A STUDY ON ANTI L-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT: In this paper, we made an attempt to study the algebraic nature of an anti L-fuzzy subhemiring of a hemiring. 2000 AMS Subject classification: 03F55, 06D72, 08A72.

KEY WORDS: L-fuzzy set, anti L-fuzzy subhemiring, pseudo anti L-fuzzy coset.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R; +, .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (R; +, .) share the same properties as a ring except that (R; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R; +, .) is said to be a semiring if (R; +) and (R; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring may have an identity 1, defined by 1.a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[12], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy Left h- ideals in Hemirings was introduced by Akram.M and K.H.Dar[1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[6]. In this paper, we introduce the some Theorems in anti L-fuzzy subhemiring of a hemiring.

1. PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1. A L-fuzzy subset A of X is a function A : X → L.

1.2 Definition: Let (R, +, .) be a hemiring. A L-fuzzy subset A of R is said to be an anti L-fuzzy subhemiring (ALFSHR) of R if it satisfies the following conditions:

(i) μA(x+y) ≤ μA(x) ∨ μA(y),
(ii) μA(x+y) ≤ μA(x) ∨ μA(y), for all x and y in R.

1.3 Definition: Let A and B be L-fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by AxB, is defined as AxB = {(x, y), μA(x,y) / for all x in G and y in H }, where μAxB(x, y) = μA(x) ∨ μB(y).

1.4 Definition: Let A be a L-fuzzy subset in a set S, the anti-strongest L-fuzzy relation on S, that is a L-fuzzy relation on A is V given by μA(x, y) = μA(x) ∨ μA(y), for all x and y in S.

1.5 Definition: Let (R, +, .) and (R', +, .) be any two hemirings. Let f : R → R' be any function and A be an anti L-fuzzy subhemiring in R, V be an anti L-fuzzy subhemiring in f(R) = R', defined by μV(y) = inf x ∈ f⁻¹(y) μA(x), for all x in R and y in R'. Then A is called a preimage of V under f and is denoted by f⁻¹(V).

1.6 Definition: Let A be an anti L-fuzzy subhemiring of a hemiring (R, +, .) and a in R. Then the pseudo anti L-fuzzy coset (aA)p is defined by (aA)p(x) = p(a)μA(x), for every x in R and for some p in P.

2. PROPERTIES OF ANTI L-FUZZY SUBHEMIRING OF A HEMIRING

2.1 Theorem: Union of any two anti L-fuzzy subhemirings of a hemiring R is an anti L-fuzzy subhemiring of R.

Proof: Let A and B be any two anti L-fuzzy subhemirings of a hemiring R and x and y in R. Let A = {(x, μA(x)) / x ∈ R} and B = {(x, μB(x)) / x ∈ R} and also let C = A ∪ B = {(x, μC(x)) / x ∈ R}, where μC(x) = μA(x) ∨ μB(x). Now, μC(x+y) ≤ [μA(x)∨μB(y)]∨ μC(x)∨μC(y). Therefore, μC(x+y) ≤
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\( \mu_c(x) \lor \mu_c(y) \), for all \( x \) and \( y \) in \( R \). And, \( \mu_c(xy) \leq \{ \mu_a(x) \lor \mu_a(y) \} \lor \{ \mu_b(x) \lor \mu_b(y) \} = \mu_c(x) \lor \mu_c(y) \).

Therefore, \( \mu_c(xy) \leq \mu_c(x) \lor \mu_c(y) \), for all \( x \) and \( y \) in \( R \). Therefore \( C \) is an anti L-fuzzy subhemiring of a hemiring \( R \).

**2.2 Theorem:** The union of a family of anti L-fuzzy subhemirings of hemiring \( R \) is an anti L-fuzzy subhemiring of \( R \).

**Proof:** It is trivial.

**2.3 Theorem:** If \( A \) and \( B \) are any two anti L-fuzzy subhemirings of the hemirings \( R_1 \) and \( R_2 \) respectively, then anti-product \( AB \) is an anti L-fuzzy subhemiring of \( R_1 \times R_2 \).

**Proof:** Let \( A \) and \( B \) be two anti L-fuzzy subhemirings of the hemirings \( R_1 \) and \( R_2 \) respectively. Let \( x_1 \) and \( x_2 \) be in \( R_1 \), \( y_1 \) and \( y_2 \) be in \( R_2 \). Then \((x_1, y_1) \) and \((x_2, y_2) \) are in \( R_1 \times R_2 \). Now, \( \mu_{AB}[ (x_1, y_1) + (x_2, y_2) ] \leq \{ \mu_A(x_1) \lor \mu_A(x_2) \} \lor \{ \mu_B(y_1) \lor \mu_B(y_2) \} = \mu_{AB}(x_1, y_1) \lor \mu_{AB}(x_2, y_2) \). Therefore, \( \mu_{AB}[ (x_1, y_1) + (x_2, y_2) ] \leq \mu_{AB}(x_1, y_1) \lor \mu_{AB}(x_2, y_2) \).

Therefore, \( \mu_{AB}[ (x_1, y_1)(x_2, y_2) ] \leq \mu_{AB}(x_1, y_1) \lor \mu_{AB}(x_2, y_2) \). Hence \( AB \) is an anti L-fuzzy subhemiring of hemiring of \( R_1 \times R_2 \).

**2.4 Theorem:** Let \( A \) be a L-fuzzy subset of a hemiring \( R \) and \( V \) be the anti-strongest L-fuzzy relation of \( R \). Then \( A \) is an anti L-fuzzy subhemiring of \( R \) if and only if \( V \) is an anti L-fuzzy subhemiring of \( R \times R \).

**Proof:** Suppose that \( A \) is an anti L-fuzzy subhemiring of a hemiring \( R \). Then for any \( x=(x_1, x_2) \) and \( y=(y_1, y_2) \) in \( R \times R \), we have \( \mu_V(x+y) = \mu_A(x_1+y_1) \lor \mu_A(x_2+y_2) \leq \{ \mu_A(x_1) \lor \mu_A(y_1) \} \lor \{ \mu_A(x_2) \lor \mu_A(y_2) \} = \mu_V((x_1, x_2) \lor \mu_V(y_1, y_2) = \mu_V(x+y) \). Therefore, \( \mu_V(x+y) \leq \mu_V(x) \lor \mu_V(y) \), for all \( x \) and \( y \) in \( R \times R \). Conversely, suppose that \( V \) is an anti L-fuzzy subhemiring of \( R \times R \), then for any \( x=(x_1, x_2) \) and \( y=(y_1, y_2) \) in \( R \times R \), we have \( \mu_V(x+y) = \mu_A(x_1+y_1) \lor \mu_A(x_2+y_2) = \mu_V(x+y) \leq \mu_V(x) \lor \mu_V(y) \). Therefore, \( \mu_V(x+y) \leq \mu_V(x) \lor \mu_V(y) \), for all \( x \) and \( y \) in \( R \times R \). Hence \( V \) is an anti L-fuzzy subhemiring of \( R \times R \).

**2.5 Theorem:** \( A \) is an anti L-fuzzy subhemiring of a hemiring \(( R, +, . ) \) if and only if \( \mu_A(x+y) \leq \mu_A(x) \lor \mu_A(y) \), \( \mu_A(xy) \leq \mu_A(x) \lor \mu_A(y) \), for all \( x \) and \( y \) in \( R \).

**Proof:** It is trivial.

**2.6 Theorem:** If \( A \) is an anti L-fuzzy subhemiring of a hemiring \(( R, +, . ) \), then \( H = \{ x / x \in R; \mu_A(x) = 0 \} \) is either empty or is a subhemiring of \( R \).

**Proof:** It is trivial.

**2.7 Theorem:** Let \( A \) be an anti L-fuzzy subhemiring of a hemiring \(( R, +, . ) \). If \( \mu_A(x+y) = 1 \), then \( \mu_A(x) = 1 \) or \( \mu_A(y) = 1 \), for all \( x \) and \( y \) in \( R \).

**Proof:** It is trivial.

**2.8 Theorem:** Let \( A \) be an anti L-fuzzy subhemiring of a hemiring \(( R, +, . ) \), then the pseudo anti L-fuzzy coset \((aA)^*\) is an anti L-fuzzy subhemiring of a hemiring \( R \), for every \( a \) in \( R \).

**Proof:** Let \( A \) be an anti L-fuzzy subhemiring of a hemiring \( R \). For every \( x \) and \( y \) in \( R \), we have \( \mu_{(aA)^*}(x+y) \leq \{ \mu_A(x) \lor \mu_A(y) \} \lor \{ \mu_A(x) \lor \mu_A(y) \} = \mu_{(aA)^*}(x+y) \). Therefore, \( (aA)^* \) \( (x+y) \leq \{ \mu_A(x) \lor \mu_A(y) \} \lor \{ \mu_A(x) \lor \mu_A(y) \} = \mu_{(aA)^*}(x+y) \). Therefore, \( \mu_{(aA)^*}(x+y) \leq \{ \mu_A(x) \lor \mu_A(y) \} \lor \{ \mu_A(x) \lor \mu_A(y) \} = \mu_{(aA)^*}(x+y) \). Hence \( (aA)^* \) is an anti L-fuzzy subhemiring of a hemiring \( R \).

**2.9 Theorem:** Let \(( R, +, . ) \) and \(( R', +, . ) \) be any two hemirings. The hemomorphic image of an anti L-fuzzy subhemiring of \( R \) is an anti L-fuzzy subhemiring of \( R' \).

**Proof:** Let \( f : R \rightarrow R' \) be a hemomorphism. Then, \( f(x+y) = f(x) + f(y) \) and \( f(xy) = f(x)f(y) \), for all \( x \) and \( y \) in \( R \). Let \( V = f(A) \), where \( A \) is an anti L-fuzzy subhemiring of \( R \). Now, for \( f(x), f(y) \) in \( R' \), \( \mu_f(x+y) \leq \mu_f(x) \lor \mu_f(y) \), which implies that \( \mu_f(x+y) \leq \mu(f(x)) \lor \mu(f(y)) \). Again, \( \mu_f(x+y) \leq \mu(f(x)) \lor \mu(f(y)) \). Hence \( V \) is an anti L-fuzzy subhemiring of \( R' \).

**2.10 Theorem:** Let \(( R, +, . ) \) and \(( R', +, . ) \) be any two hemirings. The hemomorphic preimage of an anti L-fuzzy subhemiring of \( R' \) is an anti L-fuzzy subhemiring of \( R \).

**Proof:** Let \( V = f(A) \), where \( V \) is an anti L-fuzzy subhemiring of \( R' \). Let \( x \) and \( y \) in \( R \). Then, \( \mu_f(x+y) \leq \mu_f(x) \lor \mu_f(y) \), which implies that \( \mu_f(x+y) \leq \mu(f(x)) \lor \mu(f(y)) \). Again, \( \mu_f(x+y) \leq \mu(f(x)) \lor \mu(f(y)) \). Hence \( A \) is an anti L-fuzzy subhemiring of \( R \).

**2.11 Theorem:** Let \(( R, +, . ) \) and \(( R', +, . ) \) be any two hemirings. The anti-homomorphic image of an anti L-fuzzy subhemiring of \( R \) is an anti L-fuzzy subhemiring of \( R' \).

ISSN: 2250-3021 www.iosrjen.org 1421 | P a g e
Proof: Let f : R → R' be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti L-fuzzy subhemiring of R. Now, for f(x), f(y) in R', μA(xy) ≤ μA(x) μA(y) implies that μA(f(x)f(y)) ≤ μA(f(x)) ∨ μA(f(y)). Hence V is an anti L-fuzzy subhemiring of R'.

2.12 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. The anti-homomorphic preimage of an anti L-fuzzy subhemiring of R' is an anti L-fuzzy subhemiring of R.

Proof: Let V = f(A), where V is an anti L-fuzzy subhemiring of R'. Let x and y in R. Then, μA(xy) = μA(f(x+y)) ≤ μA(f(x)) ∨ μA(f(y)) = μA(x) ∨ μA(y), which implies that μA(x+y) ≤ μA(x) ∨ μA(y). Again, μA(xy) = μA(f(x)f(y)) ≤ μA(f(x)) ∨ μA(f(y)) = μA(x) ∨ μA(y), which implies that μA(f(x)f(y)) ≤ μA(x) ∨ μA(y). Hence A is an anti L-fuzzy subhemiring of R.

In the following Theorem * is the composition operation of functions

2.13 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring H onto R. Then A*f is an anti L-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, (μA*f)(x+y) = μA(f(x)+f(y)) ≤ μA(f(x)) ∨ μA(f(y)) = (μA*f)(x) ∨ (μA*f)(y), which implies that μA*f(x+y) ≤ μA*f(x) ∨ μA*f(y). And, (μA*f)(xy) = μA(f(x)f(y)) ≤ μA(f(x)) ∨ μA(f(y)) = (μA*f)(x) ∨ (μA*f)(y), which implies that μA*f(xy) ≤ μA*f(x) ∨ μA*f(y). Therefore A*f is an anti L-fuzzy subhemiring of a hemiring R.

2.14 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring H onto R. Then A*f is an anti L-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, (μA*f)(x+y) = μA(f(x)+f(y)) ≤ μA(f(x)) ∨ μA(f(y)) = (μA*f)(x) ∨ (μA*f)(y), which implies that μA*f(x+y) ≤ μA*f(x) ∨ μA*f(y). And, (μA*f)(xy) = μA(f(x)f(y)) ≤ μA(f(x)) ∨ μA(f(y)) = (μA*f)(x) ∨ (μA*f)(y), which implies that μA*f(xy) ≤ μA*f(x) ∨ μA*f(y). Therefore A*f is an anti L-fuzzy subhemiring of a hemiring R.

2.15 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A' be a L-fuzzy set in R defined by A'(x) = A(x) + 1 - A(0), for all x in R. Then A is an anti L-fuzzy subhemiring of a hemiring R.

Proof: Let x and y in R. We have, A'(x+y) = A(x+y) + 1 - A(0) ≤ (A(x) ∨ A(y)) + 1 - A(0) = A'(x) ∨ A'(y). Therefore, A'(x+y) ≤ A'(x) ∨ A'(y), for all x, y in R. Similarly, A'(xy) = A(x+y) + 1 - A(0) ≤ (A(x) ∨ A(y)) + 1 - A(0) = A'(x) ∨ A'(y). Therefore, A'(xy) ≤ A'(x) ∨ A'(y), for all x, y in R. Hence A is an anti L-fuzzy subhemiring of a hemiring R.

2.16 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A' be a L-fuzzy set in R defined by A'(x) = A(x) + 1 - A(0), for all x in R. Then there exists 0 in R such that A(0) = 1 if and only if A'(x) = A(x).

Proof: It is trivial.

2.17 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A' be a L-fuzzy set in R defined by A'(x) = A(x) + 1 - A(0), for all x in R. Then there exists x in R such that A'(x) = 1 if and only if x = 0.

Proof: It is trivial.

2.18 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A' be a L-fuzzy set in R defined by A'(x) = A(x) + 1 - A(0), for all x in R. Then (A')* = A*.

Proof: It is trivial.

2.19 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A be a L-fuzzy set in R defined by A(x) = A(0)A(x), for all x in R. Then A is an anti L-fuzzy subhemiring of the hemiring R.

Proof: For any x in R, we have A'(x+y) = A(0)A(x+y) ≤ A(0)A(x) ∨ A(0)A(y) = A'(x) ∨ A'(y), for all x, y in R. Similarly, A'(xy) = A(0)A(x+y) ≤ A(0)(A(x) ∨ A(y)) = A(0)A(x) ∨ A(0)A(y) = A'(x) ∨ A'(y). Hence A is an anti L-fuzzy subhemiring of the hemiring R.

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