Time delay estimation of Shock wave Signal Based on Cross-
correlation algorithm

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Abstract: This article put forward the application of generalized cross-correlation algorithm to estimate the shock wave signal time delay estimation in allusion to the situation that outside noise and reverberation have little effect on shock wave signal which is produced by the supersonic object. This paper mainly studied the shock wave signal time delay estimation problem of generalized cross-correlation algorithm and the simulation verified the feasibility and effectiveness of generalized cross-correlation function in estimating time delay value; At the same time the paper simulate the delay estimation performance under different SNR, the experimental results show that for the shock wave signal PHAT weighting function has the characteristics of TDE accurately, anti-interference ability and high efficiency.

Keywords: GCC; TDE; Shock Wave signal; PHAT-GCC

I. INTRODUCTION

The passive acoustic signal acquisition and processing system of supersonic moving object Sonics moving object passive acoustic signal acquisition and processing system consists of a microphone array, precircuit, the data processor and computer system. When the supersonic objects are flying in the air, they just look like the situation that supersonic air blows objects, and then the head of the object is separated, resulting in aerodynamic turning concave and convex corner turning angle phenomenon, so that the surrounding air of the objects compress and expand, and a conical shock wave off the body will be in the head and tail of the object. Microphone array receive shock wave signal produced by a supersonic object when it is over the target, and the amplitude of shock wave and the time difference received by different pressure sensor acoustic are recorded, and other relevant parameters are determined by the delay difference, such as distance, azimuth and target state, etc. The most critical issue of this method is TDE.

In the time delay estimation algorithm, the correlation method is the most classic delay estimation method, it estimates the delay time difference between signals through the peak signal of the autocorrelation function\cite{1}. This method is simple to understand, easy to implement, but requires the signal and noise, noise and noise uncorrelated\cite{2}. However, the shock wave signal which is produced by a supersonic objects when flying in the air is unrelated to the noise, and the peripheral noise and reverberation have little effect on it. Therefore the paper proposes the application of generalized cross correlation weighting function to estimate the shock wave signal delay value\cite{3}.

This paper verified the validity and feasibility of the proposed method by simulation. It selected weighting function applied to the shock wave signal by simulation, so that the cross-correlation function has a relative sharp peak, therefore the delay can be estimated accurately

II. THE GENERALIZED CORRELATION ALGORITHM

2.1 correlation method

Correlation analysis is the basic method to compare two similar time domain signal level, as the signal from the same sound source has a certain correlation, so we can estimate the delay of sound source to both two microphones by calculating the signal cross-correlation function which is received by both two microphones\cite{4}.

According to the typical model of ideal microphone signals, $x_i(t)$, $x_j(t)$ are the signal which are received by the two sensors $M_i$, $M_j$. Their cross-correlation function is:
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\[ R_s = E[x_i(t)x_i(t-\tau)] = a_i a_j E[x(t-\tau)x(t-\tau-\tau)] + a_i E[s(t-\tau)n_j(t-\tau)] + a_j E[s(t-\tau)n_i(t-\tau)] \]

(1)

Since the sound source and noise are uncorrelated, both \( a_i E[s(t-\tau)n_j(t-\tau)] \) and \( a_j E[s(t-\tau)n_i(t-\tau)] \) are 0, the equation can be simplified as:

\[ R_s = a_i a_j E[x(t-\tau)x(t-\tau-\tau)] + a_i E[n_i(t)n_i(t-\tau)] + a_j E[n_j(t)n_j(t-\tau)] \]

(2)

Wherein \( a_i, a_j R_{ss} (\tau - (\tau_i - \tau_j)) \) is the autocorrelation function of the sound source, \( R_{nn} (\tau) \) is the noise autocorrelation function.

According to the formula (2). When the signal to noise ratio is large enough, the effect of noise autocorrelation function to peak will be relatively small, When \( \tau = (\tau_i - \tau_j) = 0 \), \( R_{ss} (\tau - (\tau_i - \tau_j)) \) will achieve maximum. So the delay \( \tau \) can be obtained from the peak of the cross-correlation function.

2.2 Generalized cross-correlation method

According to the two signals cross power spectral density function is just the cross-correlation function of the Fourier transform, we do Fourier transform to correlation function (2) to obtain two signals cross power spectrum:

\[ G_{ss}(\omega) = a_i a_j G_{ss}(\omega) + G_{nn}(\omega) \]

(3)

Wherein: \( G_{ss}(\omega) \) is the power spectrum of \( R_{ss}(\omega) \), \( G_{nn}(\omega) \) is the power spectrum of \( a_i a_j R_{ss}(\tau - (\tau_i - \tau_j)) \), \( G_{nn}(\omega) \) is the power spectrum of \( R_{nn}(\omega) \).

It can be seen from the formula (3) that, GCC method is based on the non-reverberation model. With the effect of model error, it is difficult to identify a plurality of sound sources and the direction interference of the noise correctly and it is more inclined to estimate the delay value of a strong signal, due to the low computational complexity of generalized cross-correlation method and the easy implement, this article intends to use GCC delay estimation method to estimate the value of the shock wave signal[5]. Getting power spectrum of two related signals cross and power spectral domain were weighted to highlight the relevant parts of the signal and suppress the parts of noise and reverberation, the peak of the correlation function is more prominent, and then it transform to the time domain inversely, and the delay is sought ultimately. This approach is generalized cross-correlation method. Generalized cross-correlation function is:

\[ R_{GCC}(\omega) = \int_{-\infty}^{\infty} \psi(\omega) G_{ss}(\omega)e^{i\omega t} \]

(4)

Where: \( \psi(\omega) \) is a weighting function, commonly used weighting functions are: phase transformation (PHAT), Roth processor and smooth coherent transformation (SCOT); While, for different noise and reflection, different weighting functions can be chose so that \( R_{GCC}(\omega) \) may have a sharp peak value[7]. Which kind of weighted function is more suitable for estimating the time delay of shock wave signal?

III. TIME DELAY ESTIMATION OF SHOCKWAVE SIGNAL BASED ON CROSS CORRELATION ALGORITHM

3.1 Generalized correlation Simulation with different weighting function

We will verify the validity and accuracy of the delay estimation in different weighting function in the generalized correlation algorithm in measuring the shock wave signal through following simulation experiments.

First, the two-way shock wave signal is given as shown in Figure 2, the signal’s length is 2048, the sampling frequency is 100KHz, d which is the points of two-way signal delay (d multiplyed by sampling period is the value of TDE) is 100.
Time delay estimation of Shock wave Signal Based on Cross-correlation algorithm

After we apply the GCC, PHAT-GCC, ROTH-GCC and SCOT-GCC to get the delay estimation of two-way shock wave signal, the simulation results are shown in Table 1 below:

<table>
<thead>
<tr>
<th></th>
<th>GCC</th>
<th>PHAT-GCC</th>
<th>ROTH-GCC</th>
<th>SCOT-GCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>97</td>
<td>99</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td>0.023296</td>
<td>0.012498</td>
<td>0.012498</td>
</tr>
</tbody>
</table>

It can be seen from the table that, the sampling points and delay estimates which were got by cross-correlation function and the various weighting algorithm are basically the same, the time delay value which is estimated by PHAT - GCC differs from the actual given signal delay by a sample point, and the error is very small. It can be seen from the face of simulation results of the shock wave signal with different delay estimation algorithm that, generalized cross-correlation algorithm and its various weighting algorithm can accurately determine the value of TDE shock wave signal.

3.2 Comparison of delay estimation performance of different weighting functions in the same SNR

The simulation results show that delay estimation of shockwave signal based on cross-correlation algorithm is feasible, and next we will simulate under different SNR with the above weighted algorithm, and study the performance of various algorithms in measuring the shock-weighted signal’s TDE.

First, the two signals are given, the length of signal is 2048, the sampling frequency is 100KHz, two-way signal’s delay points is 100, when SNR is equal to 0, 10, 20 and 50, we will apply the algorithm to estimate the value of delay, the results are shown in Table 2: d represents the estimated delay dots and t represents the time program run (s).

<table>
<thead>
<tr>
<th></th>
<th>GCC</th>
<th>PHAT-GCC</th>
<th>ROTH-GCC</th>
<th>SCOT-GCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>0</td>
<td>95</td>
<td>0.023296</td>
<td>0.012498</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>96</td>
<td>0.024790</td>
<td>0.012434</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>97</td>
<td>0.023049</td>
<td>0.012601</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>99</td>
<td>0.023553</td>
<td>0.013493</td>
</tr>
</tbody>
</table>

It can be seen from the simulation results that, in terms of time, the basic cross-correlation algorithm spent the most time, the running time of other related improvements such as PHAT weighting algorithm, ROTH weighting and SCOT weighted algorithm is basically the same, PHAT and scot spent less time relatively; But according to sharpening of kurtosis, TDE accuracy of this four ways deteriorate with the lower of SNR and the sharpness of cross-correlation function’s peak decreases with the decrease of the ratio of signal and noise. Although the weighted GCC, SCOTT and ROOT has some noise immunity, but with the decrease of SNR, its extent of fluctuation significantly strengthened, its reflection and limited observational data is very sensitive especially for the noise outside, it can make the peak unobvious; For PHAT weighting in higher SNR time, it shows little fluctuation and peak sharp features. When signal to noise ratio is lowered, it also showed a strong anti-interference. Therefore, in the use of the cross-correlation function of shock wave signal time delay estimation algorithms, PHAT weighting function has a relatively good performance.

IV. SUMMARY

This paper studied time delay estimation technique of shock wave signal based generalized cross correlation. The feasibility of generalized cross-correlation function to get supersonic object delay estimation was given; Through matlab simulation of GCC, PHAT weighting, ROTH weighting and SCOT weighted algorithm, it was verified that they can estimate the delay value of two-way shock wave signal; The performance of the time delay estimation of shock wave signal is simulated under different signal to noise ratio. The simulation results...
showed that PHAT weighting function having characteristics of accurate delay estimation, anti-interference ability and less time for the shock wave signal. Compared with other algorithms, this algorithm has better real-time performance.

REFERENCES


Author Profile

LuJingyi received the B.S. and M.S. degrees in Faculty of Electricity and Information Engineering from NorthEast Petroleum University in 2001 and 2005, respectively. Now he is a PhD student in the Harbin Institute of Technology. Mainly engaged in research work signal detection and acoustic signal processing.