Algorithmic Way of Teaching Numerical Methods for Ordinary Differential Equations to Engineering students

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Abstract: In this paper we present algorithmic method to teach Numerical Methods for Ordinary Differential Equations to Engineering students. We compare our tabular method of solving examples of Euler’s Modified method with existing iterative way of solving for the time taken to solve the example, comfort level of students with this way of solving.

I. Introduction

Teaching and learning process for Applied Mathematics is challenging in the sense that engineering students always wonder why they are learning mathematics, so as a teacher we must know the applications of the various methods we teach to the students and explain how these methods are used in real life applications. We must emphasize on when there is a need for learning particular mathematical topic or a concept and how it is going to help students in specific real-life problems. This approach of teaching takes the students near the subject and learning becomes fun. They start owning the subject and when that happens efficient learning becomes inevitable. Many mathematics faculties face challenge while teaching mathematics specially applied mathematics, but we feel that Mathematics itself being a tool for logical thinking and teaches systematic way of solving any problem, why not teach it logically and using algorithmic way. Which makes the understanding of mathematical methods easy and students are able to remember it for a long duration. The reason for introducing this new way of solving the example of Euler’s Modified Method, is that existing method consumes lot of time and pages. Students need to turn the pages to check the earlier answers. Whereas our tabular way of solving the example is very easy to use and all the answers are available in a table for easy access. We feel that teaching the mathematical concepts and various mathematical methods in algorithmic way has lot of benefits since it makes students remember the concepts and way of solving examples in a logical and systematic way. We have introduced this way of teaching mathematics in many topics of First Year and Second Year Engineering mathematics topics and have observed that understanding the topic and remembering the way of solving is much better than existing way of solving. Student’s results in internal test and at the university exam has improved.

II. Literature Review

We tried to find the related papers working on the similar work, but we found that there are no directly related papers for the work we are presenting so we are presenting literature review of papers presenting other methods in teaching mathematics. Diana Audi and Rim Gouia-Zarrad[1] discussed about using iPads in teaching mathematics and what are the impact of this with students understanding. Speer, N., Smith J. and Horvath A.[2] reviewed the difference between the instructional activities and teaching practice for collegiate mathematics teaching. Tezer, M. and Aktunc, E. [3] introduced drama method in teaching mathematics in 2009. The paper presented by Wang, W., Wang, S., Cui, S. and Qu, G [4] analyzed the theoretical and practice of RMI thinking method in mathematics teaching area. The paper evaluating a model based on creative mathematically founded reasoning (CMR) and comparing it with procedural way of teaching was presented by Jonsson, B., Norqvist, M., Liljekvist, Y. and Lithner, J.[5]. Bilen, K. [6] investigated the effect of micro teaching techniques on the teacher’s beliefs about teaching mathematics. Yee, S., Boyle, J., Ko, Y. and Bleiler-Baxter, S. [7] work was on
III. Algorithm and Tabular method for solving examples of Euler’s Modified Method

Euler’s Modified method/ Runge-Kutta 2nd order method

For the Ordinary Differential Equation (O.D.E.) which is of the type

\[ \frac{dy}{dx} = f(x, y), \text{ with the initial values } x_0, y_0, h \text{ and } x \]

We can find the value of \( y \) which is a solution of the above system by the Euler’s Modified Method given by.

\[ y_{n+1}^{r+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_{n+1}^r)] \]

Algorithm for solving examples of Euler’s modified method

**Step 1** Write down the given data, i.e. \( f(x, y), x_0, y_0, h \) and \( x \)

Construct the following table for calculating the values of \( y \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x_n )</th>
<th>( y_n )</th>
<th>( f(x_n, y_n) = A )</th>
<th>( r )</th>
<th>( x_{n+1} )</th>
<th>( y_{n+1} )</th>
<th>( f(x_{n+1}, y_{n+1}) = B )</th>
<th>( y_{n+1}^{r+1} = y_n + \frac{h}{2}[A + B] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( x_0 )</td>
<td>( y_0 )</td>
<td>( f(x_0, y_0) = A )</td>
<td>0</td>
<td>( x_1 )</td>
<td>( y_1 )</td>
<td>( f(x_1, y_1) = B )</td>
<td>( y_1^{r+1} = y_0 + \frac{h}{2}[A + B] )</td>
</tr>
<tr>
<td>1</td>
<td>( x_1 )</td>
<td>( y_1 )</td>
<td>( f(x_1, y_1) = B )</td>
<td>1</td>
<td>( x_2 )</td>
<td>( y_2 )</td>
<td>( f(x_2, y_2) = B )</td>
<td>( y_2^{r+1} = y_1 + \frac{h}{2}[A + B] )</td>
</tr>
<tr>
<td>2</td>
<td>( x_2 )</td>
<td>( y_2 )</td>
<td>( f(x_2, y_2) = B )</td>
<td>2</td>
<td>( x_3 )</td>
<td>( y_3 )</td>
<td>( f(x_3, y_3) = B )</td>
<td>( y_3^{r+1} = y_2 + \frac{h}{2}[A + B] )</td>
</tr>
</tbody>
</table>

**Step 2**

Continue above calculations till we get 2 consecutive values of \( y \) exactly same up to 4 decimals. Then stop incrementing \( r \) and go out of this loop to increase \( n \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x_n )</th>
<th>( y_n )</th>
<th>( f(x_n, y_n) = A )</th>
<th>( r )</th>
<th>( x_{n+1} )</th>
<th>( y_{n+1} )</th>
<th>( f(x_{n+1}, y_{n+1}) = B )</th>
<th>( y_{n+1}^{r+1} = y_n + \frac{h}{2}[A + B] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x_1 )</td>
<td>( y_1 )</td>
<td>( f(x_1, y_1) = A )</td>
<td>0</td>
<td>( x_2 )</td>
<td>( y_2 )</td>
<td>( f(x_2, y_2) = B )</td>
<td>( y_2^{r+1} = y_1 + \frac{h}{2}[A + B] )</td>
</tr>
<tr>
<td>2</td>
<td>( x_2 )</td>
<td>( y_2 )</td>
<td>( f(x_2, y_2) = B )</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 3** The number of iterations depend on the final value of \( x \) at which we want to find \( y \) and \( h \).

**Step 4** Stop the calculations when you get the desired values of \( y \) and given \( x \)

IV. Experiment

We selected 60 first year engineering students where 30 students will be solving the example for Euler’s Modified Method using existing iterative method and remaining 30 students will use our tabular method and then they will exchange the way of solving. This will allow students to solve using both methods and then compare their comfort level with each method and time taken by them to solve the example using both ways. Out of 60 students we selected the samples only for those who solve completely and with all the steps. Table for the time taken by each student for both methods was recorded and is presented in the Table 1 given below.
Then we asked students to give feedback on the same. We received following comments.
1. Tabular way of solving the example is less time consuming.
2. Although it takes time to create the table but after practicing many examples it takes less than one minute to draw the table
3. In Iterative method for solving the number of pages consumed for solving the example was more than tabular method.
4. Turning pages to check the answers of the previous iteration was making them uncomfortable since some times they took wrong value.

V. Conclusion
The algorithmic way of teaching and learning is implemented for many topics for first and second year engineering mathematics. The similar method can be implemented in many other mathematical concepts and topics. Our future work will be in the area of Pure Mathematics topics

References