Some Stronger Forms of \( g^\mu b \) –continuous Functions

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Abstract:
The purpose of this paper is to introduce new classes of functions called strongly \( g^\mu b \) –closed map, strongly \( g^\mu b \) –continuous, perfectly \( g^\mu b \)-continuous and strongly \( g^\mu b \) –irresolute functions in supra topological spaces. Some properties and several characterizations of these types of functions are obtained. Also we investigate the relationship between these classes of functions.

1. Introduction


In this paper we introduce and investigate notions of new classes of functions namely strongly \( g^\mu \)–closed, strongly \( g^\mu b \)–closed, strongly \( g^\mu \)-continuous, strongly \( g^\mu b \)-continuous strongly \( g^\mu \) –irresolute, strongly \( g^\mu b \) – irresolute, almost \( g^\mu \)–irresolute and almost \( g^\mu b \) – irre solute functions in supra topological spaces. Relations between these types of functions and other classes of functions are obtained. We also note that the class of \( g^\mu b \)–closed map is properly placed between strongly \( g^\mu b \)–closed map and almost \( g^\mu b \)–closed map.

2. Preliminaries

Definition: 2.1 [9]
A subclass \( \tau^* \subset P(X) \) is called a supra topology on \( X \) if \( X \in \tau^* \) and \( \tau^* \) is closed under arbitrary union. \((X, \tau^*)\) is called a supra topological space (or supra space). The members of \( \tau^* \) are called supra open sets.

Definition: 2.2 [9]
The supra closure of a set \( A \) is defined as \( Cl^\mu(A) = \cap \{B; B \ is \ supra \ closed \ and \ A \subseteq B\} \)
The supra interior of a set \( A \) is defined as \( Int^\mu(A) = \cup \{B; B \ is \ supra \ open \ and \ A \supseteq B\} \)

Definition 2.3 [12]
Let \((X, \mu)\) be a supra topological space. A set \( A \) is called a supra \( b \)–open set if
\( A \subseteq Cl^\mu(\int^\mu(A)) \cup \int^\mu(Cl^\mu(A)) \). The complement of a supra \( b \)–open set is called a supra \( b \)–closed set.

Definition: 2.4 [3]
Let \((X, \mu)\) be a supra topological space. A set \( A \) of \( X \) is called supra generalized - closed set (simply \( g^\mu \) - closed) if \( cl^\mu(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is supra open. The complement of supra generalized - closed set is supra generalized - open set.

Definition: 2.5 [3]
Let \((X, \mu)\) be a supra topological space. A set \( A \) of \( X \) is called supra generalized \( b \) - closed set (simply \( g^\mu b \) - closed) if \( bcl^\mu(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is supra open. The complement of supra generalized \( b \) - closed set is supra generalized \( b \) - open set.

Definition: 2.6 [14]
A function \( f:(X, \tau) \to (Y, \sigma) \) is said to be \( g^\mu b \) –continuous if \( f^{-1}(V) \) is \( g^\mu b \) – closed in \((X, \tau)\) for every supra closed set \( V \) of \((Y, \sigma)\).

Definition: 2.7 [14]
A function \( f:(X, \tau) \to (Y, \sigma) \) is said to be \( g^\mu b \) – irresolute if \( f^{-1}(V) \) is \( g^\mu b \) – closed in \((X, \tau)\) for every \( g^\mu b \) - closed set \( V \) of \((Y, \sigma)\).
Definition : 2.8 [16]
A supra topological space (X, µ) is said to be supra T
\( g \)-space if for every \( g^b \)-closed set is \( b^\circ \) - closed.

Definition : 2.9 [16]
A supra topological space (X, µ) is said to be supra T
\( g \)-space if for every \( g^b \)-closed set is \( g^\mu \) - closed.

Definition: 2.10 [13]
A function \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called Perfectly\( \mu \) continuous if \( f^{-1}(V) \) is \( c\mu \) open\( \mu \) in X for each supra open set V of Y.

Definition: 2.11 [15]
A subset A of (X, µ) is said to be supra regular open if \( A = \text{Int}^\mu(\text{Cl}^\mu(A)) \) and supra regular closed if \( A = \text{Cl}^\mu(\text{Int}^\mu(A)) \).

Definition : 2.12 [14]
A map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is said to be \( g^\mu b \) -closed map if for every supra closed F of X, \( f(F) \) is \( g^\mu b \) -closed in Y.

3. Strongly \( g^\mu \) b-closed map

Definition: 3.1
A map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is said to be strongly \( g^\mu \) closed map if for every \( g^\mu \) closed F of X, \( f(F) \) is \( g^\mu \) - closed in Y.

Definition: 3.2
A map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is said to be strongly \( g^\mu b \) closed map if for every \( g^\mu b \) closed F of X, \( f(F) \) is \( g^\mu b \) - closed in Y.

Theorem: 3.3
(i) If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is \( g^\mu b \) closed map and \( g: (Y, \sigma) \rightarrow (Z, \gamma) \) is strongly \( g^\mu b \) closed map then \( g \circ f: (X, \tau) \rightarrow (Z, \gamma) \) is \( g^\mu b \) closed map.

(ii) If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is supra-closed map and \( g: (Y, \sigma) \rightarrow (Z, \gamma) \) is strongly \( g^\mu b \) closed map then \( g \circ f: (X, \tau) \rightarrow (Z, \gamma) \) is \( g^\mu b \) closed map.

Proof: It is obvious.

Remark: 3.4
If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is strongly \( g^\mu b \) closed map and \( g: (Y, \sigma) \rightarrow (Z, \gamma) \) is supra closed map then the composite map \( g \circ f \) may not be strongly \( g^\mu b \) closed map and it is shown by the following example.

Example: 3.5
Let \( X = \{a, b, c\} \), \( \tau = \{\phi, X, \{a\}\} = \sigma \) \( \eta = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\} \). Let \( f: (X, \tau) \rightarrow (X, \sigma) \) and \( g: (X, \sigma) \rightarrow (X, \eta) \) be an identity map. Then \( f \) is strongly \( g^\mu b \) closed map and \( g \) is supra closed map but \( (g \circ f)(b) = \{b\} \) is not \( g^\mu b \) closed in \( (X, \eta) \). Therefore \( g \circ f \) is not strongly \( g^\mu b \) closed map.

Definition: 3.6
A map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is said to be almost \( g^\mu b \) closed map if for every \( \text{regular}^\mu \) closed F of X, \( f(F) \) is \( g^\mu b \) closed in Y.

Theorem: 3.7
(i) Every strongly \( g^\mu b \) closed map is almost \( g^\mu b \) closed map.
(ii) Every strongly \( g^\mu b \) closed map is \( g^\mu b \) closed map.
(iii) Every $g^\mu$-b-closed map is almost $g^\mu$-b-closed map.

Proof: It is obvious.

Remark: 3.8
The converse of the above theorem is not true and it is shown by the following examples.

Example: 3.9
Let $X=\{a,b,c,d\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$. Let $f : (X, \tau) \rightarrow (X, \tau)$ be defined by $f(a) = c; f(b) = a; f(c) = d; f(d) = b$. Here $f$ is almost $g^\mu$-b-closed but not strongly $g^\mu$-b-closed map.

Example: 3.10
Let $X=\{a,b,c,d\}, \tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$. Let $f : (X, \tau) \rightarrow (X, \tau)$ be defined by $f(a) = b; f(b) = a; f(c) = d; f(d) = c$. Here $f$ is almost $g^\mu$-b-closed but $f(b,d) = \{a,c\}$ is not $g^\mu$-b-closed. Therefore $f$ is not $g^\mu$-b-closed map.

Theorem: 3.11
If $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost $g^\mu$-b-closed map and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is strongly $g^\mu$-b-closed map then $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is almost $g^\mu$-b-closed map.

Proof: It is obvious.

Theorem: 3.12
The composite mapping of two strongly $g^\mu$-b-closed map is strongly $g^\mu$-b-closed map.

From the above theorem and example we have the following diagram

Strongly $g^\mu$-b-closed map $\rightarrow$ almost $g^\mu$-b-closed map $\rightarrow$ $g^\mu$-b-closed map

4. Strongly $g^\mu$ b-continuous and perfectly $g^\mu$ b-continuous maps

Definition: 4.1
A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $g^\mu$-continuous if the inverse image of every $g^\mu$-open set of $Y$ is supra open in $(X, \tau)$.

Definition: 4.2
A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $g^\mu$ b-continuous if the inverse image of every $g^\mu$ b-open set of $Y$ is supra open in $(X, \tau)$.

Theorem: 4.3
(i) Every strongly $g^\mu$ b-continuous function is supra-continuous.

(ii) Every strongly $g^\mu$ b-continuous function is strongly $g^\mu$-continuous

The converse of the above theorem is not true and it is shown by the following example.

Example: 4.4
Let $X = \{a, b, c\}; \tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$. Define $f : (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = a, f(b) = c$ and $f(c) = b$. Then $f$ is supra continuous but $f^{-1}\{a\} = \{a\}$ is not supra closed. Therefore $f$ is not strongly $g^\mu$ b-continuous.

Theorem: 4.5
If $f : (X, \tau) \rightarrow (Y, \sigma)$ is strongly $g^\mu$ b-continuous and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is $g^\mu$ b-continuous then $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is supra continuous.

Definition: 4.6
A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu$ b-continuous if the inverse image of every $g^\mu$ b-open set of $Y$ is $cl^\mu$ open$^\mu$ in $(X, \tau)$.

Definition: 4.7
A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu b$-continuous if the inverse image of every $g^\mu b$-open set of $Y$ is $cl^\mu open$ in $(X, \tau)$.

Theorem: 4.8

(i) If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu b$-continuous function then $f$ is perfectly $\mu$-continuous.

(ii) If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu b$-continuous function then $f$ is strongly $g^\mu b$-continuous.

(iii) If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu -continuous function then $f$ is perfectly $\mu$-continuous.

(iv) If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu -continuous function then $f$ is strongly $g^\mu -continuous function.

(v) If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $g^\mu b$-continuous function then $f$ is perfectly $g^\mu b$-continuous.

Example: 4.9

Let $X = \{a, b, c\}; \tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\emptyset, X, \{a\}\}$. Define $f: (X, \tau) \rightarrow (X, \sigma)$ be an identity map. Then $f$ is perfectly $\mu$-continuous but it is not perfectly $g^\mu -continuous and perfectly $g^\mu b$-continuous. 

Theorem: 4.10

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is perfectly $\mu$-continuous and if $Y$ is both $T^{\mu}_{\frac{1}{2}} - space but not $T^{\mu}_{\frac{1}{2}} - space, then $f$ is perfectly $g^\mu b$-continuous.

From the above theorem and examples we have the following implications:

From the above theorem and examples we have the following implications:

Here the numbers 1-5 represent the following:
1. Supra continuous  2. strongly $g^\mu b$-continuous 3. strongly $g^\mu -continuous
4. Perfectly $g^\mu b$-continuous  5. Perfectly $g^\mu -continuous  6. Perfectly $\mu$-continuous

5. Strongly $g^\mu b$ -irresolute and Almost $g^\mu b$-irresolute Functions

Definition: 5.1

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $g^\mu -irresolute if $f^{-1}(V)$ is supra open in $(X, \tau)$ for every $g^\mu -open set V of $(Y, \sigma)$.

Definition: 5.2

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $g^\mu b$-irresolute if $f^{-1}(V)$ is supra open in $(X, \tau)$ for every $g^\mu b$-open set V of $(Y, \sigma)$.

Theorem: 5.3

(i) Every strongly $g^\mu b$-irresolute function is $g^\mu b$-irresolute.
Let $V$ be $g^s$-open in $(Y, \sigma)$. Since $f$ is strongly $g^s$-irresolute, $f^{-1}(V)$ is supra open in $(X, \tau)$ and hence it is $g^s$-open in $(X, \tau)$. Therefore $f$ is $g^s$-irresolute.

**Remark:** 5.4

The converses of the above theorems are not true and it is shown by the following examples.

**Example 5.5**

(i) Let $X = \{a, b, c\}$; $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$. Define $f: (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then $f$ is $g^s$-b-irresolute. But $f^{-1}\{a\} = \{b\}$ is not supra closed. Therefore $f$ is not strongly $g^s$-b-irresolute.

(ii) Let $X = \{a, b, c, d\}$; $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$ and $\sigma = \{\emptyset, X, \{a\}\}$. Define $f: (X, \tau) \rightarrow (X, \sigma)$ be an identity function. Here $f$ is $g^s$-continuous. But $f^{-1}\{a, c\} = \{a, c\}$ is not $g^s$-b-closed. Therefore $f$ is not strongly $g^s$-b-irresolute.

**Definition 5.6**

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $g^s$-irresolute if $f^{-1}(V)$ is supra open in $(X, \tau)$ for every $g^s$-open set $V$ of $(Y, \sigma)$.

**Theorem 5.7**

(i) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly $g^s$-irresolute and $Y$ is $T_{\emptyset, \emptyset}$-space then $f$ is strongly $g^s$-b-irresolute.

(ii) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly $g^s$-irresolute and $Y$ is $T_{g^s, g^s}$-space then $f$ is strongly $g^s$-b-irresolute.

**Proof:** It is obvious.

**Theorem 5.8**

(i) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly $g^s$-b-irresolute function then $f$ is strongly $g^s$-irresolute.

(ii) If $f: (X, \tau) \rightarrow (Y, \sigma)$ is strongly $g^s$-b-irresolute function then it is strongly $g^s$-irresolute.

**Proof:** It is obvious.

**Remark 5.9**

The converse of the above theorem is not true and it is shown by the following example.

**Example 5.10**

Let $X = \{a, b, c\}$; $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$ and $\sigma = \{\emptyset, X, \{a\}\}$. Define $f: (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then $f$ is strongly $g^s$-irresolute. But $f^{-1}\{c\} = \{c\}$ is not supra open in $(X, \tau)$. Therefore $f$ is not strongly $g^s$-b-irresolute.

**Example 5.11**

Let $X = \{a, b, c\}$; $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$. Let $f: (X, \tau) \rightarrow (X, \tau)$ be an identity function. Here $f$ is strongly $g^s$-irresolute. But $f^{-1}\{a\} = \{a\}$ is not supra closed in $(X, \tau)$. Therefore $f$ is not strongly $g^s$-b-irresolute.

**Theorem 5.12**

For a function $f: (X, \tau) \rightarrow (Y, \sigma)$ if $f$ is strongly $g^s$-b-irresolute then for each $x \in X$ and each $g^s$-b-open set $V$ of $Y$ containing $f(x)$ there exist a supra open set $U$ of $X$ containing $x$ such that $f(U) \subset V$.

**Proof:** Let $x \in X$ and $V$ be any $g^s$-b-open set $V$ of $Y$ containing $f(x)$. Since $f$ is strongly $g^s$-b-irresolute then $f^{-1}(V)$ is supra open in $X$ and contains $x$. Let $U = f^{-1}(V)$ then $U$ is supra open subset of $X$ containing $x$ such that $f(U) \subset V$.

**Theorem 5.13**

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \nu)$ be any two functions then the composition $g \circ f: (X, \tau) \rightarrow (Z, \nu)$ is i) strongly $g^s$-b-irresolute if $f$ is strongly $g^s$-b-irresolute and $g$ is $g^s$-b-irresolute.
Theorem: 5
Proof:

Example: 5

Remark: 5

The converse of the above theorem is not true and it is shown by the following examples.

Example: 5.18
Let \( X = \{a, b, c, d\} \) and \( \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\} \). Define \( f: (X, \tau) \to (X, \tau) \) by \( f(a) = b, f(b) = c \). Hence \( f \) is b\(^\mu\)-continuous. But \( f^{-1}(\{a, b, c\}) = \{a, b, c\} \) is not b\(^\mu\)-closed. Therefore \( f \) is not almost g\(^\mu\)-irresolute.

Example: 5.19
Let \( X = \{a, b, c\} \) and \( \sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\} \). Define \( f: (X, \tau) \to (X, \sigma) \) by \( f(a) = b, f(b) = a, f(c) = c \). Hence \( f \) is g\(^\mu\)-irresolute. But \( f^{-1}(\{a, c\}) = \{a, c\} \) is not b\(^\mu\)-closed. Therefore \( f \) is not almost g\(^\mu\)-irresolute.

Theorem: 5.20
Let \( f: (X, \tau) \to (Y, \sigma) \) and \( g: (Y, \sigma) \to (Z, \nu) \) be any two functions then the composition \( g \circ f: (X, \tau) \to (Z, \nu) \) is i) almost g\(^\mu\)-irresolute if \( f \) is almost g\(^\mu\)-irresolute and \( g \) is g\(^\mu\)-irresolute
(ii) almost g\(^\mu\)-irresolute if \( f \) is b\(^\mu\)-irresolute and \( g \) is almost g\(^\mu\)-irresolute.

Proof: It is obvious.

Theorem: 5.21
For a function \( f: (X, \tau) \to (Y, \sigma) \) if \( f \) is almost g\(^\mu\)-irresolute then for each \( x \in X \) and each g\(^\mu\)-b-open set \( V \) of \( Y \) containing \( f(x) \) there exist a supra open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subseteq V \).

Proof: It is obvious.

From the above theorem and examples we have the following diagram:

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1  2  3     4  5  6     7  8  9
|   |   |    |   |   |    |   |   |    |
|   |   |    |   |   |    |   |   |    |
|   |   |    |   |   |    |   |   |    |
|   |   |    |   |   |    |   |   |    |
|   |   |    |   |   |    |   |   |    |
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Here the numbers 1-9 represent the following implication:
1. Strongly g\(^\mu\)-irresolute  2. g\(^\mu\)-b-irresolute  3. g\(^\mu\)-irresolute
4. Strongly $g^\mu$-irresolute
5. Strongly $b^\mu$-irresolute
6. $g^\mu$ $b$-continuous
7. almost $g^\mu$ $b$-irresolute
8. $b^\mu$-continuous
9. almost $g^\mu$ -irresolute

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