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Effect of Initial Compression on Love Wave Propagation in a Rotating Orthotropic Elastic Solid half-space with Impedance Boundary Conditions

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Abstract:
The main objectives of this paper is to investigate the effects of rotation and initial stress on Love wave propagation in an orthotropic elastic solid with impedance boundary conditions. Basic equations has been solved to derive the secular equations of Love wave propagation. Under MATLAB software, the frequency curves of Love waves are drawn for a particular solid material.
Key Words: Love waves, Orthotropic Solid, Initial stress, Impedance boundary conditions.

I. Introduction

The study of Love waves at interface and half-space materials are great importance in many engineering fields like Civil, Mechanical, Aerospace, Navel, Chemical and Nuclear Engineering. Propagation of surface waves in a homogeneous and non- homogeneous elastic half-spaces are very important in the study of wave theory. The Mathematical model of a surface waves known as Love waves developed by Love [1]. Propagation of elastic waves in a isotropic, fluid saturated porous solids are discussed by Biot [2-4]. Wave propagation problems in an orthotropic elastic solids are studied by Abd-Alla [5]. El-Naggar et.al. [6] investigated the rotation effect on non- homogeneous orthotropic infinite cylinder. The effect of initial stress, gravity on surface waves in an orthotropic elastic solids are studied by Abd-Alla et.al. [7].

The boundary conditions in SH-type wave problems are considered as a traction free surfaces i.e., stress vanished surfaces. The other type of boundary conditions are taken in Seismology or geophysics. The impedance boundary conditions are commonly used in the fields of acoustics and electromagnetism. The impedance boundary conditions are linear combination of unknown function and their derivatives.

II. Formulation of the Problem

We consider the wave propagation in an orthotropic elastic solid under an initially stress ***P*** along the *x*-axis. Consider the origin of the coordinate system (*x*, *y*, *z*) at any point on the plane surface. Choose *z*-axis pointing vertically down ward into the half-space, so, that is represented by $z \geq 0$ and $z = o$ is stress free surface. Let us assume that the wave is propagating along *x*-axis. Assume that the medium is rotating about *z*-axis with angular rotation speed Ω, so, that angular rotation vector $\vec{\Omega}$ is given by $\vec{\Omega} = (0,0,\Omega)$ with Centripetal acceleration $\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)$ and Corilois acceleration $2\left(\vec{\Omega}\times\vec{u}\right)$, where $\vec{u}$ is the macro-displacement vector of the solid.

The equations of motion in three-dimensional form of a rotating orthotropic elastic medium under initial compression stress *P* are given by Datta [12] as:

$$\sigma_{11,1} + \sigma_{12,2} + \sigma_{13,3} + P\left(\phi_{12,2} - \phi_{13,3}\right) = \rho\left[\ddot{u} - \Omega^2 u - 2\Omega\dot{v}\right] \tag{1}$$

$$\sigma_{12,1} + \sigma_{22,2} + \sigma_{23,3} - P\phi_{12,1} = \rho\left[\ddot{v} - \Omega^2 v + 2\Omega\dot{u}\right] \tag{2}$$

and

$$\sigma_{13,1} + \sigma_{32,2} + \sigma_{33,3} - P\phi_{13,1} = \rho\ddot{w} \tag{3}$$

where $\vec{u} = (u, v, w)$ is macro displacement vector, ρ is density of the solid, ϕ_1, ϕ_2, ϕ_3 are micro-rotational vector components respectively, with

$$\phi_{ij} = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right); i, j = 1,2,3.$$

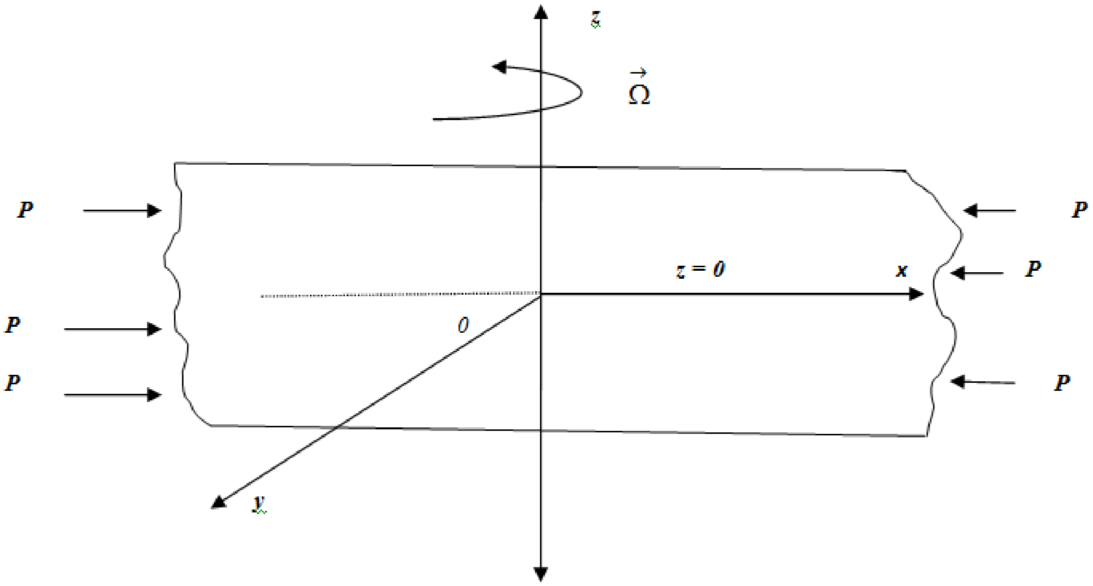

Fig. (1): Geometry of the Problem

The comma followed represents the partial derivative with respect to coordinate axis and superpose dot represents the partial derivative with respect to time variable *t*.

The stress σ_{ij} ; $j, j = 1,2,3$ are given by

$$\sigma_{11} = \left(C_{11} + P\right)u_{1,1} + \left(C_{12} + P\right)u_{2,2} + \left(C_{13} + P\right)u_{3,3}$$
(4)

$$\sigma_{22} = C_{12}u_{1,1} + C_{22}u_{2,2} + C_{32}u_{3,3}$$
(5)

$$\sigma_{33} = C_{13}u_{1,1} + C_{23}u_{2,2} + C_{33}u_{3,3}$$
(6)

$$\sigma_{12} = C_{44}\left(u_{1,2} + u_{2,3}\right)$$
(7)

$$\sigma_{13} = C_{55}\left(u_{1,3} + u_{3,1}\right)$$
(8)

$$\sigma_{23} = C_{66}\left(u_{2,3} + u_{3,2}\right)$$
(9)

where C_{ij} ; $j, j = 1,2,3,......,6$ are the stiffness tensor components in the contraction notation with

$$C_{44} = \frac{1}{2}\left(C_{11} - C_{12}\right); \quad C_{55} = \frac{1}{2}\left(C_{11} - C_{13}\right);$$

$$C_{66} = \frac{1}{2}\left(C_{22} - C_{23}\right)$$
(10)

and

$$\left(u_1, u_2, u_3\right) = \left(u, v, w\right)$$

III. Methodology

Love waves are SH-type waves, so for propagation of Love waves along *x*-direction in *xz*-plane, the displacement components (*u, v, w*) taken as $u = w = 0$, $v = v(x, y, t)$; $\frac{\partial}{\partial y} \equiv 0$, and $C_{11} = C_{33} = C_{13} = 0$.

With these assumptions and with the help of equations (4) to (10); the equations (1) to (3) reduces to

$$(P - 2C_{12})\frac{\partial^2 v}{\partial x^2} + 2(C_{22} - C_{23})\frac{\partial^2 v}{\partial z^2} = 4\rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v\right)$$

(11).

Let the solution of eq.(11) be

$$v(x, z, t) = V(z)e^{ik(c-ct)}$$

(12)

where *V(z)* is an amplitude decay, *k* is a wave number, and angular frequency ω and wave velocity *c* are related as, $c = \omega k$. After substituting eq. (12) in eq.(11) we obtain,

$$(D^2 - r^2)V = 0$$

(13)

where

$$D^2 \equiv \frac{d^2}{dz^2}$$

and

$$r^2 = \frac{k^2(P - 2C_{12} - 4\rho c^2) - 4\rho\Omega^2}{2(C_{22} - C_{23})}$$

(14). Let the solution of eq.(13) be

V(z) = A cosh(rz)+B Sinh(rz)

Therefore, the displacement component $v(x, z, t)$ is given by

$$v(x, z, t) = [A\cosh(rz) + B\sinh(rz)]e^{ik(x-ct)}$$

(15)

IV. Boundary Conditions and Secular Equations

The impedance boundary conditions in terms of displacements and stresses are given by Malischewsky [13] as: $\sigma_{j2} + \in_j u_j = 0$ for $z = 0$, where $\in_j$ are impedance parameters with the dimensions of stress/ length. For elastic half-space, Godoy, Duran and Nedelac [8], expressed $\in_j$ as $\in_j = \omega Z_j$, where Z_j are impedance real-valued parameters with dimensions stress / velocity and angular frequency ω is given by $\omega = cv$. So, impedance boundary conditions at the surface $z = 0$ of a orthotropic elastic solid are

$\sigma_{i3} + \omega Z_i u_i = 0$ at $z = 0, i = 1,2,3$

reduces to

$\sigma_{13} + \omega Z_1 u = 0$, $\sigma_{23} + \omega Z_2 v = 0$, $\sigma_{33} + \omega Z_3 w = 0$.

For Love wave propagation, the appropriate boundary condition is

$$\sigma_{23} + \omega Z_2 v = 0 \quad \text{at} \quad z = 0$$

(16).

On using equations (9) and (15) in equation (16), we obtain for ratio $A/B = 1$ as,

$$2\omega^2\left[Z_2^2 + 2\rho(C_{22} - C_{23})\right] = (C_{22} - C_{23})\left[K^2(P - 2C_{12}) - 4\rho\Omega^2\right]$$

(17).

Equation (17) is called dispersion relation of Love waves under the influence of impedance boundary with an initial stress in a rotating orthotropic elastic solid half-space.

V. Special Cases

(i) In the absence of initial stress *P*, the equation (17) reduces to

$$\omega^2 = \frac{(C_{23}-C_{22})(K^2C_{12}+2\rho\Omega^2)}{Z_2^2+2\rho(C_{22}-C_{23})}$$

(18)

is known as dispersion relation of Love waves under the influence of impedance boundary conditions in a non-stressed rotating orthotropic elastic solid half-space.

(ii) In the absence of rotation Ω, equation (17) reduces to

$$\omega^2 = \frac{K^2(C_{22}-C_{23})(P-2C_{12})}{2[Z_2^2+2\rho(C_{22}-C_{23})]}$$

(19)

is known as dispersion relation of Love waves under the influence of impedance boundary conditions with an initial stress in a non-rotating orthotropic elastic solid half-space.

(iii) On free boundary, i.e., $Z_2 = 0,$ equation (17)) reduces to

$$\omega^2 = \frac{1}{4}\left[K^2(P-2C_{12})-4\rho\Omega^2\right] \tag{20}$$

known as dispersion relation of Love wave propagation on free boundary of rotating initially stressed orthotropic elastic half-space.

(iv) In the absence of rotation Ω, initial stress *P* on free boundary (i.e., $Z_2 = 0$), the equation (17) reduces to the dispersion relation of Love waves as,

$$\omega^2 = \frac{-C_{12}k^2}{2\rho} \tag{21}$$

Equation (21) also called dispersion relation of Love waves in a non-rotating orthotropic elastic solid half-space.

VI. Numerical Application

Consider a particular example to discuss the theoretical results of orthotropic medium, by considering material values as:

$C_{23} = 2.694;$ $C_{22} = 2.363;$ $C_{12} = 0.661;$ and density $\rho = 6.$

The frequency curves of Love waves for different wave numbers of rotating, initially stressed solid versus the non-dimensional impedance parameter values Z_2 with $0 \le Z_2 \le 0.8$ are shown in fig.(2). From this figure we observed that the frequencies are proportional to the wave number in the given range of impedance boundary parameters. Also we observed that constant frequencies are occurs at zero wave number in an initially stressed rotating solid.

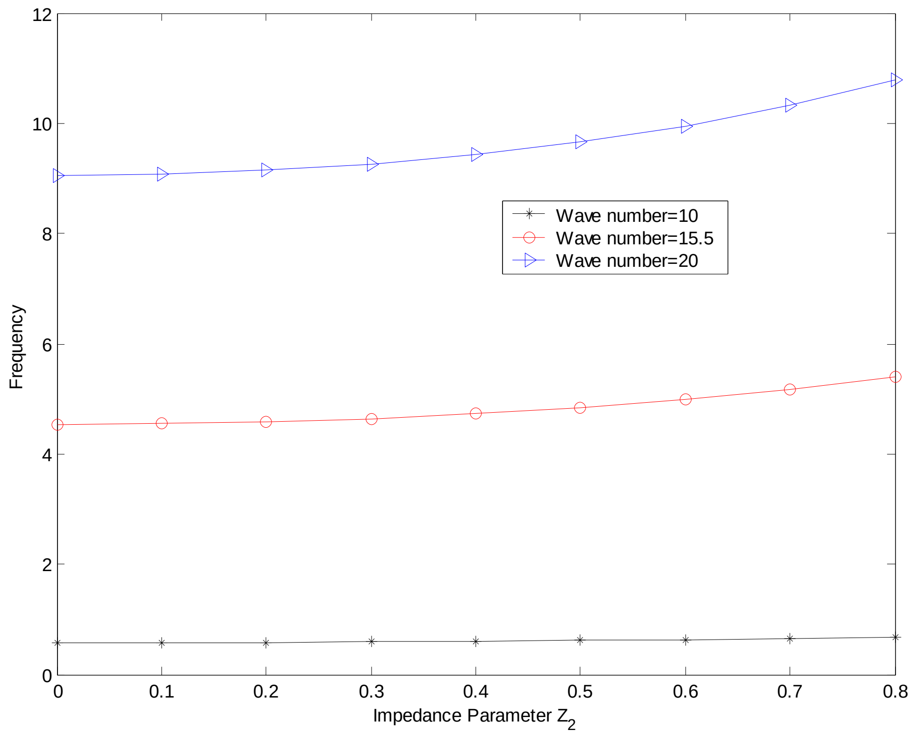

Fig.2. Impedance parameter vs. Frequency

The effects of initial stresses (or, compression) on Love waves in a rotating orthotropic elastic solid with impedance boundary are shown in fig.(3). Also initial stresses are proportional to the Love waves. Love waves are propagate with constant frequency/ velocity at free surface (i.e., non-stressed surface).

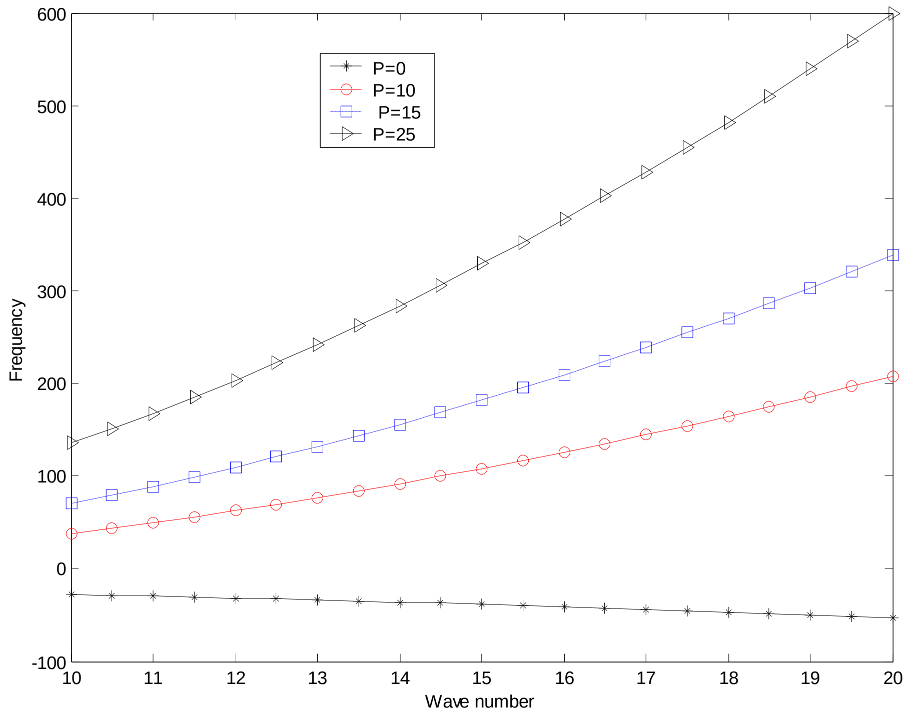

Fig.3. Wave number parameter vs. Frequency

The effects of rotations on Love waves of initially stressed orthotropic elastic solid with impedance boundary are shown in fig. (4). It is observed that rotation speeds are inverse proportional to the Love waves.

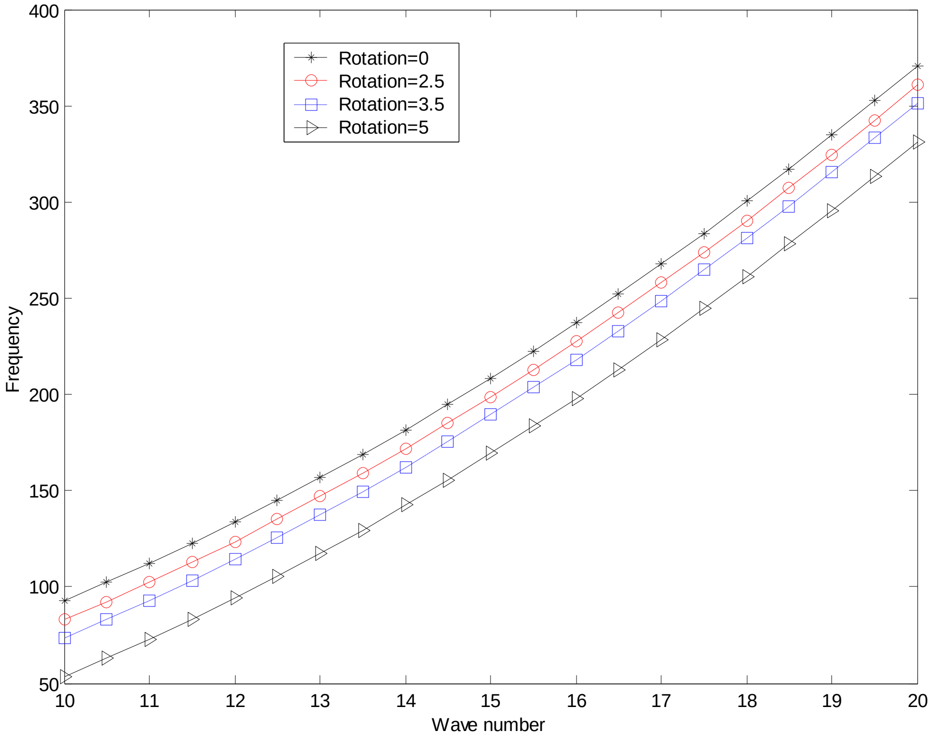

Fig.4. Wave number parameter vs. Frequency

The effects of impedance parameters on Love wave propagation in a rotating initially stressed are shown in fig.(5). From this figure one can observed that impedance parameters are proportional to the frequency / speed of Love waves.

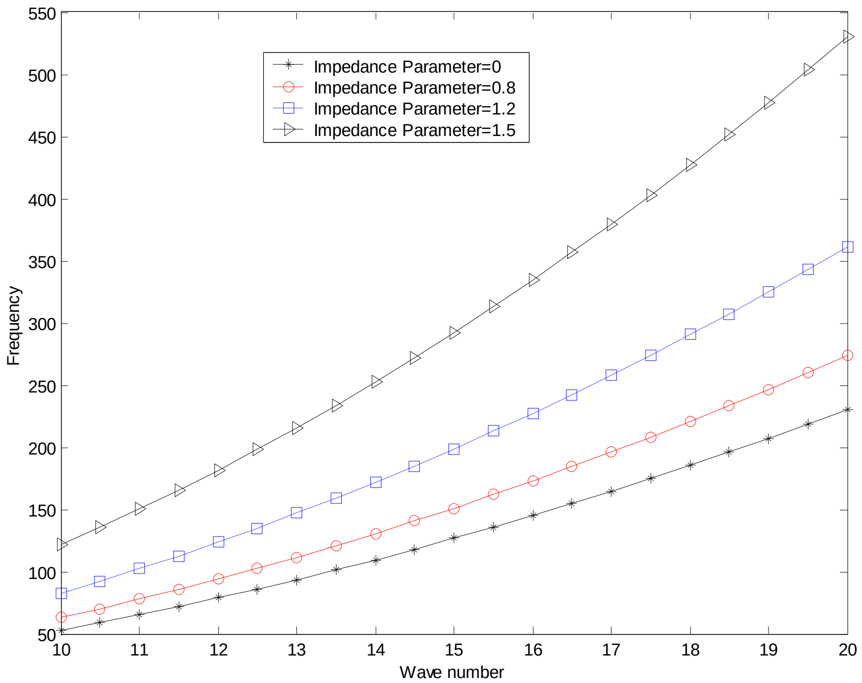

Fig.5. Wave number parameter vs. Frequency

VII. Conclusion

The effects of rotation and initial stress on Love waves in an orthotropic elastic half-space medium are investigated with its impedance boundary conditions. With traditional techniques, the basic equations are solved to obtain frequency equations of Love waves. With theoretical illustrations and particular numerical example, one can observed that:

(i) Love waves of initially stressed rotating orthotropic elastic solid are proportional to the wave number in the increasing impedance boundary parameters. Also they are constant at vanishing wave number.

(ii) Initially stresses are proportional to Love waves in a rotating orthotropic elastic solids on its impedance boundary. Also they are constant on free surface (i.e., non-stressed surface) with impedance boundary.

(iii) The rotation speeds of the solid are inverse proportional to the Love waves in an initially stressed orthotropic elastic solid with impedance boundary.

(v) The speed/frequencies of Love waves are proportional to the impedance boundary parameters in a rotating initially stressed orthotropic elastic solids.

References

[1]. Love A. E. H., 1944. A treatise on the Mathematical Theory of elasticity, Dover Publications, New York, 4th edition, ISBN 9780486601748, 0486601749

[2]. Biot M. A., 1955. Theory of elasticity and consolidation for a porous anisotropic solid, Journal of Applied Physics, 26, 182-185. doi: 10.1063/1.1721956.

[3]. Biot M. A., 1956 a. Theory of deformation of a porous viscoelastic anisotropic solid , Journal of Applied Physics, 27, 459-467. https://doi.org/10.1063/1.1722402

[4]. Biot M. A., 1956 b. Theory of propagation of elastic waves in fluid saturated porous solid, Journal of the Acoustical Society of America, 28, 168-178. https://doi.org/10.1121/1.1908239

[5]. Abd-Alla A. M., 1999. Propagation of Rayleigh waves in an elastic half-space of orthotropic material, Applied Mathematics and Computation, 99 (1), 61-69. https://doi.org/10.1016/S0096-3003(97)10170-9

[6]. EI- Naggar A. M., Abd-Alla A.M, and Ahmed, S.M., 1995. On the rotation of a non-homogeneous composite infinite cylinder of orthotropic material, Applied Mathematics Computation, 69, 2-3, 147-157. https://doi.org/10.1016/0096-3003(94)00074-E

[7]. Abd-Alla A. M, Hammed, H. A. H and Abo Dahab, S. M., 2004. Rayleigh waves in a magneto-elastic half space of orthotropic material under influence of initial stress and gravity field, Applied Mathematics and Computation,154 (2,5), 583-597. https://doi.org/10.1016/S0096-3003(03)00767-7

[8]. Godoy E., Duran, M and Nedelec J. C., 2012. On the existence of surface waves in an elastic half-space with impedance boundary conditions, 49 (6), 585-594. http://dx.doi.org/10.1016/j.wavemoti.2012.03.005

[9]. Vinh, P. C. and Hue, T. T. T., 2014. Rayleigh waves with impedance boundary conditions in anisotropic solids, 51 (7), 1082-1092. http://dx.doi.org/10.1016/j.wavemoti.2014.05.002

[10]. Singh B., 2016. Rayleigh wave in a thermo-elastic solid half-space with impedance boundary conditions, Meccanica, 51 (5), 1135-1139. https://doi.org/10.1007/s11012-015-0269-y

[11]. Somaiah K., and Ravi Kumar A ., 2021. Rayleigh type wave propagation in a rotating micro-polar elastic solid with stretch in the gravity field, Advances and Application in Mathematical Sciences, 20 (12), 3397-3412.

[12]. Datta B. K., 1986. Some observation on interaction of Rayleigh waves in an elastic solid with gravity field , Rev. Roumaine Sci. Tech. Ser. Mec. Appl., 31 (4), 369-374.

[13]. Malischewsky P., 1988. Surface waves and Discontinuities Elsevier, Amsterdam.

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