Experimental and Analytical Modal Analysis of a Crankshaft

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ABSTRACT: The paper presents the experimental and analytical modal analysis of a crankshaft. The effective material and geometrical properties are measured, and the dynamic behavior is investigated through impact testing. The three-dimensional finite element models are constructed and an analytical modal analysis is then performed to generate natural frequencies and mode shapes in the three-orthogonal directions. The finite element model agrees well with the experimental tests and can serve as a baseline model of the crankshaft.

Keywords: Experimental modal analysis (EMA), finite element analysis (FEA), FFT (Fast Fourier Transformation), crankshaft.

1. Introduction
The experimental modal analysis (EMA) means the extraction of modal parameters (frequencies, damping ratios, and mode shapes) from measurements of dynamic responses (Rao, 2004). Basically, it is carried out according to both input and output measurement data through the frequency response functions (FRFs) in the frequency domain or impulse response functions (IRFs) in the time domain. For mechanical engineering structures, the dynamic responses (output) are the direct records of the sensors that are installed at several locations (Ren, 2004).

The finite element analysis (FEA) is currently a common way to perform an analytical modal analysis of crankshafts. However, some problems always occur when establishing an accurate FEA model of the existing structure. The problem arises not only from the errors resulting from simplified assumptions made in modeling of the complicated structures but also from parameter errors due to structural damage and uncertainties in the material and geometric properties (Ren, 2004).

The FEA is analytical, the EMA is experimental and modes are the common ground between the two. In fact the EMA is still used to validate FEA models, but it is also heavily used for troubleshooting noise and vibration problems in the field. Once an FEA model has been validated, it can be used for a variety of static and dynamic load simulations.

This paper concentrates on both experimental and analytical modal analysis of a crankshaft. Analytical work involved the development of a three-dimensional FE model. A modal analysis was performed to provide frequencies and mode shapes. Results of the FE modal analysis were compared with those obtained from the EMA.

2. Crankshaft Description
The crankshaft is that of a Peugeot 80’s model (Fig. 1). It is made of cast iron.
To construct the geometry of the crankshaft and in order to have precise measurements, we have used the three-dimensional metrology (Fig. 2)

![Fig. 1. Facade view showing the crankshaft](image-url)
Fig. 2. Crankshaft on the three-dimensional metrology device

Fig. 3 shows the dimensions of the crankshaft from the measurements done using the three-dimensional metrology device.

Fig. 3. Dimensions of the crankshaft (mm)
To measure the Young’s modulus of the material of the crankshaft, the ultrasonic method was used (Fig. 4).

![Crankshaft and the ultrasonic device](image)

**Fig. 4.** Crankshaft and the ultrasonic device

A sonic wave is emitted in the material of the crankshaft and it took 5.77 $10^{-6}$ seconds for the wave to traverse 32.6 mm (2x16.3 mm; back and forth). Knowing that the velocity equals the distance divided by the time, it was found that the velocity of propagation of the sonic wave is 5719 m/s. Using this number, given that the material is isotropic and homogenous we have:

$$V_{OL} = \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} \frac{E}{\rho}$$

Then

$$E = \frac{(1 + \nu)(1 - 2\nu)\rho V_{OL}^2}{1 - \nu}$$

Where $\nu$ = Poisson coefficient = 0.31, $E$ = Young’s Modulus; value to be found, $\rho$ = density = 7800 Kg/m$^3$, $V_{OL}$ = velocity of the longitudinal wave = 5719 m/s.

We can find $E = 184.05$ GPa.

3. **Finite Element Modeling**

Now that the geometrical and mechanical properties of the crankshaft are found, we can proceed with the finite element modeling. Three-dimensional linear elastic finite element model has been constructed using Visual Nastran 4D®FEA software. The crankshaft is modeled using solid ten-noded tetrahedral elements (each node has 3 degrees of freedom UX, UY and UZ). (Fig. 5)
Fig. 6 shows the full three-dimensional (3D) view of the finite element model of the crankshaft:

![Finite Element Model of Crankshaft](image)

**Fig. 6.** The finite element model of the crankshaft

The full model has a total of 67,657 tetrahedral solid elements with more than 120,000 nodes. The unit mesh size is 5 mm. The crankshaft is analyzed in free-free position, so rigid body modes are expected in the results. With 6 modes to extract, the results of the modal analysis are shown in Table 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequencies FEA (Hz)</th>
<th>Description of the mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>367.7</td>
<td>First vertical deflection (Bending in xz plane)</td>
</tr>
<tr>
<td>2</td>
<td>496.1</td>
<td>First horizontal deflection (Bending in xy plane)</td>
</tr>
<tr>
<td>3</td>
<td>859.2</td>
<td>Second vertical deflection (Bending in xz plane)</td>
</tr>
<tr>
<td>4</td>
<td>972.6</td>
<td>First Longitudinal (along x axis)</td>
</tr>
<tr>
<td>5</td>
<td>991.2</td>
<td>First twisting mode (around x axis)</td>
</tr>
<tr>
<td>6</td>
<td>1284.0</td>
<td>Second Longitudinal (along x axis)</td>
</tr>
</tbody>
</table>

The mode shapes of the crankshaft are shown in Fig. 7 sorted from the lowest frequency to the highest:

![Mode Shapes](image)

(a) $f_1 = 367.7$ Hz
Fig. 7. Modes shapes of the crankshaft
4. **Experimental Modal Analysis (EMA)**

EMA has grown steadily in popularity since the advent of the digital FFT (Fast Fourier Transformation) spectrum analyzer in the early 1970’s (Schwarz & Richardson). In this paper, we will make FRF measurements with a FFT analyzer, modal excitation techniques, and modal parameter estimation from a set of FRFs (curve fitting). Experimental modal parameters (frequency, damping, and mode shape) are also obtained from a set of FRF measurements. The FRF describes the input-output relationship between two points on a structure as a function of frequency. Since both force and motion are vector quantities, they have directions associated with them. Therefore, an FRF is actually defined between a single input DOF (point & direction), and a single output DOF. FRF is defined as the ratio of the Fourier transform of an output response \(X(\omega)\) divided by the Fourier transform of the input force \(F(\omega)\) that caused the output (See Fig. 8). An FRF is a complex valued function of frequency. Actually FRF measurements are computed in a FFT analyzer.

\[
\begin{align*}
\text{Mechanical System} & \quad \text{F(t)} \quad \text{X(t)} \\
\text{Time:} & \quad \text{Mechanical System} \\
\text{Frequency:} & \quad \text{H(\omega)} \quad X(\omega) = H(\omega) \times F(\omega)
\end{align*}
\]

**Fig. 8.** Time and Frequency Domain

5. **Exciting Modes with Impact Testing**

Impact testing is a fast, convenient, and low cost way of finding the modes of machines and structures. All the tests were performed at the University of PAUL SABATIER, Toulouse, in the Mechanical engineering LAB, at LGMT - CRITT.

The following equipment is required to perform an impact test:
1. An **impact hammer** with a load cell attached to its head to measure the input force (Fig. 9).
2. An **accelerometer** to measure the response acceleration at a fixed point & direction (Fig. 9).
3. A 2 channel **FFT analyzer** to compute FRFs.
4. **Post-processing modal software** for identifying modal parameters and displaying the mode shapes in animation.

**Fig. 9.** The accelerometer to the left, the impact hammer to the right
The whole process of the impact testing is depicted in Fig. 10.

In general a wide variety of structures and machines can be impact tested. Of course, different sized hammers are required to provide the appropriate impact force, depending on the size of the structure; small hammers for small structures, large hammers for large structures.

6.  Roving Hammer Test
A roving hammer test is the most common type of impact test. In this test, the accelerometer is fixed at a single DOF, and the structure is impacted at as many DOFs as desired to define the mode shapes of the structure. Using a 2-channel FFT analyzer, FRFs are computed one at a time, between each impact DOF and the fixed response DOF.

7.  Testing the reliability of the EMA
Before applying the EMA, its reliability was tested on four steel bars, two of them with circular section and the other ones with rectangular section. For such simple bars the natural frequencies are known analytically.

Again the bars are suspended on elastic cables as if they are in free-free position.

The analytical formula of the frequency of the lateral vibration for a free-free beam is given by:

$$ f (Hz) = \frac{\lambda^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} $$

Where: $E$ = Young’s Modulus; $I$ = inertia of the bar, $\rho$ = density = 7800 Kg/m$^3$, $A$ = cross-sectional area of the bar, $L$ = length of the bar and the values of $\lambda$ are given in Table. 2.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.730</td>
</tr>
<tr>
<td>2</td>
<td>7.853</td>
</tr>
<tr>
<td>3</td>
<td>10.995</td>
</tr>
<tr>
<td>4</td>
<td>14.137</td>
</tr>
<tr>
<td>5</td>
<td>17.278</td>
</tr>
<tr>
<td>6</td>
<td>20.420</td>
</tr>
</tbody>
</table>

When comparing the frequencies of the EMA to the frequencies of the analytical solution, we have found an average difference of 1.5% (See Appendix Table. 5).

Now that the theoretical values are close to the experimental ones, hence the EMA is quite reliable, thus we can move for the experiment on the crankshaft.

In this experiment, the crankshaft is suspended on elastic cables (Fig. 11), so that rigid body modes have very small frequencies compared to those of the deformation modes.
Fixing the accelerometer at a single DOF, the crankshaft was impacted at many DOF to excite all modes (see Appendix, Fig. 13 for the position and directions of all DOF, and Appendix, Table. 6 for the coordinates of all points). After every impact the measurements were taken and saved. The software used is LMS® (Leuven Measurement System).

From the measured FRFs, the software evaluates natural frequencies and mode shapes as well as damping ratios, but the latter are not shown. Table. 3 lists the identified frequencies from the EMA using LMS software.

**Table. 3 Calculated frequencies from the EMA**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequencies EMA (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>350.7</td>
</tr>
<tr>
<td>2</td>
<td>481.8</td>
</tr>
<tr>
<td>3</td>
<td>799.6</td>
</tr>
<tr>
<td>4</td>
<td>874.5</td>
</tr>
<tr>
<td>5</td>
<td>965.3</td>
</tr>
<tr>
<td>6</td>
<td>1127.8</td>
</tr>
</tbody>
</table>

Animation of different modes is also available (see Fig. 12)

**Fig. 11. Crankshaft suspended on elastic cables**

**Fig. 12. First mode of vibration using LMS software**

8. **Results and Comparison**

The FE analytical modal analysis was validated by EMA in terms of natural frequencies and mode shapes. Theoretically, a perfect model would match all experimentally determined mode shapes and frequencies exactly. In practice, it is not expected to be a perfect match between all analytical and measured modal properties. Therefore, only the most structurally significant modes and frequencies are used in the comparison process.
Table 4 summarizes the frequencies of both methods, EMA and FEA. \( \Delta \) is the relative difference between the frequencies of both methods for the given mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequencies (Hz)</th>
<th>Frequencies (Hz)</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EMA (Hz)</td>
<td>FEA (Hz)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>350.7</td>
<td>367.7</td>
<td>4.62%</td>
</tr>
<tr>
<td>2</td>
<td>481.8</td>
<td>496.1</td>
<td>2.88%</td>
</tr>
<tr>
<td>3</td>
<td>799.6</td>
<td>859.2</td>
<td>6.94%</td>
</tr>
<tr>
<td>4</td>
<td>874.5</td>
<td>972.6</td>
<td>10.09%</td>
</tr>
<tr>
<td>5</td>
<td>965.3</td>
<td>991.2</td>
<td>2.61%</td>
</tr>
<tr>
<td>6</td>
<td>1127.8</td>
<td>1284.0</td>
<td>12.17%</td>
</tr>
</tbody>
</table>

9. Conclusions
The analytical modal analysis with 3D finite element models of the crankshaft is compared with the EMA. The results from finite element model agree well with the experimental values. This model is suitable for the dynamic analysis of the crankshaft. The validated finite element model can be used for further dynamic analysis and evaluation of structural performance from loadings.

10. Appendix
In this appendix:
- Table 5 lists the frequencies (Hz) of the four bars from both methods the experimental and the analytical.
- Fig. 13 shows the impact points (1 to 37 DOF) on the crankshaft.
- Fig. 14 shows a 3D view of the position of the impacted points.
- Table 6 lists the coordinates of all impacted points on the crankshaft. O (0, 0, 0) is the origin of the axes, A (12, 12, 121) is where the accelerometer is attached, point 1 to point 21 are the impacted points and point 22 to point 37 are complementary points used for the visualization of the deformed shapes.

Table 5. Analytical and Experimental frequencies (Hz) of the four test bars

<table>
<thead>
<tr>
<th>Mode</th>
<th>Circular Section L = 200 mm</th>
<th>Circular Section L = 1000 mm</th>
<th>Rectangular Section L = 200 mm</th>
<th>Rectangular Section L = 1000 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ana</td>
<td>EMA</td>
<td>( \Delta )</td>
<td>Ana</td>
</tr>
<tr>
<td>1</td>
<td>1708</td>
<td>1790</td>
<td>4.6%</td>
<td>68</td>
</tr>
<tr>
<td>2</td>
<td>4750</td>
<td>4960</td>
<td>4.2%</td>
<td>190</td>
</tr>
<tr>
<td>3</td>
<td>9336</td>
<td>9595</td>
<td>2.7%</td>
<td>373</td>
</tr>
<tr>
<td>4</td>
<td>615</td>
<td>621</td>
<td>1.0%</td>
<td>7189</td>
</tr>
<tr>
<td>5</td>
<td>7316</td>
<td>7210</td>
<td>-1.5%</td>
<td>293</td>
</tr>
<tr>
<td>6</td>
<td>11846</td>
<td>11606</td>
<td>-2.1%</td>
<td>474</td>
</tr>
<tr>
<td>7</td>
<td>575</td>
<td>572</td>
<td>-0.5%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>948</td>
<td>941</td>
<td>-0.7%</td>
<td></td>
</tr>
</tbody>
</table>

- Ana = Analytical, i.e. from the formula of natural frequency (transverse vibration) of a beam in free-free position.
- \( \Delta \) is the relative difference between the frequencies of both methods for the given mode.
Fig. 13. Impact points (1 to 37 DOF) on the crankshaft

Fig. 14. A 3D view of the position of some impacted points (here only points 2, 3 and 4 are shown)

Table. 6. Coordinates of all impacted points on the crankshaft

<table>
<thead>
<tr>
<th>Node Number</th>
<th>X (mm)</th>
<th>Y (mm)</th>
<th>Z (mm)</th>
<th>Impact Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>X, Y, Z</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>0</td>
<td>-70</td>
<td>X, Z</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td>70</td>
<td>-35</td>
<td>X, Y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
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<td>---</td>
<td>---</td>
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<td>4</td>
<td>48</td>
<td>-70</td>
<td>-35</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>68</td>
<td>0</td>
<td>45</td>
<td>X, Y, Z</td>
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<td>6</td>
<td>108</td>
<td>0</td>
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<td>Y, Z</td>
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<td>148</td>
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<td>X, Y, Z</td>
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<tr>
<td>8</td>
<td>172</td>
<td>0</td>
<td>70</td>
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<tr>
<td>9</td>
<td>172</td>
<td>70</td>
<td>35</td>
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<td>12</td>
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<td>*</td>
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<td>342</td>
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<td>45</td>
<td>*</td>
</tr>
</tbody>
</table>

* The corresponding direction (DOF) is interpolated from adjacent directions

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