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Quanta Maths Models

Gudrun Kalmbach H.E.
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ABSTRACT
In the MINT-Wigris Tool bag are 11 models available for the quantum range. Figure 1 is for fusion, figure 4 for weak bosons decays and figure 8 has eight more tools. One more figure wheel is not shown, it presents the spaces Euler angle for generating as transformation matrices the three quaternionic symmetries for xyz-space rotational axes. More can be constructed where the energy exchange between systems uses different quasiparticles P from a list of about 32. Bases of a
4-dimensional real Hilbert space H_4 are used for them and a GF measuring apparatus for P reduces in a Boolean subspace block 2^4 of H_4 one base vector for projective geometrical use. Some GF triples are listed in figure 10. A proof in [1] demonstrates that in figure 9 for hypergraphs of blocks a *rgb*-graviton in nucleon exists as an additional GF for a 3-cycle of blocks belonging to the three quarks in a nucleon. Another 4-cycle of blocks requires a central astroid consisting of 4 blocks. The astroid blocks generating base vectors must be different. This is interpreted as doubling quaternion coordinates to octonians. To the boson decays is added a brief interpretation of the Hopf SU(2) geometry for the leptons generated by a deaying weak boson. Some comments to the 2^4 blocks follow where it is explained why the 2^4 blocks subspace lattice can also be used when H_4 is not real, H = R, but a complex or quaternionic Hilbert space with H = C or H = Q. This allows more blocks in a subspace lattice than for H = R and more GF's. In the last section, the Hopf geometry is used for proposing another fiber bundle geometry, due to the strong interactions geometry. For the new octonian coordinates is quoted a table which uses the factorization of the nucleon tetrahedron symmetry where six octonian coordinates, six energies, six color charges and the D_3 symmetry are collected in the six columns as a row listing. Why two more octonian coordinates are added later on in the early dark development of the universe is due to the electromagnetic interaction when spectral series can escape from atoms. The tables listing gives as geometrical tool the hedgehog for a deuteron or atomic kernel. It acts for energy exchanges of the kernel with its environment.

Two examples: fusion and weak boson decays

In 2020 the author published open access mathematical models for the quantum range. For states of systems the infinite dimensional Hilbert space is replaced by models in eight dimensions. Observable are in physical experiments real datas and complex imaginary remain undetermined.They are often modeled by rotations with no measurable fixed angle as for spin or fixed for changing states of systems.

Example: Take in the fusion model two protons (at left in figure 1)where a Higs boson has set as center of the two tetrahedrons a common barycenter. The parallel location of the observable quark triangles means that two opposite u-quarks exist. In fusion the upper u-quark is decaying acording to the rules. It first releases in a longer time interval a weak boson W+and becomes a d-quark. W+ decays in the observable positron and neutrino which as energy are emitted from the system. This sets an energy for rotation of the upper tetrahedron free such that their central projection forms then a hexagon in the right figure. Now Cooper pairs of u-, d-quarks are located on the single xyz-space axes and an energy exchange cycle is presenting the D_3 symmetry of the triangle. In conic rotations of paired triangles sides with center a quark vertex, barycenterical coordinates as rotation axes of the generated cone are set. Their intersection is a nucleon barycenter where Higgs can set a revised mass m for the nucleon. The sum of the three quarks mass is observed as 10 percent m_q , inner frequencies as speeds m_f or interaction energies m_i count for about 80 and 10 percent which add to

$m = m_q + m_f + m_i$. For m_f is used the $mc^2 = hf$ formula which allows Higgs to measure frequencies in kg,mass. For m_i and the wave package description of the nucleon it is necessary to make a rescaling of mass m by for getting a common group speed v<c with which the wave package moves in its environment.The momentum is p = mv. Applying special relativistic scaled mass m =
$m_q /\cos \varphi$ with $\sin \varphi = v/c$ uses differentiation and the optical way to compute speed as
(v =) $u = \partial\omega/\partial k$, $k = 2\pi/\lambda$ wave number (substituted by $2\pi p/h$), $\omega = 2\pi f$, λ wave length.

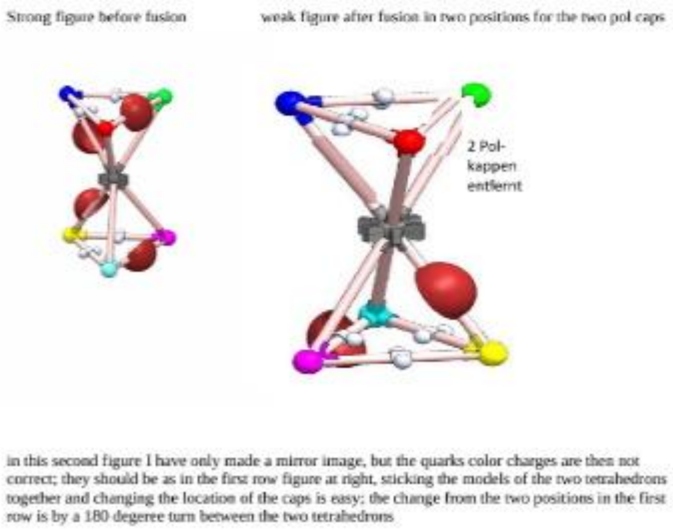

Figure 1 nucleon tetrahedrons with two polar caps for a positron of a proton; at right one positron is emitted; the gluon exchange between Cooper paired quarks is shown as two small balls attached on their connecting interval; the dual color charges are also used for one nucleon, but is imaginary and not observable, in observations this is changed to rgb by applying the conjugation C operator

As a proof for the existence of the tetrahedons *rgb*-graviton whirl at the tip of the nucleon tetrahederon with base the quark triangle is quoted the neutral color charge red-green-blue of all nucleons and base choices in a complex or real 4-dimensional Hilbert space H_4 .

Looking at subspace structures of H_4 it has already a new Gleason frame as 3-dimensional orthogonal base triple abc, a kg-GF which as operator T sets Higgs mass newly by adding the three weights m_q ,m_f ,m_i of the spin-like triple as new Higgs mass. In contrary to spin with three equal length for its xyz vectorial triple, GF's can add to its triples abc different non-negative real, complex of quaternionic numbers. They are then also the coordinates of H_4 provided with a Hilbert space metric $<w,w>$ and the GF changes the metric to $<wT,w>$. T can have different interpretations, for instance as a Minkowski rescaling of Euclidean $<w,w>$, $w = (x,y,z,t)$ to $wT = (x,y,z,-c^2t)$, generating for radius r the Minkowski cone $r^2 - c^2t^2 = 0$, as differentials metric $ds^2 = dr^2 – c^2dt^2$ in the tangent space.

For the *rgb*-graviton GF is used from [1] the 4-dimensional real H_4 subspace orthomodular lattice L where projection operators P are replaced by their subspace $P(H_4) = U$. The lattice structure requiers for the subspace Boolean blocks 2^4 of L that for an rgb-nucleon triangle consisting of three blocks

2^4_{ab} , ab = rg, rb,gb, that a fourth block exists 2^4_{rgb} for the postulated *rgb*-graviton which generates in conic rotations barycentrical coordinates for the Higgs mass setting. Can physics accept a mathematical proof for the L of H_4, similat to a mathematical general relativistic tensor calculation of Einstein?

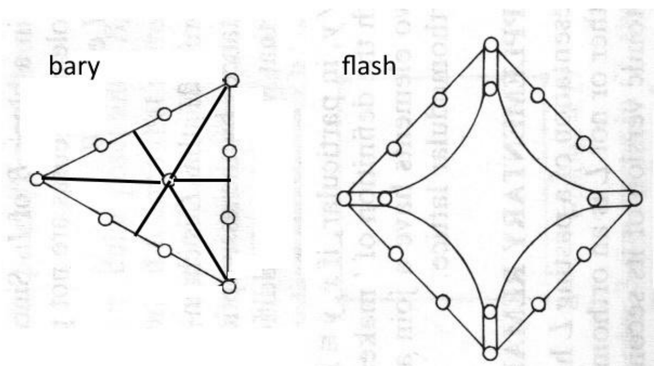

Figure 2 bary at left, flash at right for a central astroid, proved in [1]; blocks 2^4 are drawn as intervals with 4 points marked for their base line vectors, except for the central block at left where for demonstational porposes the fourth point is drawn as barycenter of the triangle the block 2^4_{rgb} contains beside the barycenter the three rgb vertices of the triangle

Example: A weak boson decay model is described. Octonians or complex C^4 extensions of real H_4 coordinates are shown in the part right of figure 2. The flash astroid of L has to be inserted in a 4-cycle of blocks 2^4, drawn as outer quadrangle. It is a modular, projective geometrical motivated block diagram with four blocks. They overlap in adjacent pairs in two base vectors, called in [1] atoms. The eight necessary atoms are pairwise different and present the doubling of quaternionic or spactime coordinates to octonians. A similar figure 4 is shown as figure 1, using the weak, not strong color charge, interaction and leptons with weak bosons, replacing quarks with gluons. Useful is the magnetic symmetry, replacing the triangle D_3 symmetry in figure 3. A reduced notation on the quadrangles sides as a,a on the left and right sides and b,b on the vertical sides shows the leptonic torus genus 1 identification of their Hopf geometry (at right). The magnetic group is considered as replacing a condensor plate in figure 4 and has at its quadrangle upper (lower) vertices at 1 an electrial (neutral) charge, at 4 a magnetic momentum (momentum), at 3 a (rotational) induction momentum and at 2 a kg-mass GF. The upper condensor plate is for an electron,

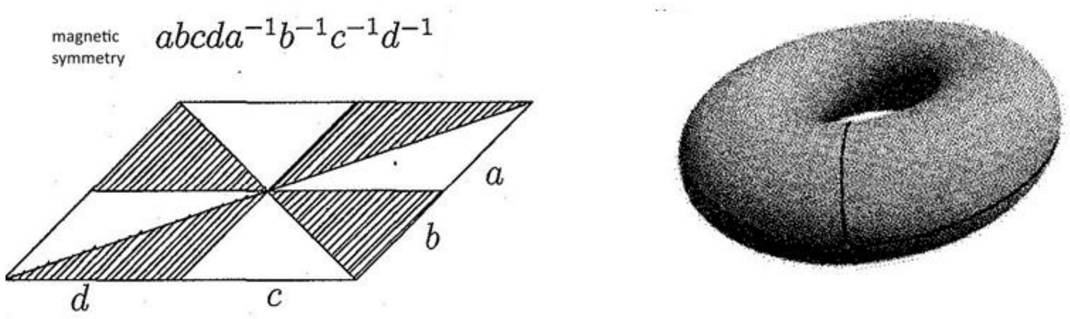

Figure 3 magnetic symmetry at left, leptonic Hopf torus at right

the lower for a neutrino. They are Hopf joined in a 3-dimensional unit sphere W-boson S^3 at the center of the 4 diagonals, drawn as a common 4-dimensional W- xyzt-coordinate system. Weak bosons decay like Higgs bosons. They have short lifetimes. One reason in this W- case is that Heegard decompositions allow for the two joined electron and antineutrino tori that one antineutrino transversal torus T_n circle can split into a rotation axis for the electron torus T_e at projective infinity such that T_e gets a vertical rotation z-axis for its Hopf inverse h^{-1} (S^2) image, showing equipotential and field lines (figure 6) while the Hopf image S^2 shows on a latitude circle a rotating electrical (or neutral) charge for its spin eigenrotation, coupled with magnetic momentum (momentum) in figure 5.

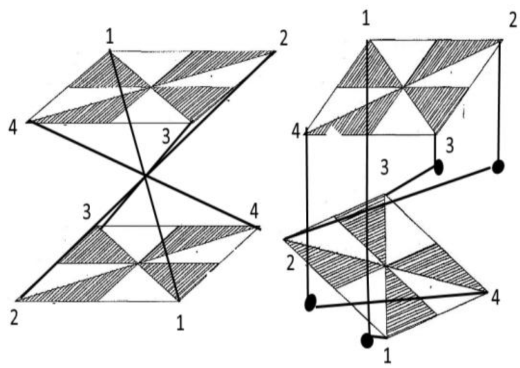

Figure 4 W- boson at left with coordinates 1234/xyzt and two electron/antineutrino condensor plates; at right a rotational twist where the coordinates are complex doubled in four planes, containing two upper/lower vectors Ou_j, u = 1,2,3,4; weak decay can alternatively use two pyramids for the two leptons in form of an octahedronwhere the condensor plates are in the middle

In figure 4 right the barycenter of the W- boson is annihilated and Higgs sets for the upper and lower pyramid two disjoint barycenters with other kg-GF mass scalars than the W- mass. The W+ decay and figures are similar for a positron and neutrino replacing and electron and an antineutrino. Weak decays of Z^0 together with a hyperphoton shows decays into two finite photon cylinders,The leptonic geometry shows that spactime coordinates have to be replaced by octonians which have another multiplication table than the eight SU(3) GellMann matrices. Physics uses for matrix/transformation presenations different multiplications, also in form of spinors or tensors where quasiparticles are used for exchanging energies between systems. Extending the

2-dimensional complex space coordinates by the conjguation operator to transformation matrices identity id for x·id and σ_2 for $y\cdot\sigma_2$ is the first step. Then the presentation of xyz-coordinates by Euler

angle transformations generate noncommutative Pauli quaternion matrices which are not the above Euclidean metrical spacetime coordinates of spacetime as H_4. Octonians double quaternion coordinates.

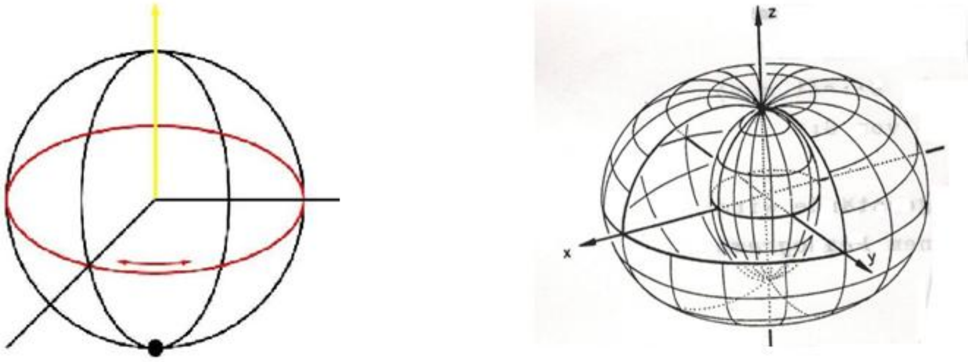

Figure 5 electron Hopf sphere S^2, magnetic momentum plus spin vectors on the rotation z-axis, electrical charge rotating with a latitude circle cw clockwise for e^- and mpo counterwlockwise for e^+; mass at the south pole as barycenter; at right a (anti-)neutrino spindle torus with momentum sitting at the south pole on the z-axis and a central retraction of the transversal torus circle for the spindle axis

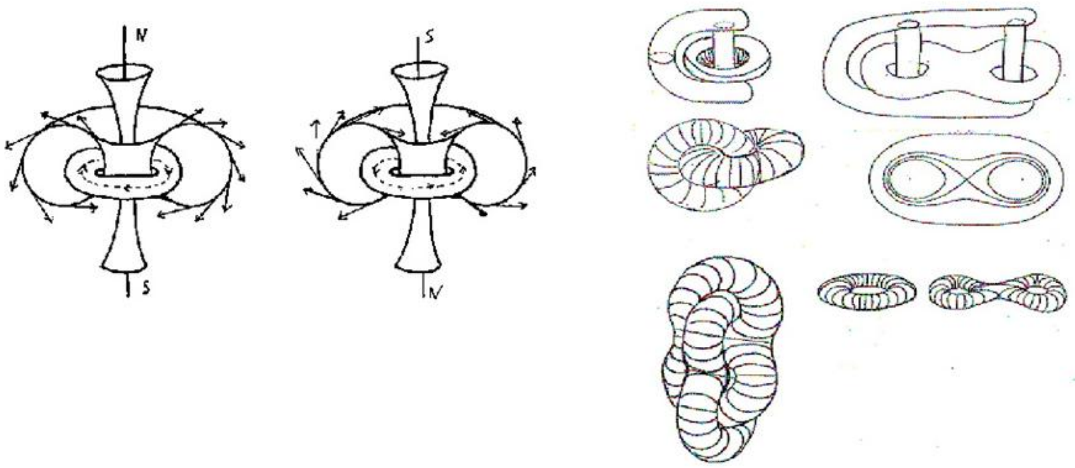

Figure 6 Hopf sphere S^3 for an electron, the points of S^2 are fiber S^1 blown up to circles, the mass S^2 point to a circular tori core which can be interpreted as a Schwarzschild radius carrying mass at rest, the rotating leptonic charge is a leaning circle on the rotating torus and fills out the torus surface like a condensor plate, the SN pole notation should be replaced by a leaning magnetic momentum plus spin vector towards the central z-axis, orthogonal to the leaning leptonic charge circles plane; at right some Heegard decompositions of S^3, two leptonic tori of genus 1, two quark brezels of genus 2; the genus arises from two orthogonal Lissajous figure frequencies, hitting for genus 1 (2) in the proportion 1:1 (1.2)

The weak boson decays can occur at an equator or a surface of genus n = 1,2,… . For genus 3 a quasiparticle trion can be responsible with the *rgb*-graviton GF for generating a brezel of genus 3 for nucleon states. The different mass distributions after the decay and the annihilating of a weak boson barycenter with its coordinate system can be attributed to not equilibrium fitting inner

frequency rotations, reset by Higgs as new mass and coordinate distributions for the separated two deay particles. New barycenters and barycentrical coordinate have to be generated for them too.

For the photon decays is mentioned that their 7th octonian coordinate e_7 is rolled to a Kaluza-Klein

circle U(1) with a projective stereographic point at infinity ∞ added from which it is stereographic projected down to e_7. Cylindrical coordinates apply for their energy expansion in time along a cylindrical helix line. In case a planar cut is taken and finfinite strips of width 2π are drawn, the helix line is a line in this complex plane. In the MINT-Wigris tool bag, which should be used for MINT(-Wigris) or STEM Quanta courses, the model can be drawn by the templates in figure 8.

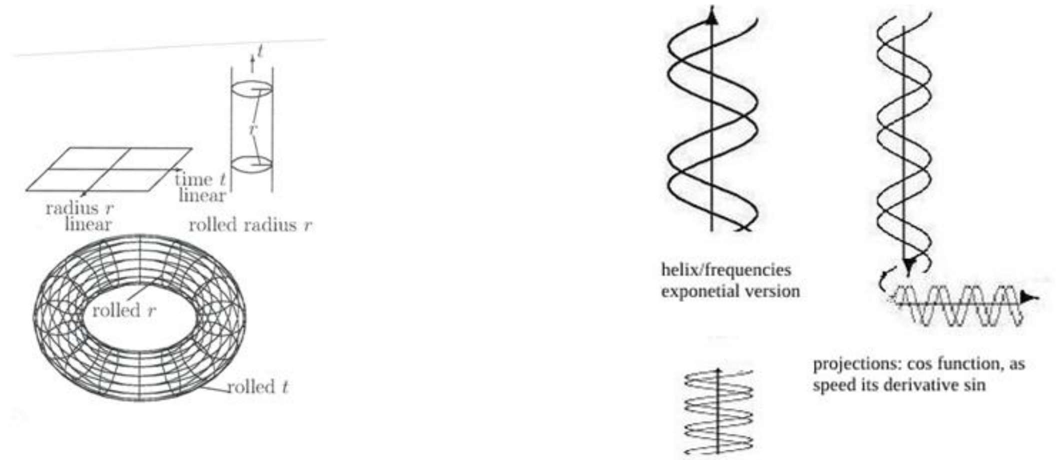

Figure 7 rolling of coordinates for periodic functions like the complex polar exponential exp(iφ) function, for double periods the cyclinder is closed by a circle; for the electromagnetic interaction (with photons) the cylinder is closed at projective infinity by a point to a pinched torus with dark energy inside where one transversal torus circle is retracted to a point

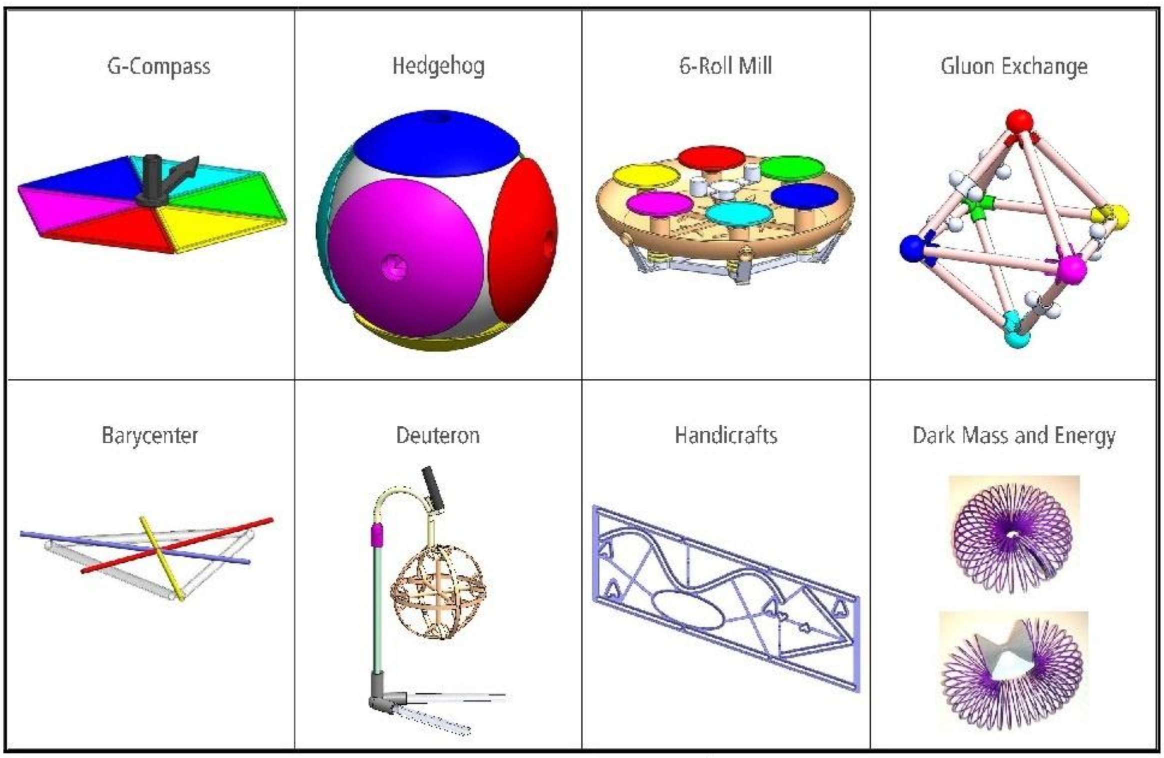

Figure 8 MintWigris Tool bag with 8 models, not explained in this article in detail; the template is named handcrafts where different shapes can be drawn with pencil on paper; when cut out and glued suitably together they present different surfaces

Blocks in 4 dimensions

In two H_4 subspace L figures the Boolean 4-dimensional base blocks are drawn as intervals containing 4 points while in the GF Fano octonian memo the intervals have 3 points for the GF triple orthogonal base. In figure 2 are the parts for bary(centrical coordinates and barycenters) and the flash/astroid. They are a requirement, proved in [1], for L having an orthocomplement lattice OML structure where the orthomodular law replaces Boolean logical reasoning in the blocks. Deductions are no OML law which should be also revising the logicians paradoxies for quanta. Quanta have neither the usual implication and modus ponens reasoning as can be read in the [1] proofs. Boolean reasing inside blocks need no bary nor flash.

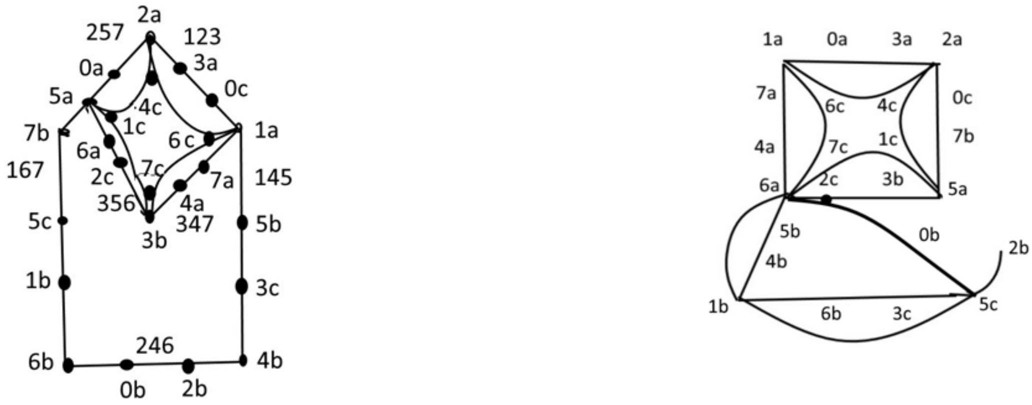

Figure 9 left: an OML sublattice of L for an existing 4-cycle of the weak/electromagnetic cases and interactions; it requieres the added octonian flash; the seven Fano GF'S are extended to
4-dimensional Boolean blocks where the additional coordinate can be time for including rotations or a projective dummy variable which makes them to projective spaces P^3; for marking the many bases of H_4 the author decided to add to octonians listed by their indices 0,,,…,7 a letter a,b,c which means only that this coordinate belongs to another block of L; right part: to a 4-cycle of L is added a *rgb*-graviton/nucleon quark triangle where the additional block is differently drawn as in figure 2, as block not as barycenter, also the atoms are not marked as black points on the intervals

In both parts of figure 9 are 24 atoms of L used. They correspond 1:1 to the nucleon tetrahedrons symmetries of S_4 . The symmetries need further explanations in models. Here is mentioned that S_4
factors by its normal CPT subgroup (adding id) to the D_3 triangle symmetry which has as dynamical representation the strong SI rotor (see the open access articles of the author or her book *MINT-Wigris*) which sets barycentrical coordinates and acts for integrations of force vectors to speeds or potentials (as examples). The use of 24 atoms can be extended. The Hilbert space H_4
has in these models real coefficients, presenting a real vector space R^4 . Setting its coordinates to
H = C or H = Q for a 4-dimensional complex or quaternionic space needs no extension of the Boolean block structure 2^4 . The extension of coordinates is attached to atoms of the blocks. Each atom in C^4 has a plane as two-dimensional base attached. With a matrix transformation the base vectors can be turned by 90 degrees. If this is written as a transformation (1,i)→(i,-1) it means that the real part is getting imaginary and the imaginary part gets real. In looking at observabels of complex valued functions like exp, the real part is observable as its cosine projection while its imaginary sinus part is not an observable. Differentiating for instance for an imaginary scaled time means that the exp wave speed sinus is an observable, differentiating twice, an accelerating force has an observable negative sign for the scaled cosine. The atoms added complex plane can adapt this for its base: reals are obersvable as 1 or -1 in (1,i) or (i,-1), imaginary i not. The atom has not to be split into two dimensions for complex base lines. This provides for the choice of blocks atoms 8 not 4 numbers, the octonians are used in figure 9. For the quaternionic Q^4 a similar method sets for the GF's Pauli matrices base triple the cubic roots replacing 1,i. In the Copenhagen interpretation only one of the GF base vector is observable in a measurent experiment, the other two are undetremined and give no output. In matrix or imaginary cubic roots multiplication the measurable vector can be replaced by the obtained third vectorial value. For an atom in Q^4 there are then three choices and the octonians j in figure 9 are enumerated newly by adding ja, jb, jac for the quaternionic extension of coordinates. In the above mentioned case of the complex exp function also a sign for the vectors is used. If this is applied for figure 9, there are then 48 choices for the atoms blocks 2^4 . This number can be sufficient to accomodate GF states of a physical systems by using blocks. Counting particle series, there are two fermion series with 12 members. The list of quasiparticles has about 32 members such that the different 48 blocks can also account for their number. A possible symmetry is O_h , the symmetry of a cube.

Symmetries and the projective Hedgehog

The factoring generates for a color charge class a coordinate, one or several symmetry transformations and an energy as listed in an old 20th century table of the author in her book on *Quantum Measures and Spaces* which is out of print. Energies treated in this article are electrical (as charge in the first column GF 123, octonian x-coordinate 1) , heat (second column, GF 246, octonian y-coordinate 2), rotational energy (third column, GF 347, octonian space z-coordinate 3), magnetic energy(fourth column, GF 145, octonian t-coordinate 4 time), mass (fifth column, GF 247, octonian coordinate 5), frequency as inverse time-interval (sixth column, GF 356, octonian

coordinate 6). For the Moebius transformations in this table are responsible the six complex cross ratios where beside a complex variable z are used reference points 0,1,∞ on a Riemannian sphere
S^2. They are D_3 permuted and z is kept fixed. As an example 1/z is used as mathematical inversion at a circle like the Schwarzschild radius Rs of a black hole (dark matter) where the quark/matter radius r is inverted to dark matter radius r‘ in $r'r = Rs^2$. Also speeds are

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-)dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the D_3 (SU(2)/Pauli) MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, C (conjugation for quantum numbers), T (time reversal) and P (space parity) of physics.

r or $re^{i\varphi_1}$	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	$\overline{g}$	$\overline{b}$	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	$-\frac{1}{z}$	$-z$	z		
$id;\sigma_1$	$\alpha\sigma_1;\sigma_2$	$\alpha^2;\sigma_3$	$\alpha^2\sigma_1;id$	$\alpha;\delta$	$\sigma_1;id;\beta$
1	6	4	2	5	3
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
EM_{pot}	E_{heat}	E_{rot}	E_{magn}	E_{kin}	E_{pot}
c, e_0, ϵ_0	k,C	N_A,T	μ_0	h	γ_G, R_S,P

Table 1 in the first and second row ar listed coordinates, third row color charges, fourth row Moebius transformations fifth row symmetries, ninth row energies, other rows are not used in this article

inverted this way in $v'v = c^2$ at a Minkowski cone for dark energy speeds v‘>c inside a pinched torus and matter wave packages require a speed v<c. Time in this cross ratio list is acting like a translation, frequency as inverse time interval, and heat/rotation have inverse Moebius transformations, arising as projective quotients of linear transformations. The list of D_3 members in the sixth row shows that id, α, α² are a cyclic subgroup of order 3, rotating the nucleon traingle while the other three D_3 members are the reflections for its barycentrical coordinates, generated by the *rgb*-graviton. As a strong interactions projection operator it projects the first three SU(3) matrices down to the three Hopf/Pauli matrices of quaternionic spacetime. The SU(3) 3x3-matrices are extentions of the Pauli 2x2-matrices by adding a zero entry row and column. Disregarding that the three extended σ_3 matrices are linearly dependent, in z_3-extended complex vectorial triples (z_1,z_2,z_3) the three complex C_j^3 spaces have coordinates in the 3x3-matrix rows (z_1 z_2 0) (c(z_2)c(z_1)0)
(0 0 0) wherr c(z) means conjugation, (z_1 0 z_3) (0 0 0) (-c(z_3) 0 c(z_1)) and (0 0 0) (0 z_2 z_3)
(0 c(z_3) (z_2)). There are only 8 gluons as field quantums for the strong interaction. The quaternionic spacetime coordinates are $z_1 = z + it$, $z_2 = x + iy = r\cdot\exp(i\varphi)$. As research project is proposed to look
for applications of their three strong extensions, also in a projective setting. As projective CP^2 space for nucleons was proposed by the author the hedgehog presentation as in figure 19.

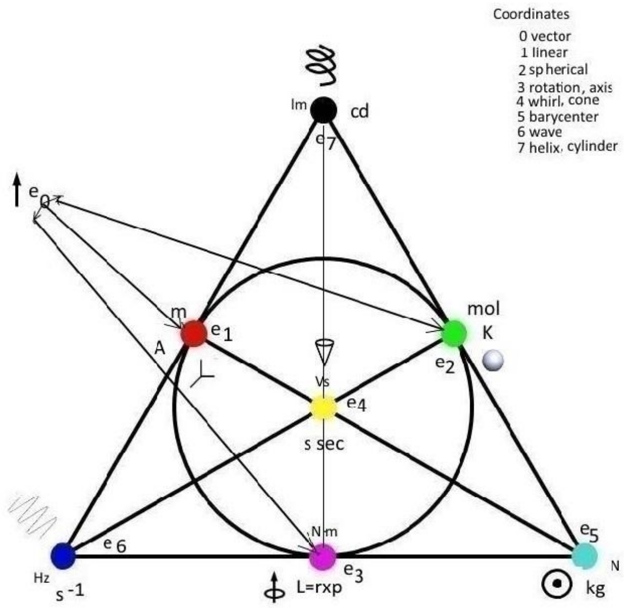

Figure 10 Fano memo for seven base octonian Pauli matrix triples

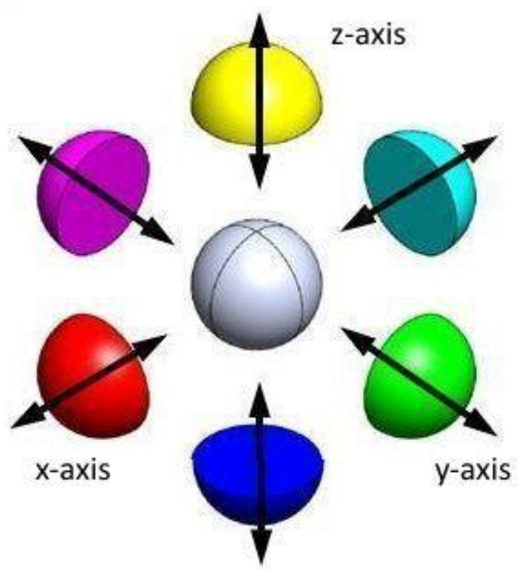

Figure 11 hedgehog as CP² for deuteron and atomic kernels, having three atmospheric Bohr shells for color charges; they are Heegard splitted in two hemispheres D, each carrying an energy force vector for the kernels energy exchange with its environment (see the table listing)

For the energy in/out vectorial direction, for quasiparticles is suggested that D is closed to a projective plane P² by identifying on its boundary diamentrical points p, -p. The central vector can rotate by 360 degrees on a Moebius strip inside P² for changing its normal direction according to existing thresholds. Energies change quantized as proposed in the following figure and can set thresholds. A leptonic example for this is when the angular frequency rotation of a lepton is not fitting a Bohr radius of an electron in an atoms shell for instance, the electron radius changes to another possible solution for its wave presentation and from atoms spectral series are emitted or absorbed.

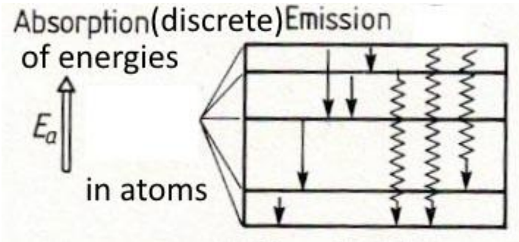

Figure 12 quantized energies

For the hedgehog the energy thresholds open (similar to a valve) a directed energy flow from an atomic kernel to its outside or reversely by turning the in/output directed normal energy vector on the polar cap. The system changes it state. For a deuteron with two nucleons is postulated that it has two inner weak WI and strong SI rotors which have each 6 states. This makes together with the hedgehog 24 states. Since the changes of states are oriented they cannot have a sign. This corresponds then to half of the 48 possible blocks in the former section, desregarding the +,- sign.

As research project, a computer program can be written which extends the two block examples in figure 9 to more than its 11 or 12 blocks. The two rotors are described in the reference articles. For the hedgehog as deuteron space CP^2 is mentioned that beside the old Hopf fiber bundle for the weak interactions SU(2) geometry S^3 exists a strong interaction fiber bundle with space S^5 , a

5-dimensional unit sphere with a norming of a fiber S^1 for the bundle map g: $S^5 \rightarrow CP^2$. There is a

4-dimensional space for the kernel C^2 and a boundary projective S^2. For the hedgehog caps three such bounding spheres are postulated. The sphere S^5 is a factor of the twisted fiber bundle S^3 x S^5 which belongs to the SU(3) strong geometry. Its first factor S^3 can be taken in the above C_1^2 subspace having z_1 ,z_2 coordinates and a last row and column with entries 0. The *rgb*-graviton acts as projection operator, deleting the last row and column. The strong S^3 copy is mapped to the Hopf S^3. The Hopf map structure of this sphere provides with the three Pauli matrices the $h(S^3)$ coordinates in space R^3 as first coordinate the complex dot product, as second coordinate the complex cross product and as third coordinate a toroidal version where one circle is rotated independent of a second orthogonal circle in the R^4 space with x_1 ,x_2 and x_3 , x_4 coordinates.

As research project a similar symmetries bound map g can be found. The GellMann SU(3) matrices for the spaces C_2^2, C_3^2 can serve for this purpose, the SI rotor symmetry D_3 provides maybe also a description for g coordinates in CP^2. Open is also the question what the generated coordinates for CP^2 represent. Some ideas why D_3 may be not good: The three barycentrical coordinates of the nucleon triangle can use as symmetries the three reflections of D_3 and set a barycenter for the nucleon mass. Then g would map the S^5 coordinates directly to these three reflections with the following action of setting these barycentrical coordinates. There are two rotations of D_3 α, α² of order 3 for two more CP^2 coordinates. As projective space it has complex u_1 , u_2 coordinates where u_1 can be taken for the triangle plane and one coordinate w of u_2 for the 3-dimensional extension by the tetrahedrons of deuteron. The second u_2 coordinate is then for a change of states in time given by the SI rotor. For the tetrahedron extension w possibly the generating *rgb*-graviton can be quoted, no symmetry exists, but an additional whirl w-coordinate. The rotations of the SI rotor use the 2-gluon exchange between paired quarks which can generate an angular frequency ω for an inverse time interval and α, α² serve for the cw and mpo direction for the ω oscillation. The two 6-gluons of the eight gluons can act for setting the boundary S^2 of CP^2. Concerning the spaces C_2^2, C_3^2, one could use the coordinates z_1, z_2, refering to matrix presentations through $\lambda_{4,5,6,7,}$ as coordinates for C^2 and z_3 for S^2 without a GellMann matrix, and the z_3 coordinate used for closing the boundary plane by a projective point at infinity to S^2. This means that the projective coordinates for CP^2 are $[z_1 ,z_2 ,z_3]$ with $z_3 =1$ for C^2 and $z_3 = 0$ generates the boundary sphere S^2 as $[z_1/z_2,1,0]$, $z_2 \neq 0$, with the point [1,0,0] added for $z_2 = 0$. This version looks better for the CP^2 coordinates as D_3.
That z_3 has only one λ_8 GellMann matrix, not two attached may be not as inportant since it is not a projective complex line S^2 at infinity, but serves only for the projective norming, z_3 could be replaced by one real coordinate $w = |z_3|$.

Critical Remarks

In the standard modelling of quantum physics, the underlying coordinate space is kept 4-dimensional. In a 5-dimensional extension the Kaluza-Klein KK theory [7] shows that a projective extension allows to unify gravity GR with electromagnetic EM potential. In projection, there are then three 4-dimensional fields, one for EM, one for GR and a scalar field. The approach in this article, introducing higher dimensions, is different. The quantum use of Ψ functions or approximate solutions of spacetime coordinate bound differential equations for functions in space and time coordinates is given up. Systems like nucleons or partices like leptons can have inner dynamics descibed by symmetries, not by functions in four variables and their differentials dx, dy, dz, dt, including their repetitions. Changing states of a system are then not related to a matrix eigenvector, -values

calculus. Having an additional 4-dimensional Higgs field in KK with a fifth rolled U(1) coordinate remaining massless for the electromagnetic interactions photons has no meaning for the photon geometry. MINT-Wigris postulates that this coordinate is carrying the function Ψ value in form of the complex polar exponential function description exp(iφ) where φ can be substituted using spacetime coordinates. Photons not having a Higgs mass attached miss a barycenter at which Higgs can set time independent its mass scalar. The situation equals the missing fixed angle for spin rotations. At no time an angle is fixed for spin. Photons energy is measured as frequency helix line expanding on a cylinder. In projection onto the U(1) transversal circle, there is one or full windings are needed in order to store energy E = hf. No Higgs barycenter can be set for a computed (relativistic) kg-scalar by $mc^2 = hf$. In an operator description, the Higgs operator T has a probability distribution on Hilbert subspaces in higher dimensions than 4 where on a fifth or higher dimension the support for T has an orthocomplemented subspace where energy in this subspace has mass value m = 0, but f ≠ 0. The Einstein line is not existing in this space. This motivates the authors view that an [m,f,w] projective plane P^2 has to be added to spacetime coordinates which cannot be treated in spacetime coordinates (x,y,z,t) of a Ψ function. In P^2 it is possible to choose the line at infinity differently by using projective transformations. Then m = 0 gives a line [0,f/w,1] for w ≠ 0 closed by a projective point q = [0,1,0] to a P^1 circle U(1). The point q has at no time a fixed location which could be used as barycenter, but rotates with an angular frequency $\omega = 2\pi f$ on P^1 . A similar frequency rotation exists for leptons in the Hopf geometry where on latitude circle of radius r in a Riemannian sphere S^2 an electrical or neutral leptonic charge is rotating. In the Hopf blow up of S^2, this point q is extended to a fiber S^1 and the latitude circle is a 2-dimensional toroidal rotating surface S^1 x S^1 . Since leptons are extended 3-dimensional by their spin in spacetime, on the core of the radial shrinking tori a Higgs field can place a mass scalar. In projection mass sits then at the Hopf S^2 south pole and spin at the north pole. Since photons show in experiments also a particle character, its relativistic mass can mean that the transversal circle of its cyclinder is shrinked to a point q and the statistical distribution of them shows up as for particles while for double slits Young experiment shows also interference wave distributions, the circle is not shrinked to a point; in the Stern-Gerlach experiment lights quantized momentum character shows up where spin directions change. The projective w-coordinate U(1) and the mass, frequency coordinates in [m,f,w] have to be added to spacetime coordinates for these quantum effects. In the above table m,f are listed in the two last rows and U(1) has to be added as a 7th octonian coordinate. The first octonian coordinate e_0 is added to (x,y,z,t) coordinates for setting vectorial units of energies wherever they are attached to an initial point in space, having on a line a direction for its action as speed, force,… or setting an angle towards a rotational z-axis for energy systems or as angle between two rays for normal projections, rescaling measuring units as in the Minkowski or Schwarzschild metrics case. Beside a Higgs field, setting mass scalars at possible barycenters which is according to physics everywhere available in space, e_0 is similary a vector units setting vectorial field. Octonians are needed, 4-dimensional calculus has to be extended. As energy carrier, to e_0 are attributed color charges, to U(1) candela cd, to f Hz, to mass kg. The vector calculus of physics is introduced by octonians, having 8 not 4 digit base numbers for their vectorial directed coordinates, eight energies or for measures. The authors claim for Hilbert space H dimensions is that it can remain 4-dimensional bases (u,v,w,p) for Boolean blocks in its subspace lattice L and adds for atoms in different blocks of L the new dimensions as scalars q = u,v,w,p, for H = R and q real,
H = C complex (8 for C^4 , octonians, SU(3)) and q complex or H = Q quaternionic in case 16 are needed for q a quaternionic Q^4 Hilbert space. The listing of names for atoms is not bound to 4 ior different systems bases set in H. A suggestion is that for L astroids atoms are enumerated by octonians and triangles barycentrical drawn coordinates as tetrahedron GF‘s like *rgb*-gravitons with a base triangle replacing the quark triangle. Describing states of systems need no countably infinite Hilbert space. Some geometrical (projective and projections, fiber bundles), measuring (GF‘s with triple bases in the Copenhagen interpretation) and symmetry (S_4 ,D_3 ,...) replacements are presented in this article.

REFERENCES

[1]. Kalmbach, G. (1983): Orthomodular Lattices. – London New York: Academic Press 390p.
[2]. G. Kalmbach H.E. (2017), MINT-WIGRIS, MINT Verlag, Bad Woerishofen.
[3]. G. Kalmbach H.E. (Chef-Hrsg), MINT (Mathematik, Informatik, Naturwissenschaften, Technik), MINT Verlag, Bad Woerishofen 1997-2020, vol. 1-65.
[4]. Internet video under YouTube: Moebius Transformations Revealed 2014.
[5]. G. Kalmbach, U. Eberspaecher, MINT-Wigris Tool Bag, Bad Woerishofen 2019
[6]. G. Kalmbach, MINT-Wigris Postulates, in: researchgate.net 2020 under *MINT-Wigris Project*
[7]. E. Schmutzer, Projektive einheitliche Feldtheorie, Harry Deutsch, Frankfurt, 2004,
[8]. K. Stierstadt, Physik der Materie, VCH, Weinheim, 1989
[9]. MINT-Wigris project (G. Kalmbach H.E.), in the internet under: researchgate.net
[10]. MINT, PF 1533, D-86818 Bad Woerishofen, Germany, email: mint-01@maxi-dsl.de