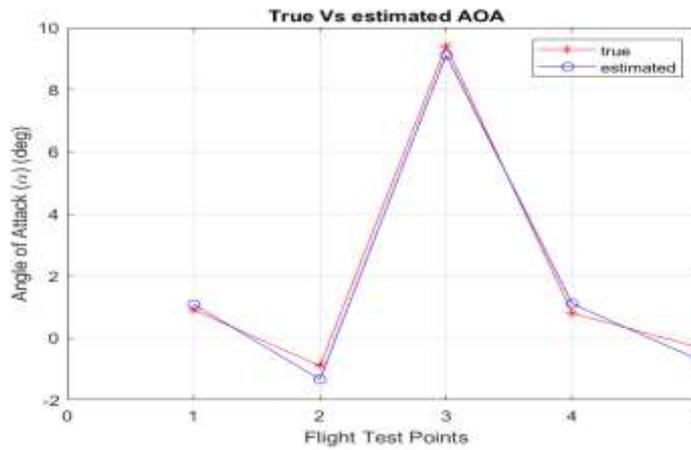


Fig 1. True Vs Estimated □

**Second Methodology (State Observer /Kalman Filter).**To derive and validate the linear longitudinal and lateral mathematical model (state equations) of the test aircraft, stabilised runs were done by trimming the aircraft at different flight conditions (AOA, Mach Number, Altitude). At each test point aircraft was stabilised for 20 seconds. All the required state and control variables were recorded through on-board flight data recorder. Elevator doublet and throttle inputs were given to excite short period and phugoid mode. Eigen values computed were compared with actual flight data. A program in MATLAB was developed for modelling of longitudinal and lateral motion. Input to the program consisted of flight conditions (altitude, Mach number, air density etc), control input (elevator, throttle), initial conditions and stability derivatives. The program computed A & B matrices and outputs the associated eigen values and eigen vectors which were verified with actual flight test data. Forced system response to control surface step inputs were calculated using a Taylor series expansion to second order. Step response was calculated through state transition matrix (using MATLAB Control Design tool Box). Correlation was found very good with different elevator input. Simulations were also done using wind gust model as given in equation (4).Computed Gust spectral density is shown in figure 2.

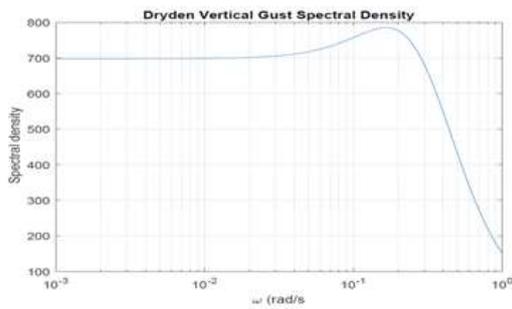


Fig 2. Vertical gust spectral density

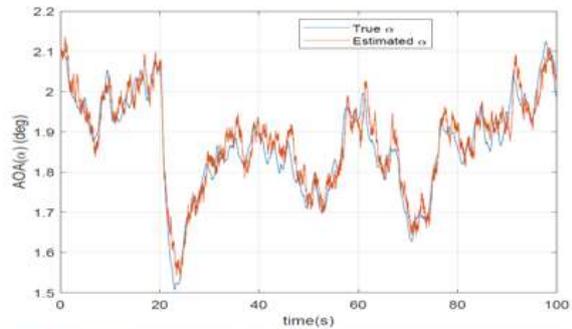


Fig 3. AOA Recovery Under Gust

Kalman filter performance was found good in simulated gusts. Filter was able to recover AOA very satisfactorily as shown in figure 3. As could be seen in figure 4, Kalman filter performed very well in estimation of gust velocities. The estimation of normal acceleration in presence of measurement noise is shown in figure 5. The performance of the filter in terms of measurement and estimated co-variance are tabulated in table-3.

**Table 3: Error Co-variance of Kalman Estimator**

Parameter	Normal Acceleration ( $N_z$ )	Pitch rate $q$
Measurement error co-variance	.0051	.0020
Estimated error co-variance	$8.4 \times 10^{-5}$	$1.95 \times 10^{-7}$

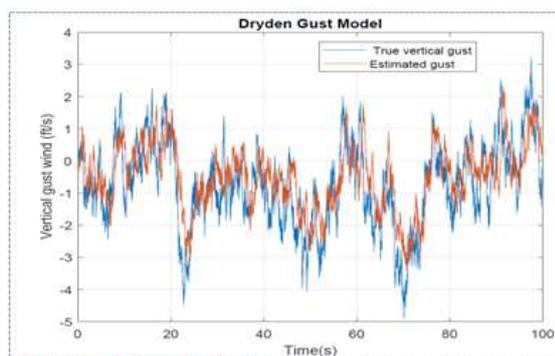


Fig 4. Vertical Gust Estimation

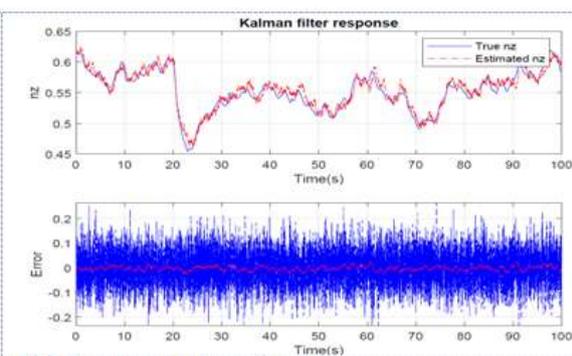


Fig 5. Normal acceleration estimation

#### IV. Conclusion

I have presented the result of flight tests done on a fighter aircraft during Avionics upgrade toward estimation of angle of attack and angle of side slip using Laser Gyro based INS and air data computer. This will provide redundancy and improve the reliability and flight safety. In addition, mission performance can be improved with accurate measurement of angle of attack and wind velocities.

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