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Fuzzy approach on minimization using hungarian method

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Abstract: Many researchers worked on Fuzzy time minimization Assignment problem, which is optimizing the time of assigning the *m* jobs to the *n* workers. In this paper, the fuzzy time minimization assignment problem is solved by using Hungarian method and Robust's Ranking technique for the Fuzzy trapezoidal numbers. The algorithm of the method is presented and a numerical example explains the mentioned procedure.

Keywords: Fuzzy number, Hungarian method, Robust's ranking technique, Time minimization assignment problem.

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I. INTRODUCTION

The transportation model which deals with transportation costs, supply and demand was originally developed by Hitchcock (1941). After the transportation problem, the next important topic of research is the assignment problem. Any assignment problem is a particular type of transportation problem where specific tasks (jobs) are to be assigned to an equal number of machines (workers) on one to one basis, such that the assignment cost (or profit) is minimum (or maximum).

In day to day life, there are many diverse situations due to uncertainty like- lack of data, computational errors, measuring inaccuracy or any other factors due to which parameters cannot be expressed in precise form. To deal with imprecise information in making decisions, Zadeh [6] introduced the notion of fuzziness. Further studies about decision making in fuzzy environment was done by Bellman and Zadeh [1] . Later Zimmermann [7] showed that solutions obtained by fuzzy linear programming were effectively accurate. Dinagor and Palanive [2] studied fuzzy transportation problem with aid of trapezoidal fuzzy numbers.Different methods have been presented for assignment problem and various articles have been published on the subject. R. Nagarajan and A. Solairaju [5] had given an algorithm to solve fuzzy assignment problem using Robust ranking technique. This paper attempts to adopt a method to obtain effective solution by using Hungarian method and Robust's Ranking technique for solving Assignment problem on trapezoidal fuzzy numbers.

II. PRELIMINARIES

A fuzzy set is characterized by a membership function mapping elements of a domain, or the universe of discourse X to the unit interval [0, 1] (i.e.) $A = \{x, \mu_A(x); x \in X\}$. Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the Fuzzy set A and $\mu_A(x)$ is called the membership value of x ∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

For a trapezoidal fuzzy number A(x), it can be represented by A (m, n, a, b ; 1) with membership function μ(x) is given by

$$\mu(x) = \begin{cases} \dfrac{x-m}{n-m}, m \le x \le n \\ 1, n \le x \le a \\ \dfrac{b-x}{b-a}, a \le x \le b \\ 0, otherwise \end{cases}$$

The α-cut of a fuzzy number A(α) is defined as , $A(\alpha) = \{ x: \mu(x) \ge \alpha , \alpha \in [0,1] \}$

III. ROBUST RANKING TECHNIQUE

Robust ranking technique satisfies linearity, and additive properties and provides results which are consistent. Given a convex fuzzy number ã , the Robust Ranking Index is defined by

$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U) d\alpha$, Where (a_α^L, a_α^U) is the α- level cut of the fuzzy number ã

and $(a_\alpha^L, a_\alpha^U) = \{(n - m)\alpha + m), (a - (a - n)\alpha)\}$.

The Robust ranking index R(ã) gives the representative value of the fuzzy number ã .

IV. HUNGARIAN METHOD

step 1. If the matrix is n x n, proceed to next step. For m x n, a dummy row or column is added to make the matrix square.
step 2. Subtract the entries of each row by the row minimum. These operations create atleast one zero in each rows.
step 3. Subtract the entries of each column by the column minimum. These operations create at least one zero in each rows and each column.
step 4. Draw the minimum number of lines to cover all the zero's of the matrix. If the number of drawn lines less than n, then the complete assignment is not possible, while if the number of lines is exactly equal to n, then the complete assignment is obtained.
step 5. If a complete assignment is not possible in step 4 , then select the smallest element (say k) which is not covered by the line in step 4. Now subtract k from all uncovered elements and add k to all elements which are covered by both horizontal and vertical line. If still a complete optimal assignment is not achieved , then use step 4 and 5 iteratively until the number of lines crossing all zero's becomes equal to the order of matrix, which indicates an optimal assignment is obtained.

V. MAIN RESULT

We explain by solving a fuzzy assignment problem, using the above mentioned algorithms.
Problem: Let therows of the below matrix represent Machines, say A, B, and C and D and the columns represent four Tasks, say job1, job2, job3 and job4. Considering the past average records, the cost that each machine takes to perform a task isrepresented by fuzzy trapezoidal numbers and the data is shown below.

Fuzzy costs (in rupees)				
Task ↓ Machines →	A	B	C	D
1	(7,8,10,11)	(4,7,8,10)	(2,3,10,14)	(5,7,9,13)
2	(2,3,5,7)	(4,7,10,14)	(3,5,6,7)	(3,5,7,10)
3	(4,6,9,12)	(5,7,12,15)	(6,8,10,12)	(4.10,13,15)
4	(2,3,5,9)	(9,10,11,15)	(7,9,10,13)	(8,11,13,15)

We will find the assignment of tasks to machines that will minimize the Total Fuzzy Cost.
Solution: In conformation to model, the fuzzy assignment problem can be formulated in the following way,
Min { R(7,8,10,11)a_{11} + R(4,7,8,10)a_{12} + R(2,3,10,14) a_{13} + R(5,7,9,13)a_{14} +
R(2,3,5,7)a_{21} + R(4,7,10,14)a_{22} + R(3,5,6,7)a_{23} + R(3,5,7,10)a_{24} +
R(4,6,9,12)a_{31} + R(5,7,12,15)a_{32} + R(6,8,10,12)a_{33} + R(4.10,13,15)a_{34} +
R(2,3,5,9)a_{41} + R(9,10,11,15)a_{42} + R(7,9,10,13)a_{43} + R(8,11,13,15)a_{44} }
Subject to

$a_{11} + a_{12} + a_{13} + a_{14} = 1$ $\quad a_{11} + a_{21} + a_{31} + a_{41} = 1$
$a_{21} + a_{22} + a_{23} + a_{24} = 1$ $\quad a_{12} + a_{22} + a_{32} + a_{42} = 1$
$a_{31} + a_{32} + a_{33} + a_{34} = 1$ $\quad a_{13} + a_{23} + a_{33} + a_{43} = 1$
$a_{41} + a_{42} + a_{43} + a_{44} = 1$ $\quad a_{14} + a_{24} + a_{34} + a_{44} = 1$ where $a_{ij} \in [0,1]$

By applying Robust Ranking technique, we calculate the value of R(7,8,10,11).
The membership function of the trapezoidal fuzzy number (7,8,10,11) is

$$\mu(x) = \begin{cases} \frac{x-7}{8-7}, & 7 \le x \le 8 \\ 1, & 8 \le x \le 10 \\ \frac{11-x}{11-10}, & 10 \le x \le 11 \\ 0, & otherwise \end{cases}$$

The α – cut of the fuzzy number (7,8,10,11) is (a_α^L, a_α^U) where
$a_\alpha^L = x - 7 / 1 = \alpha \Rightarrow x = \alpha + 7$
$a_\alpha^U = 11 - x / 1 = \alpha \Rightarrow x = 11 - \alpha$
$(a_\alpha^L, a_\alpha^U) = (\alpha + 7, 11 - \alpha) = \alpha + 7 + 11 - \alpha = 18$ for which
R ($\tilde{a}_{11}$) = R(7,8,10,11) = $\int_0^1 (0.5)(a_\alpha^L, a_\alpha^U) d\alpha = \int_0^1 (0.5)(18)\, d\alpha = 9\int_0^1 d\alpha = 9$
Similarly, the Robust's Ranking indices for the fuzzy costs are calculated and we get,
R ($\tilde{a}_{11}$) = 9, R ($\tilde{a}_{12}$) = 7.25, R ($\tilde{a}_{13}$) = 7.25, R ($\tilde{a}_{14}$) = 8.5.
R ($\tilde{a}_{21}$) = 4.25, R ($\tilde{a}_{22}$) = 8.75, R ($\tilde{a}_{23}$) = 5.25, R ($\tilde{a}_{24}$) = 6.25.

R ($\tilde{a}_{31}$) = 15.5, R ($\tilde{a}_{32}$) = 9.75, R ($\tilde{a}_{33}$) = 9, R ($\tilde{a}_{34}$) = 10.5.
R ($\tilde{a}_{41}$) = 4.75, R ($\tilde{a}_{42}$) = 11.25, R ($\tilde{a}_{43}$) = 9.75, R ($\tilde{a}_{44}$) = 11.75.
We now replace the values in their corresponding a_{ij}, which results in convenient assignment problem in the Linear programming form.

$$\begin{bmatrix} 9 & 7.25 & 7.25 & 8.5 \\ 4.25 & 8.75 & 5.25 & 6.25 \\ 15.5 & 9.75 & 9 & 10.5 \\ 4.75 & 11.25 & 9.75 & 11.75 \end{bmatrix}$$

We now solve it by Hungarian method . We use the steps mentioned in the algorithm to get the following optimal solution.
step 1:The matrix is square matrix, so we proceed to next step.
step 2: We find the row minimum element, write it on the right of matrix and then subtract each element of the row with corresponding row minimum element.
Min

$$\begin{bmatrix} 9 & 7.25 & 7.25 & 8.5 \\ 4.25 & 8.75 & 5.25 & 6.25 \\ 15.5 & 9.75 & 9 & 10.5 \\ 4.75 & 11.25 & 9.75 & 11.75 \end{bmatrix} \begin{matrix} 7.25 \\ 4.25 \\ 9 \\ 4.75 \end{matrix}$$

On subtracting we get,

$$\begin{bmatrix} 1.75 & 0 & 0 & 1.25 \\ 0 & 4.5 & 1 & 2 \\ 6.5 & 0.75 & 0 & 1.5 \\ 0 & 6.5 & 5 & 7 \end{bmatrix}$$, as we see each row has atleast one zero.

step 3:Similarly now find column minimum and write below respective columns and then subtract each element of column with corresponding minimum element.

$$\begin{bmatrix} 1.75 & 0 & 0 & 1.25 \\ 0 & 4.5 & 1 & 2 \\ 6.5 & 0.75 & 0 & 1.5 \\ 0 & 6.5 & 5 & 7 \end{bmatrix}$$

Min 0 0 0 1.25

On subtracting we get,

$$\begin{bmatrix} 1.75 & 0 & 0 & 0 \\ 0 & 4.5 & 1 & 0.75 \\ 6.5 & 0.75 & 0 & 0.25 \\ 0 & 6.5 & 5 & 5.75 \end{bmatrix}$$

step 4:We now draw the minimum number of lines to cover all the zero's of the matrix.

$$\begin{bmatrix} 1.75 & 0 & 0 & 0 \\ 0 & 4.5 & 1 & 0.75 \\ 6.5 & 0.75 & 0 & 0.25 \\ 0 & 6.5 & 5 & 5.75 \end{bmatrix}$$

Since the number of lines are less than n, the complete assignment is not possible. If the number of lines would had been equal to n, then complete assignment is obtained.
step 5: we now select a smallest element k(= 0.25),which doesn't lie on any of the lines and thensubtract it from each entry not covered by the lines and add it to each entry which lies on both vertical and horizontal line together. Then draw minimum number of lines covering all zero's .

$$\begin{bmatrix} 2 & 0 & 0.25 & 0 \\ 0 & 4.25 & 1 & 0.5 \\ 6.5 & 0.5 & 0 & 0 \\ 0 & 6.25 & 5 & 5.5 \end{bmatrix}$$

We see that optimal solution is not obtained, as number of lines drawn are less than n.
Using step 5 again, taking k (= 0.5) we get

$$\begin{bmatrix} 2.5 & 0 & 0.25 & 0 \\ 0 & 3.75 & 0.5 & 0 \\ 7 & 0.5 & 0 & 0 \\ 0 & 5.75 & 4.5 & 5 \end{bmatrix}$$

now draw the minimum number of lines to cover all the zero's of the matrix,

$$\begin{bmatrix} 2.5 & 0 & 0.25 & 0 \\ 0 & 3.75 & 0.5 & 0 \\ 7 & 0.5 & 0 & 0 \\ 0 & 5.75 & 4.5 & 5 \end{bmatrix}$$

We can see that the number of lines are equal to n and hence complete assignment is obtained.
We now examine rows and columns where exactly one zero is found which indicates the assignment will be made there.

$$\begin{bmatrix} 2.5 & 0 & 0.25 & 0 \\ 0 & 3.75 & 0.5 & 0 \\ 7 & 0.5 & 0 & 0 \\ 0 & 5.75 & 4.5 & 5 \end{bmatrix}$$

Make assignment in terms of zero's and the solution is:
Assign, Machine A→ task 4, Machine B→ task 1, Machine C→ task 3, Machine D→ task 2,
The fuzzy optimal total cost is $\tilde{a}_{12}+\tilde{a}_{24}+\tilde{a}_{33}+\tilde{a}_{41}$
= R(2,3,5,9)+ R(4,7,8,10) + R(6,8,10,12) + R(3,5,7,10) = R(15,23,30,41) = 27.25

Conclusion: In this paper we used a simple method to solve fuzzy assignment problem. This method can be used for all fuzzy assignment problems whether triangular or trapezoidal fuzzy numbers. It can also be used either to maximize or minimize the objective function.

REFERENCES

[1]. Bellman.R.E, Zadeh.L.A, " Decision making in a fuzzy environment " Management Sci.17 (1970), pp.141-164
[2]. D.S. Dinagar, K. Palanivel (2009), " The transportation problem in fuzzy environment", International journal of algorithms, computing and mathematical pp 55-71.
[3]. HadiBasirzadeh," Ones Assignment Method for Solving Assignment Problems", Applied Mathematical Sciences, Vol. 6, 2012, no. 47, 2345 - 2355,
[4]. Hamdy A. Taha, Operations Research, an introduction, 8th Ed. (2007).
[5]. Nagarajan . R. and Solairaju (2010). A. " A computing improved fuzzy optimal Hungarian assignment problem with fuzzy cost under Robust ranking technique, internal journal of computer applicationvolume.6, No.4. pp 6-13.
[6]. Zadeh. L. A(1965); " Fuzzy sets" information and control volume 08. pp 338-353.
[7]. Zimmerman. H. J (1978), " Fuzzy programming and linear programming with several objective functions".Fuzzy sets and systems pp 45-55.
[8]. H. J. Zimmermann, Fuzzy set theory and its Applications, third Ed., kluwerAcademic, Boston, 1996.

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