Design Of Different Controllers In Quadcopter

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Abstract: Quadcopter is an unmanned aerial vehicle with four propulsors. Design of controller is necessary for the proper and safe movement of quadcopter under any adverse conditions like climatic changes, air bumps etc. For attain stability under these conditions we design different controllers. For design the controllers we choose 3 control variables such as pitch, yaw and angle. As part of derive the equations of mathematical modelling first define and explained the coordinate system. By using this coordinate system the controller parameters are designed. First derive all the parameters by using Newton–Euler's equations. Then by using these we design controllers using different control techniques. Here mainly using PID controller, Adaptive controller, Model Reference Adaptive Controller (MRAC), Self Tuning Regulator (STR) and Kalman filter based controller. The final stage is to analyse and compare the stability of the system using these controllers and choose the most stable controller for quadcopter.

Keywords: Newton-Euler's Equation, PID controller, Adaptive controller, Model Reference Adaptive Controller (MRAC), Self Tuning Regulator (STR), Kalman filter based controller.

I. Introduction

Our project is aimed at developing the different controllers for minimize the accidents in quadcopter due to the climatic changes, air bumps etc. The quadcopter and helicopter is basically more or less same operations. The structural difference is makes them different. We design controller for quadcopter because it is less stable compared to helicopter. This paper describes a detailed modelling of controllers for the stable operation of a quadcopter. The quadcopter stability is analyzed by different controllers using different control techniques. The basic equations for each controller will be same. Mainly we need to derive the basic equations and we need to form the transfer function. By using this transfer function as base, we design all the controllers and checking their stability. Here we mainly use PID controller, Adaptive Controller and MRAC [4]. A quadcopter with four rotors, and each has variable speed, allows the rotors to have a balanced variation in operation in terms of thrust, speed and acceleration in the desired directions. Generally a quadcopters have six degrees-of-freedom (DoF), which means that they can move along the space axes X, Y and Z and the control variables roll (φ), pitch (θ) and yaw (ψ) angles [1].

II. Methodology

Quad copter, also called as the quadrotor is a type of helicopter with four rotors. All the rotors are directed in the upward direction and then forms in the shape of a square with equal distance from the centre of mass of the quadcopter. The main application of the quadcopter are used in surveillance, rescue and search, construction inspection and other applications [2].

Fig.1 shows the simple structure of a quadcopter. As Quad copter is an unmanned aerial vehicle with four propulsors which enables them for the vertical takeoff and landing. It has three degrees of freedom 3 DOF and four control variables which determines the stability of the system. The number of the control variables and the number of propulsors and they affect the position and attitude of the quadcopter in space [3].

<table>
<thead>
<tr>
<th>FACTORS</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>F₁, F₂, F₃, F₄</td>
<td>Forces created by rotors</td>
</tr>
<tr>
<td>ω₁, ω₂, ω₃, ω₄</td>
<td>Angular velocities</td>
</tr>
<tr>
<td>ℰ₁, ℰ₂, ℰ₃, ℰ₄</td>
<td>Torque</td>
</tr>
</tbody>
</table>

Table 1: Parameters of quadcopter
1. Mathematical Modelling

From the quadcopter structure, we get all the parameters required for the designing. That is the force created, angular velocities, torque, and their corresponding directions. The linear position is obtained from coordinate axes X, Y, and Z. The attitude is defined in the coordinate system with three angles. Pitch angle $\theta$ specifies the rotation of quadcopter around the Y axis, $\phi$ is the roll angle represents the quadcopter rotation around the x-axis and $\psi$ is the yaw angle around the z-axis.$^{[1]}$

The origin of the coordinate system is situated at the middle part of the quadcopter. From the system the $V_B$ represents linear velocities and angular velocities by $V$.

$$V_B = \begin{bmatrix} V_x, B \\ V_y, B \\ V_z, B \end{bmatrix}, \nu = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

The $Sx$ and $Cx$ represents the sine and cosine angles.

$$R = \begin{bmatrix} C_\theta C_\phi & C_\phi S_\theta - S_\phi C_\theta & C_\phi S_\theta + S_\phi C_\theta \\ S_\theta & C_\theta & S_\phi \\ -S_\phi C_\theta & S_\phi S_\theta + C_\phi C_\theta & -S_\phi C_\phi \end{bmatrix}$$

By solving these equation we obtain the state space model as,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + T/m \begin{bmatrix} C_\psi C_\phi C_\theta + S_\psi S_\phi \\ S_\phi C_\phi S_\theta - C_\psi S_\phi \\ C_\phi C_\theta \end{bmatrix}$$
1.1 Newton-Euler equations

The Newton-Euler equations are used to find out the basic parameters of the quadcopter. In the system, necessary force required to accelerate the mass $m \dot{V}_B$ and the centrifugal force $v \times (mV_b)$ are equal to the gravity $R^T G$ and the total thrust acting on the rotors $T_B$.

$$m\dot{V}_B + (mV_B) = R^T G + T_B$$  \hspace{1cm} (5)

In the system, the effect of centrifugal force is negligible. Hence the gravitational force and the magnitude and direction of the thrust adds to the acceleration of the quadcopter.

$$m\ddot{V}_B + (mV_B) = R^T G + T_B$$  \hspace{1cm} (6)

$$m\ddot{V} = G + RT_B$$  \hspace{1cm} (7)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m} \begin{bmatrix} C_\phi C_\psi C_\theta + S_\psi S_\phi \\ S_\psi C_\phi S_\theta - C_\psi S_\phi \\ C_\phi C_\psi \end{bmatrix}$$ \hspace{1cm} (8)

In the system, the angular acceleration of the inertia $I\dot{\omega}$, the centripetal forces $v \times (I)$ and the angular forces $T$ are equal to the external torque $\tau$.

$$\dot{\omega} = I^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = I_T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w_r + \lambda$$ \hspace{1cm} (9)

1.2 Euler-Lagrange equations

In this equation we mainly consider the rotational, potential and translational energies. The Lagrangian $L$ can be obtained by adding the rotational and translational energy and subtract it with potential energy.

$$L(q, \dot{q}) = E_{trans} + E_{rot} - E_{pot} = m/2 \dot{v}^T \dot{v} + 1/2 v^T T v - mgz$$ \hspace{1cm} (10)

The Jacobian matrix $J$ from to is,

$$J(q) = J = W^T \eta \frac{\partial \eta}{\partial \eta}$$ \hspace{1cm} (11)

$$= \begin{bmatrix} I_{xx} & 0 & -I_{X X} S_\theta \\ 0 & I_{YY} C_\phi C_\theta & (I_{YY} - I_{ZZ}) C_\phi S_\theta \\ -I_{X X} S_\theta & (I_{YY} - I_{ZZ}) C_\phi S_\theta & I_{X X} S_\theta + 2 I_{YY} C_\phi C_\theta \end{bmatrix}$$

Hence, the rotational energy $E_{rot}$ of the system is given by,

$$E_{rot} = \frac{1}{2} V^T I v = \frac{1}{2} \eta^T J I \eta$$ \hspace{1cm} (12)

From these we can write,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m} \begin{bmatrix} c_\phi c_\theta s_\theta + s_\phi s_\theta \\ s_\phi c_\theta s_\theta - c_\phi s_\theta \\ c_\phi c_\theta \end{bmatrix} - \frac{y}{m} \begin{bmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$ \hspace{1cm} (13)

By solving the equations 1 and 2 we get the transfer function for the controller.

3. Design of PID

In the simulation part we mainly deals with the PID controller and its stability analysis. For that we are using the equation.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{M} \begin{bmatrix} c_\phi c_\theta s_\theta + s_\phi s_\theta \\ s_\phi c_\theta s_\theta - c_\phi s_\theta \\ c_\phi c_\theta \end{bmatrix} - \frac{y}{m} \begin{bmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$ \hspace{1cm} (14)

From this equation we can write,
Design Of Different Controllers In Quadcopter

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = 
\begin{bmatrix}
0 & -g/m & 0 \\
0 & 0 & 0 \\
-g & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0.246 & 0.25 & 0 \\
-0.185 & 0.25 & 0 \\
0.951 & 0.25 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\begin{bmatrix}
T & 0 & 0 \\
0 & M & 0 \\
0 & 0 & m
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}
\]  
(15)

By using this we get the transfer function as,

\[
y_1(s) = \frac{0.525}{s^3 + 0.534s} \\
y_2(s) = \frac{0.395}{s^3 + 0.534s} \\
y_3(s) = \frac{2.032}{s^2 + 0.534s}
\]

Here we got three different transfer functions, from this we can find the controller values using PID equations.

\[
s^4 + 0.534s^3 + 0.524s^2k_d + 0.525sk_p + 0.525k_i \geq 0 \quad (19)
\]

Table 2: RH analysis for first system

<table>
<thead>
<tr>
<th>(s^4)</th>
<th>(s^3)</th>
<th>(s^2)</th>
<th>(s^1)</th>
<th>(s^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.534</td>
<td>0.2798k_d - 0.525k_p</td>
<td>-0.27k_dk_p - 0.516k_p^2 - 0.28k_i</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: RH analysis for second system

\[
s^4 + 0.534s^3 + 0.395s^2k_d + 0.395sk_p + 0.395k_i \geq 0 \quad (20)
\]

Table 4: RH analysis for third system

\[
s^4 + 0.534s^3 + 2.032s^2k_d + 2.032sk_p + 2.032k_i \geq 0 \quad (21)
\]

By using the values in table 5.1, 5.2 and 5.3 we can find the ranges of \(k_p\), \(k_d\) and \(k_i\). So here we take the value of \(k_d = 5\) then we get values of \(k_p = 0.5k_d\) and \(k_i = 0.5k_d\) for the first system. For the second system we get \(k_d = 5\), \(k_p = 0.6k_d\) and for third system, we get the input as stable. So we do not want to design the controller. Here we mainly simulate all the system using Bode plot in MATLAB.

4. Design of Adaptive controller

In adaptive control, the controller will adapt to a system by varying the range of control parameters and thereby obtain the system in a stable mode. From the basic state space model we design the transfer function for adaptive controller. The design is as follows,
Design Of Different Controllers In Quadcopter

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{M} \begin{bmatrix} c_\phi c_\theta + s_\phi s_\theta \\ s_\phi c_\theta - c_\phi s_\theta \end{bmatrix} \left( \begin{array}{c} \frac{A_{xx}}{M} \\ \frac{A_{yy}}{M} \\ \frac{A_{zz}}{M} \end{array} \right) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}
\]

\[S^2 x(s) = ASX(s) + Bu(s)\]  
(23)

\[\left[ S^2 - AS \right] = Bu(s)\]  
(24)

\[S \left[ SI - A \right] X(s) = Bu(s)\]  
(25)

\[\frac{x(s)}{u(s)} = \frac{B}{S} \left[ SI - A \right]^{-1}\]  
(26)

\[SI - A = \begin{bmatrix} s + \frac{A_{xx}}{M} & 0 & 0 \\
0 & s + \frac{A_{yy}}{M} & 0 \\
0 & 0 & s + \frac{A_{zz}}{M} \end{bmatrix}\]  
(27)

\[\left[ SI - A \right]^{-1} = \]
\[
\begin{bmatrix}
(s + \frac{A_{xx}}{M})(s + \frac{A_{yy}}{M}) & 0 & 0 \\
0 & (s + \frac{A_{yy}}{M})(s + \frac{A_{zz}}{M}) & 0 \\
0 & 0 & (s + \frac{A_{yy}}{M})(s + \frac{A_{zz}}{M}) \\
\end{bmatrix}
\]
(28)

Taking inverse,

\[
\frac{B}{S} \left[ SI - A \right]^{-1} = \frac{B}{S} \left[ SI - A \right]^{-1} \frac{T}{SM} \begin{bmatrix} c_\phi c_\theta s_\theta + s_\phi s_\theta \\ s_\phi c_\theta s_\theta - c_\phi s_\theta \\ c_\phi c_\theta \end{bmatrix}
\]
(29)

The transfer function is,

\[
\frac{X_1(s)}{T(s)} = \frac{(s + \frac{A_{yy}}{M})(s + \frac{A_{zz}}{M})}{(s + \frac{A_{yy}}{M})(s + \frac{A_{zz}}{M})(s + \frac{A_{xx}}{M})} \cdot \frac{c_\phi c_\theta s_\theta + s_\phi s_\theta}{SM}
\]
(30)

\[
\frac{X_1(s)}{T(s)} = \frac{c_\phi c_\theta s_\theta + s_\phi s_\theta}{S^3M + A_xS}
\]
(31)

The generalised expression for PID is,

\[
G_c(s) = K_p + \frac{K_i}{S} + K_ds
\]  
(32)

\[
G_c(s) = \frac{SK_p + K_i + K_ds^2}{S}
\]
(33)

\[
1 + G_c(s)G(s) = 0
\]
(34)

From this we can find the PID values as:
From the above equations we need to find the stable ranges of variable parameters, Ranges of φ, θ, ϕ are,

\[ 0 < \phi < 90, \quad 0 < \theta < 90 \quad \text{and} \quad 0 < \phi < 180. \]

Similarly for the second transfer function the ranges are as follows,

\[ K_P = \frac{c_\theta s_\phi c_\theta + c_\phi s_\phi}{S} \]

\[ K_i = c_\phi s_\phi c_\theta + c_\phi s_\phi \]

\[ K_P = \frac{c_\theta s_\phi c_\theta + c_\phi s_\phi}{S^2} \]

Range of φ, θ, ϕ are

\[ 0 < \phi < 90, \quad 0 < \theta < 90 \quad \text{and} \quad 0 < \phi < 180. \]

Similarly for the third transfer function,

\[ \frac{x_3(s)}{T(s)} = \frac{-g c_\phi c_\theta}{S^2 M + A_s S} \]

\[ K_P = \frac{-g c_\phi c_\theta}{S} \]

\[ K_i = -g c_\phi c_\theta \]

\[ K_d = \frac{-g c_\phi c_\theta}{S^2} \]

5. Design of MRAC

MRAC is the model reference adaptive controller is another type of adaptive controller that works on the principle of adjusting the control parameters. The main parameters included in MRAC are Reference model, controller, adjustment mechanism. The adjustment mechanism is developed by using MIT rule, Lyapunov theory and theory of augmented error. The design is done by using the actual and reference system. By compare the output of the two system we check the stability. The actual system is,

\[ \frac{x_1(s)}{y_1(s)} = \frac{0.525}{s^3 + 0.534s} \]

From the design of PID we get the \( K_p \), \( K_i \) and \( K_d \) values as, \( k_p = 0.5k_d \), \( K_d = 5 \), \( K_i = 0.5 \).

Reference system is,
Design Of Different Controllers In Quadcopter

\[
\frac{x_1(s)}{T(s)} = \frac{0.94}{0.468s^2 + 0.25s}
\]  

(47)

Then \( k_p = \frac{0.941}{s} \), \( K_d = \frac{0.941}{s^2} \). \( K_i = 0.941 \). Similarly for the second system actual transfer function is,

\[
\frac{x_2(s)}{y_2(s)} = \frac{-0.395}{s^3 + 0.534s}
\]  

(48)

\( k_p = 0.6k_d \), \( K_d = 5 \), \( K_i = 80.45 \). For the reference system the transfer function is,

\[
\frac{x_2(s)}{T(s)} = \frac{0.966}{0.468s^2 + 0.25s}
\]  

(49)

Then \( k_p = \frac{0.966}{s} \), \( K_d = \frac{0.966}{s^2} \). \( K_i = 0.966 \).

For the third system the transfer function is,

III. Simulation And Results

The typical simulation result of PID, Adaptive controller and MRAC are shown below. The simulations are done in MATLAB. Fig 4 shows the bode plot of the first system of a PID controller with 2 cases. In the first case we get an unstable system with PID values as: \( K_p = 0.5K_d \), \( K_d = 0.1 \), \( K_i = 0.2 \). For second case we get a stable system with the range of values of PID as \( K_p = 0.5K_d \), \( K_d = 5 \), \( K_i = 0.5 \).

The fig 5 shows the simulation for the second system with two different PID values, one represent the unstable system and other is the stable system. The range of PID as follows, \( K_p = 0.1K_d \), \( K_d = 0.1 \), \( K_i = 8.94 \) for unstable system and \( K_p = 0.6K_d \), \( K_d = 5 \), \( K_i = 80.45 \) for stable system.

Fig 6 shows the simulation result of the third system and which is already a stable system because the poles in the transfer function is lies on the left half of s plane.

![Fig. 5. Bode plot of system 2, case(a): unstable system case(b): stable system](image-url)
The simulations of an adaptive controller also gives the stability range of each system. Here we shows the stable and unstable range of variable parameters of each system. For the first system we have 2 different cases. For the first case we have an unstable system with Fig 7 depicts the unstable system. $\phi = 120, \phi = 110, \theta = 200, m = 0.468, A_x = 0.25$. The transfer function is,

$$\frac{x_1(s)}{T(s)} = \frac{1.1}{0.468s^2 + 0.25s} \quad (50)$$

Then $k_p = \frac{11}{2}$, $K_d = \frac{13}{2}$, $K_i = 1.1$.
For the adaptive controller in the design part we find the ranges of PID controller. By putting values for the constants among these ranges we get a stable output as a result from simulation in system 1. Fig 8 gives the stable region in system 1.

$\phi = 50, \psi = 60, \theta = 120, m = 0.468, A_x = 0.25$ The transfer function is,

$$\frac{x_1(s)}{T(s)} = \frac{0.941}{0.468s^2 + 0.25s}$$  \hspace{1cm} (51)

Then $k_p = \frac{0.941}{s}, K_d = \frac{0.941}{s^2}, K_i = 0.941$

For stable range in system 2, Fig 10 shows the stable system for the second transfer function.
φ = 60, ϕ = 30, \( r = 120 \), \( m = 0.468 \), \( A_x = 0.25 \). The transfer function is,

\[
\frac{x_1(s)}{T(s)} = \frac{0.966}{0.468s^2 + 0.25s} \quad (53)
\]

Then \( k_P = \frac{0.966}{s} \), \( K_d = \frac{0.966}{s^2} \), \( K_i = 0.966 \).

For the third system we can find it as stable. That is the poles on the transfer function is,

\[
\frac{x_1(s)}{T(s)} = \frac{2.90}{0.468 + 0.25s} \quad (54)
\]

In the case of MRAC, we check the stability for the three systems. For the first system we have the actual and reference systems. Using Simulink model we find the stability of the system.

Here fig. 11 represents the system with stable and unstable outputs for system 1. Here we combine both the PID as well as adaptive for each system.

![Bode plot of system 1](image)

**Fig 11**: Bode plot of system 1 - a: unstable system  b: Stable System

Fig 12 shows the plot of second system with unstable region having the transfer functions of the PID as well as the adaptive controller. Same for the stable region.
In the third system the transfer function is a stable function. So we get a stable output.

![Bode Diagram](image)

**Fig 13:** Bode plot of system 3.

**IV. Conclusion**

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SYSTEM1</th>
<th>SYSTEM2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain margin</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>Phase margin</td>
<td>0.142</td>
<td>180</td>
</tr>
<tr>
<td>Gain crossover frequency</td>
<td>2.29</td>
<td>0.731</td>
</tr>
<tr>
<td>Phase crossover frequency</td>
<td>Inf</td>
<td>Inf</td>
</tr>
</tbody>
</table>

By comparing the values of phase margin and gain margin we can analyze the stability of the system. In the bode plot for a system to be stable, the gain margin and phase margin should be positive values and which should be a greater value. By comparing the simulation results of the PID and the Adaptive we can find

<table>
<thead>
<tr>
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<th>SYSTEM1</th>
<th>SYSTEM2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain margin</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>Phase margin</td>
<td>123</td>
<td>78.8</td>
</tr>
<tr>
<td>Gain crossover frequency</td>
<td>0.956</td>
<td>1.34</td>
</tr>
<tr>
<td>Phase crossover frequency</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In the third system the transfer function is a stable function. So we get a stable output.
that the phase and gain margin values of adaptive PID is greater than the corresponding PID values. Hence we can conclude that by comparing PID and adaptive, the adaptive controller is the most efficient controller for the stable operation of a quad copter.

References