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Stability study of inhomogeneous porous single-layered nanoplates using nonlocal elasticity theory

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Abstract: In the present paper, the stability study of porous inhomogeneous single- layered nanoplates are presented. The nonlocal elasticity theory with one nonlocal parameter is developed to examine buckling behaviour. Based on the Eringen's differential nonlocal elastic law and new first order shear deformation theory the equations of equilibrium are obtained from the principle of minimum potential energy. To simplify the equations of equilibrium and removing the bending-extension coupling, the buckling behaviours of FG nanoplates are investigated based on physical neutral surface concept. The equations of equilibrium are solved for simply supported boundary conditions using Navier's method. The effects of nonlocal parameter, porosity volume fraction, power-law index and Winkler parameter on critical buckling load are presented.

Keywords: Buckling, Porous Inhomogeneous Single-Layered Nanoplates,Nonlocal Elasticity Theory,Navier's method

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I. INTRODUCTION

Both experimental and molecular dynamic simulation (MD simulation) show that in nanostructural elements with very small dimensions their mechanical properties and behaviour change when these dimensions become very small. However, the classical elasticity theory cannot take into account the size effect in the analysis of mechanical behaviour of micro- and nanostructures. The nonlocal elasticity theoryis the most commonly used non-classical elasticity theory in which the influence of long-range interatomic forces has been incorporated into constitutive equations by means of the nonlocal scale parameter(Eringen &Edelen, 1972),(Eringen, 1983). According to nonlocal elasticity theory the stress in the observed point within a body is the function of the strains in all other points of the body.

The first paper in which the nonlocal elasticity theory was applied in the analysis of bending of Euler-Bernoulli nanobeams has been published by (Peddiesson *et al.*, 2003),(Simsek& Yurtcu, 2013) studied the bending and buckling of functionally graded nanobeams using the nonlocal Timoshenko beam. (Ansari& Sahmani, 2013) applied the classical plate theory (CLPT), first-order shear deformation theory (FSDT), higher-order shear deformation theory (HSDT) and MD simulation to investigate the biaxial buckling of single-layered graphene sheet. (Sobhy, 2015) investigated the bending response of single-layered graphene sheet in thermal environment using the two-variable plate theory and Levy type solution. (Ansari *et al.*2010) studied the vibrational behaviour of multi-layered graphene sheets with different boundary conditions using the the finite element method and nonlocal Mindlin plate theory. (Golmakani & Rezatalab, 2015) applied the nonlocal Mindlin plate theory to investigate the nonuniform biaxial buckling of orthotropic nanoplates embedded in an elastic medium. (Radić*et al.*, 2014) examined the buckling behaviour of double-orthotropic nanoplates with simply supported boundary conditions in Pasternak elastic medium using the nonlocal classical plate theory.(Pouresmaeeli *et al.*2012)studied the free vibration of double-orthotropic nanoplates embedded in an elastic medium using the nonlocal classical plate theory.(Karličić*et al.*, 2015) presented the vibration and buckling analysis of the multi-layered graphene sheets embedded in elastic medium using the nonlocal Kirchhoff-Love plate theory. (Radić& Jeremić,2017) carried out the free vibration and buckling of orthotropic double-layered graphene sheets with different boundary conditions in hygrothermal environment using nonlocal elasticity theory and new first-order shear deformation theory. (Hosseini & Jamalpoor, 2015) presented the thermomechanical vibration behaviour of double-viscoelastic nanoplates made of functionally graded materials.

(Shahverdi & Barati, 2017) studied on vibration analyisis of porous functionally graded nanoplates employing a general nonlocal strain-gradient theory. (Barati & Shahverdi, 2018) investigated the forced vibration of porous inhomogeneous nanoplatesusing the general nonlocal stress-strain gradient theory.(Shaat & Abdelkefi, 2016) was devoted to studying the buckling behaviour of porous nanobeams based on the classical couple stress theory. (Shafiei & Kazemi, 2017) carried out the buckling analysis of functionally graded porous nano-/micro-scaled beams based on the modified couple stress theory.

To the authors' best knowledge, the buckling analysis of a single-layered inhomogeneous nanoplates rested on Pasternak foundation has never been studied before.Besides that, it can be concluded that this is the first paper which deals with the buckling behaviour of single-layered inhomogeneous nanoplates using nonlocal elasticity theory.

II. THEORY AND FORMULATION

Fig. 1 shows a schematic diagram of rectangular single-layered porous functionally graded (inhomogeneous) nanoplate of uniform thickness *h*,length L_x and width L_y associated with the *z*, *x* and *y*-axes of the coordinate system. As it can be seen, single-layered FG nanoplate on elastic Pasternak substrate, where k_w and k_G are Winkler modulus parameter and shear modulus parameter respectively.

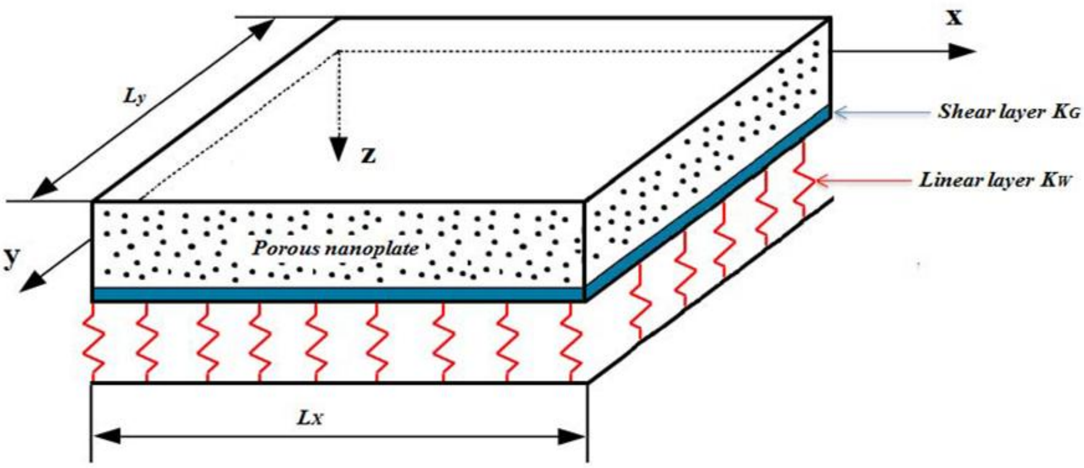


Figure 1: Configuration of porous single-layered inhomogeneous nanoplate

Kinematic relations

According to the new first-order shear deformation theory, the displacement field of the single-layered inhomogeneous nanoplate is expressed by

$$u_x(x,y,z)=u(x,y)-(z-z_0)\frac{\partial\theta}{\partial x}$$
$$u_y(x,y,z)=v(x,y)-(z-z_0)\frac{\partial\theta}{\partial y} \quad (1)$$
$$u_z(x,y,z)=w(x,y)$$

where u and v are the displacement of mid-plane along *x*- and *y*-axis respectively, w is transverse displacement of a point on the mid-plane of the inhomogeneous nanoplate, and θ is rotation parameter.

The position of the physical neutral surface z_0 can be simply determined from the condition that the integral of the first momentum of elasticity modulus $E(z)$ in the direction of thickness is equal to zero.

$$\int_{-h/2}^{h/2} E(z)(z-z_0)dz=0 \quad (2)$$

The inhomogeneous nanoplate is assumed to be made of mixture of ceramic and metal. It means that the Young's modulus *E* is changed continuously in the thickness direction.

$$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+E_m-\frac{\xi}{2}(E_c+E_m) \quad (3)$$

where *p* is the non-homogeneity or power-law index (*p* is a non-negative parameter) which determine the material distribution across the plate thickness.

Nonzero strains of the new first-order shear deformation theory model are expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} - (z - z_0) \begin{Bmatrix} \dfrac{\partial^2 \theta}{\partial x^2} \\ \dfrac{\partial^2 \theta}{\partial y^2} \\ 2\dfrac{\partial^2 \theta}{\partial x \partial y} \end{Bmatrix}, \quad (4)$$

The principle of minimum of potential energy can be defined as:

$$\delta U + \delta V = 0 \quad (5)$$

here, is the variation of strain energy and is the variation of virtual work done by external loads.
The variation of strain energy for the new first-order shear deformation theory can be written as

$$\delta U = \int_V (\sigma_{xx}\delta\varepsilon_{xx} + + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} + + \sigma_{yz}\delta\gamma_{yz})dV \quad (6)$$

The nonlocal stress resultants, and are obtained from the following expression

$$N_i = \int_{-h/2}^{h/2} \sigma_i dz$$
$$Q_j = K_s \int_{-h/2}^{h/2} \sigma_i dz \quad (7)$$
$$M_i = \int_{-h/2}^{h/2} z\sigma_i dz \qquad (i = xx, yy, xy;\ j = xz, yz)$$

where is the shear correction factor. The approximate value for the shear correction factor is =5/6.
By substituting Eq. (4) into Eq. (6) and using Eq. (7) we can write Eq. (16) in the following form

$$\delta U = \int_A \Bigg[N_{xx}\frac{\partial \delta u}{\partial x} - M_{xx}\frac{\partial^2 \delta\theta}{\partial x^2} + N_{yy}\frac{\partial \delta u}{\partial y} - M_{yy}\frac{\partial^2 \delta\theta}{\partial y^2} + N_{xy}\left(\frac{\partial \delta u}{\partial y} + \frac{\partial \delta u}{\partial x}\right) - 2M_{xy}\frac{\partial^2 \delta\theta}{\partial x \partial y}$$
$$+ Q_{xz}\frac{\partial \delta(w-\theta)}{\partial x} + Q_{yz}\frac{\partial \delta(w-\theta)}{\partial y}\Bigg] dA \quad (8)$$

The variation of the virtual work, done by external loads, can be written as

$$\delta V = \int_A \left(-N_{xx}^m \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} - N_{yy}^m \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y} - k_w \delta w + k_G \left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y} \right) \right) dA \quad (9)$$

where, and k_G are Winkler and Pasternak parameter of elastic foundation.

By inserting Eqs. (8) and (9) into Eq. (5) , using some mathematical operations and setting the coefficients of δw and $\delta\theta$ to zero, the following equations of equilibrium can be obtained

$$\delta w: \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} - k_w w + k_G\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - N_{xx}^m \frac{\partial^2 w}{\partial x^2} - N_{yy}^m \frac{\partial^2 w}{\partial y^2} = 0$$
$$\delta\theta: \frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} - \frac{\partial Q_{xz}}{\partial x} - \frac{\partial Q_{yz}}{\partial y} = 0 \quad (10)$$

III. THE NONLOCAL ELASTICITY THEORY

Making certain assumptions presented by (Eringen & Edelen, 1972),(Eringen, 1983) we will assume the nonlocal differential constitutive laws as

$$\left[1 - (e_0\ell)^2\nabla^2\right]\sigma_{ij}(x) = C_{ijkl}\varepsilon_{kl} \qquad i, j, k, l = x, y, z \quad (11)$$

where ∇^2 is the Laplacian operator which is defined by $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$,and $e_0\ell$ is the nonlocal parameter that takes into account the small scale effects into the differential constitutive law.

Based on Eqs. (11) the stress-strain constitutive equations of a rectangular inhomogeneous nanoplates are expressed as follows:

$$[1-(e_0\ell)^2\nabla^2]\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{Bmatrix}=\frac{E(z)}{1-\nu^2}\begin{Bmatrix}1 & \nu & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0\\ 0 & 0 & (1-\nu)/2 & 0 & 0\\ 0 & 0 & 0 & (1-\nu)/2 & 0\\ 0 & 0 & 0 & 0 & (1-\nu)/2\end{Bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \quad (12)$$

where ν denote Poisson's ratios.

Integrating Eq. (12) over the nanoplate's thickness and using Eq. (3), the stress resultants can be obtained as follows:

$$\begin{aligned}
&M_{xx}-(e_0\ell)^2\nabla^2M_{xx}=-D\left(\frac{\partial^2\theta}{\partial x^2}+\nu\frac{\partial^2\theta}{\partial y^2}\right)\\
&M_{yy}-(e_0\ell)^2\nabla^2M_{yy}=-D\left(\nu\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)\\
&M_{xy}-(e_0\ell)^2\nabla^2M_{xy}=-(1-\nu)D\frac{\partial^2\theta}{\partial x\partial y}\\
&Q_{xz}-(e_0\ell)^2\nabla^2Q_{xz}=K_SF\left(\frac{\partial w}{\partial x}-\frac{\partial\theta}{\partial x}\right)\\
&Q_{yz}-(e_0\ell)^2\nabla^2Q_{yz}=K_SF\left(\frac{\partial w}{\partial y}-\frac{\partial\theta}{\partial y}\right)
\end{aligned} \quad (13)$$

where

$$D=\int_{-h/2}^{h/2}\frac{E(z-z_0)^2}{1-\nu^2}dz,\ \ F=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}dz$$

The equations of equilibrium of an inhomogeneous nanoplate can be derived by inserting Eqs. (13), into Eqs. (10) as follows:

$$K_SF\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2\theta}{\partial x^2}-\frac{\partial^2\theta}{\partial y^2}\right)+(1-(e_0\ell)^2\nabla^2)\left[-k_ww+k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-N_{xx}^m\frac{\partial^2 w}{\partial x^2}-N_{yy}^m\frac{\partial^2 w}{\partial y^2}\right]=0 \quad (14)$$

$$D\left(\frac{\partial^4\theta}{\partial x^4}+\frac{\partial^4\theta}{\partial x^2\partial y^2}+\frac{\partial^4\theta}{\partial y^4}\right)+K_sF\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2\theta}{\partial x^2}-\frac{\partial^2\theta}{\partial y^2}\right)=0 \quad (15)$$

The equations (14) and (15) present the equations of equilibrium for porous inhomogeneous nanoplate on Pasternak foundation.

IV. SOLUTION BY NAVIER'S METHOD

In this section, the equations of equilibrium (14) and (15) are solved analytically using Navier's method to obtain the value of critical buckling load of porous inhomogeneous nanoplate subjected to in-plane preload. Before solving the equations of equilibrium, the boundary conditions should be defined. In this paper the nanoplate is assumed to have simply supported edges (S).

Simply supported (S):

$$\frac{\partial\theta}{\partial y}=w=M_{xx}=0,\qquad \text{at } x=0,L_x$$

$$\frac{\partial\theta}{\partial x}=w=M_{yy}=0,\qquad \text{at } y=0,L_y \quad (16)$$

To satisfy the above boundary conditions, the displacement quantities can be written in the following form:

$$\begin{aligned}&w(x,y)=W_{mn}\sin(\alpha x)\sin(\beta y)\\ &\theta(x,y)=\theta_{mn}\sin(\alpha x)\sin(\beta y)\end{aligned} \quad (17)$$

where (W_{mn},θ_{mn}) are arbitrary parameters, and $\alpha=m\pi/L_x,\beta=n\pi/L_y$.

Substituting Eq. (17) into Eqs. (14) and (15) the analytical solution of the equations of equilibrium in terms of parameters W_{mn}, θ_{mn}, can be obtained from

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ \theta_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (18)$$

V. NUMERICAL RESULTS AND DISCUSSIONS

In this research the material properties of inhomogeneous nanoplate (ceramic and metal) are given as:

$E_c = 380GPa,\ \nu_c = 0.3,\ E_m = 70GPa,\quad \nu_m = 0.3$

In the present study the following dimensionless parameters are used:

$\hat{N} = \frac{NL_x^{\ 2}}{E_m h^3},\ k_{wN} = \frac{k_w L_x^4}{D},\ k_{GN} = \frac{k_G L_x^2}{D}$

where

$N_{xx}^m = N,\ N_{yy}^m = kN$

In Fig. 2 (a) and (b) the nondimensional critical buckling load is plotted as a function of the porosity volume fraction for various values of the nonlocal parameter for the case of biaxial buckling. From Fig. 2 (a) and (b) it can beeasily noticed that the value increase of the volume porosity fraction has a decreasing effect on the nondimensional critical buckling load. It can also be seen that the value of nondimensional critical buckling load decreases with the increase in the non-local parameter value.

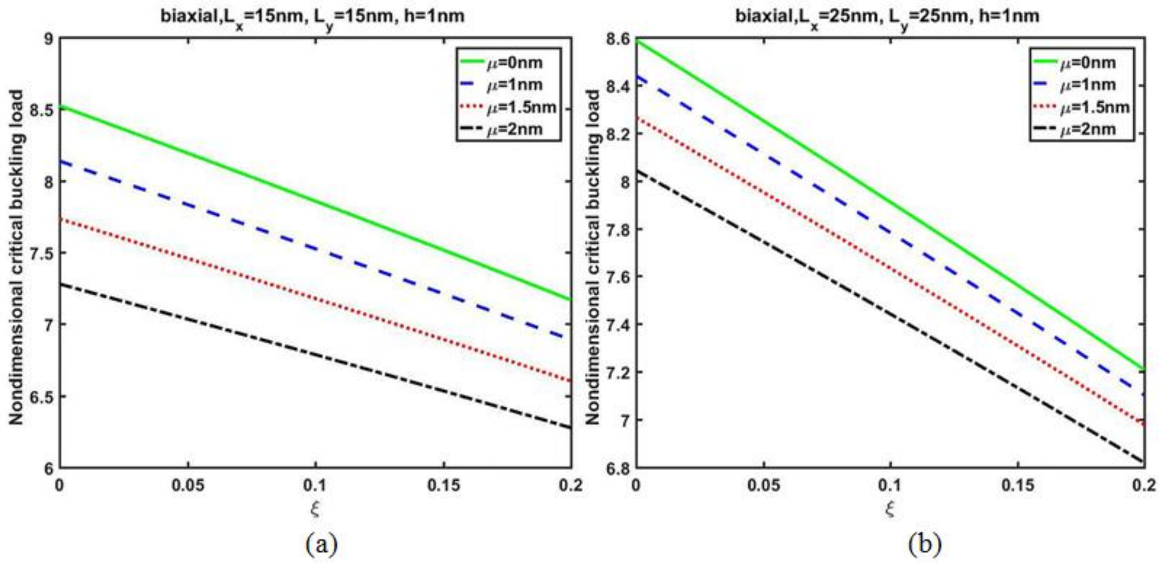
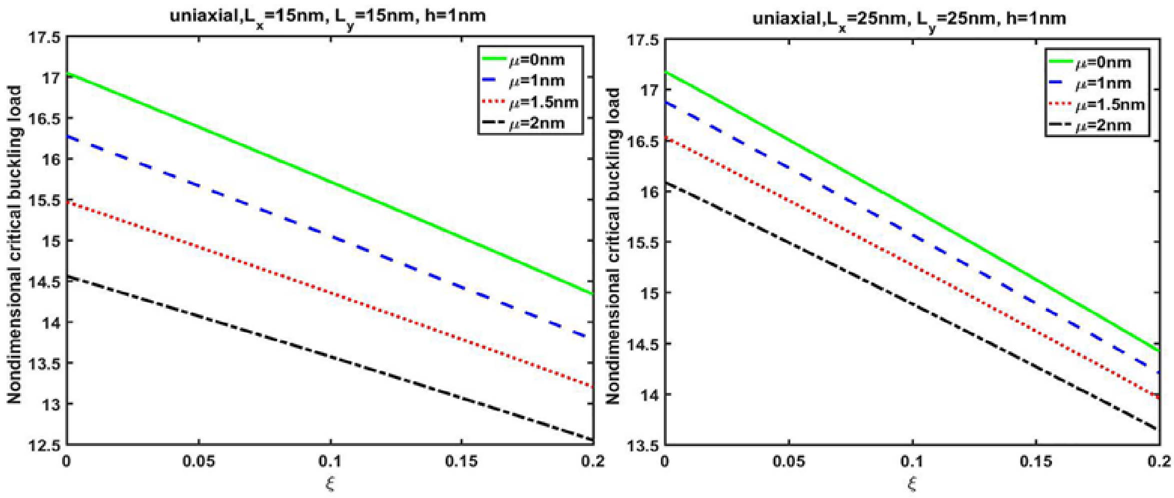


Figure 2: Effect of porosity volume fraction on the nondimensional critical buckling load for different nonlocal parameter and biaxial buckling (k=1)

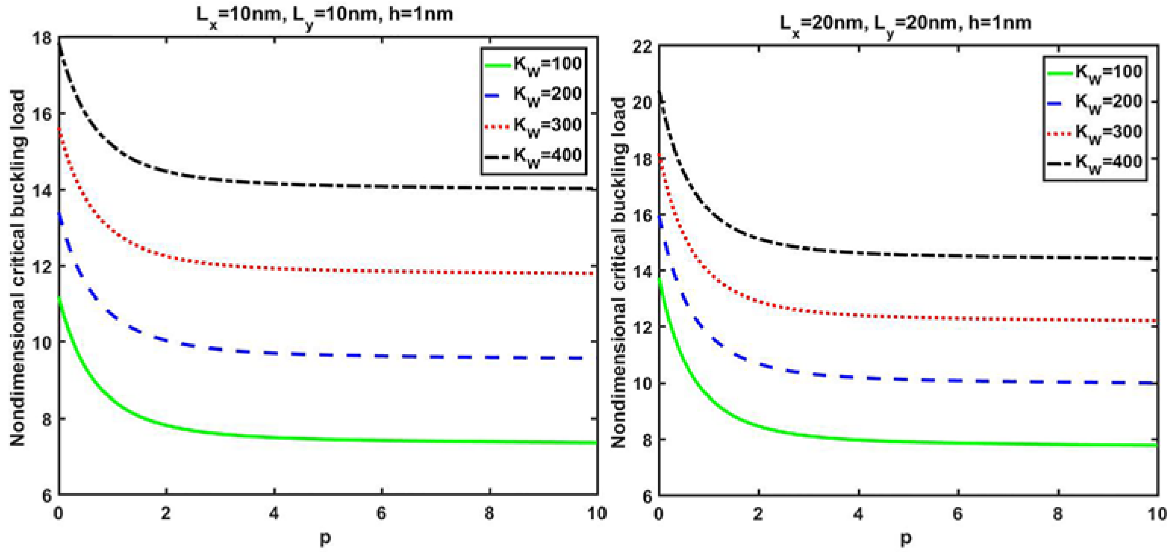
From Fig. 3 (a) and (b) it can be seen that in the case of uniaxial buckling the behaviour of a square inhomogeneous single-layered nanoplate with the change of value of porosity volume fraction and nonolocal parameter is the same as in the case of biaxial buckling. It can easily be seen that in the case of an uniaxial buckling, the value of nondimensional critical buckling load is higher than the case of biaxial buckling.



(a) (b)

Figure 3: Effect of porosity volume fraction on the nondimensional critical buckling load for different nonlocal parameter and uniaxial buckling (k=0)

Fig. 4(a) and (b) demonstrates the effects of the nondimensional Winkler parameter on the nondimensional critical buckling load versus the power-law index. It can be concluded that the value of the nondimensional critical buckling load is reduced when the power- law index *p* rises. It is evident that the value of nondimensional critical buckling load increases with the increase in the nondimensional Winkler parameter value.



(a) (b)

Figure 4: Effect of power-law index on the nondimensional critical buckling load for different value of nondimensional Winkler parameter

VI. CONCLUSIONS

In this paper, biaxial and uniaxial buckling of the inhomogeneous porous single-layered nanoplates subjected to initial in-plane preload were presented. By using the principle of the minimum potential energy, the equations of equilibrium were obtained base on the new first order shear deformation theory in the framework of the Eringen's nonlocal elasticity theory. The Navier's method has been used to solve the equations of equilibrium for SSSS boundary conditions. Numerical results are presented to investigate the effects of nonlocal parameter, porosity volume fraction, power law index and Winkler parameter on nondimensional critical buckling load. It is observed that increasing the porosity volume fraction will decrease the value of nondimensional critical buckling load for the case of biaxial and uniaxial buckling. Also, the value of nondimensional critical buckling load decrease with the increase of the power law index.

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