

Application of Homotopy Analysis Method to the Integro – Differential Equation for RLC Circuit of Non-Integer Order

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Abstract. In the present article, the solution of a fractional integro-differential equation model associated with a RLC circuit with $0 < \alpha \leq 1$ is derived. By introducing the fractional derivative in the sense of Caputo, homotopy analysis method is applied to find the analytic approximate solution of the model. The result obtained is shown graphically with the effect of varying the auxiliary parameter h and α .

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I. INTRODUCTION

Fractional calculus has attracted the interest of many researchers in the recent years being a new mathematical method for solving the diverse problems due to the frequent appearance of the fractional differential equations which are generalization of classical integer order equations in various fields like viscoelasticity, biology, mechanics, fluid flow, physics, engineering and other models. Fractional order derivatives and integrals are proved to be more appropriate for formulating certain mathematical models as compared to the classical models [1-6]. For the fractional differential equations as such no method exists which provides the exact solution. Although some effective perturbative and nonperturbative techniques are there to derive the approximate solutions for the same. The perturbation techniques such as homotopy perturbation method involve small perturbation parameters [7]. Hence when applied to a problem it involves the expansion around the perturbation parameter which leads to small convergence region of the result. Whereas in the non-perturbative techniques like the optimized perturbation theory (OPT) [8] and linear δ -expansion method (LDE) [9, 10] the convergence of the region is controlled by some artificial parameters not existing in the original problem and so the approximations have a least dependence on these parameters. Among the non-perturbative techniques the homotopy analysis method is a powerful analytical method proposed by Shi-Jun-Liao [11-13] to find the solutions of linear and non-linear differential and integral equations. The HAM contains the auxiliary parameter h to control the convergence region of the solution. The HAM was successfully applied to solve many nonlinear problems such as nonlinear Riccati differential equation with fractional order [14], nonlinear Vakhnenko equation [15], the Glauert-jet problem [16], fractional KdV-Burgers-Kuramoto Equation [17], nonlinear heat transfer [18], to projectile motion with the quadratic law [19], to boundary layer flow of nanofluid past a stretching sheet [20], to the system of Fractional differential equations [21], to the nonlinear flows with slip boundary condition [22] and so on.

This paper is organized to have some important definitions in section 2. Section 3 deals with reviewing the basic idea of HAM briefly. Section 4 is concerned with Mathematical Model for RLC electrical circuit. In section 5 the Formulation of fractional differential equation model and its solution applying the HAM is presented graphically.

II. DEFINITIONS

In this section we mention some basic definitions of the fractional calculus theory: the Caputo and its reverse operator Riemann-Liouville and their properties which we will use to get the solution [1-3].

Def.2.1: A real valued function $f(x)$, $x > 0$ is said to be in the space $C_{\mu, \mu} \in R$ if there exists a real number $p > \mu$ such that $f(x) = x^p f_1(x)$, where $f_1(x) \in C[0, \infty)$.

Def.2.2: A function $f(x)$, $x > 0$ is said to be in the space C_{μ}^n , $m \in N \cup \{0\}$, if $f^{(m)} \in C_{\mu}$.

Def.2.3 The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, for a function $f \in C_{\mu}$, $\mu \geq 1$ is defined as

$$J^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt; \quad \alpha > 0, x > 0$$

$$J^0 f(x) = f(x)$$

For the convenience of establishing the results, we have the following properties

$$J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x)$$

$$J^\alpha x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)} x^{\alpha+\beta}$$

Def2.4: The fractional derivative of $f \in C_{-1}^n$ in the Caputo's sense is defined as

$$D^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^n(\tau) d\tau, & n-1 < \alpha < n, n \in N^*, \\ \frac{d^n}{dt^n} f(t), & \alpha = n. \end{cases}$$

According to the Caputo's derivative, we can easily obtain the following expressions:

$$D^\alpha K = 0; K \text{ is a constant,}$$

$$D^\alpha t^\beta = \begin{cases} \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} t^{\beta-\alpha}, & \beta > \alpha - 1, \\ 0, & \beta \leq \alpha - 1. \end{cases}$$

III. HOMOTOPY ANALYSIS METHOD

In order to show the basic idea of HAM, consider the following differential equation

$$N[u(t)] = 0, \tag{3.1}$$

where N is a nonlinear operator, t denotes the independent variables and u is an unknown function. For simplicity, we ignore all boundary or initial condition, which can be treated in the similar way. By means of the HAM, we first construct the zeroth-order deformation equation as

$$(1-q)L[\phi(t,q) - u_0(t)] = qhH(t)N[\phi(t,q)], \tag{3.2}$$

where $q \in [0, 1]$ is an embedding parameter, $h \neq 0$ is an auxiliary parameter, L is an auxiliary linear operator, $\phi(t, q)$ is an unknown function, $u_0(t)$ is an initial guess of $u(t)$ and $H(t) \neq 0$ denotes a nonzero auxiliary function. It is obvious that when the embedding parameter $q = 0$ and $q = 1$, equation (3.2) becomes $\phi(t, 0) = u_0(t)$, $\phi(t, 1) = u(t)$

3.3)

respectively. Thus as q increases from 0 to 1, the solution varies from the initial guess $u_0(t)$ to the solution $u(t)$. Expanding $\phi(t, q)$ in Taylor series with respect to q ,

$$\phi(t, q) = u_0(t) + \sum_{m=1}^{\infty} u_m(t) q^m,$$

$$\text{Where } u_m(t) = \frac{1}{m!} \frac{\partial^m \phi(t, q)}{\partial q^m} \Big|_{q=0}. \tag{3.4}$$

The convergence of the series in (3.4) depends upon the auxiliary parameter h and the auxiliary function $H(t)$. If it is convergent at $q = 1$, we have

$$u(t) = u_0(t) + \sum_{m=1}^{\infty} u_m(t)$$

(3.5)

Which is definitely one of the solutions of the nonlinear equation considered in (3.1), as proven by Liao.

Defining the vector

$$\vec{u}_n = (u_0(t), u_1(t), \dots, u_n(t)).$$

By differentiating the zeroth order deformation equation m -times, dividing it by $m!$ and then setting $q=0$, we get the m -th order deformation equation as.

$$L[u_m(t) - \chi_m u_{m-1}(t)] = hH(t) R_m[\vec{u}_{m-1}(t)]$$

3.6)

Subject to the initial condition

$$u_m(0) = 0$$

Where

$$R_m [\vec{u}_{m-1}(t)] = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(t, q)]}{\partial q^{m-1}} \Big|_{q=0}$$

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1. \end{cases}$$

If we choose $L[\phi(t, q)] = D^\alpha$, then according to (3.6) we have

$$J^\alpha D^\alpha [u_m(t) - \chi_m u_{m-1}(t)] = hJ^\alpha [H(t)R_m \{\vec{u}_{m-1}(t)\}]$$

Which further gives

$$u_m(t) = \chi_m u_{m-1}(t) + hJ^\alpha [H(t)R_m \{\vec{u}_{m-1}(t)\}],$$

3.6)

Subject to the initial condition:

$$u_m(0) = 0$$

For the special case when $\alpha = 1$, then () reduces to

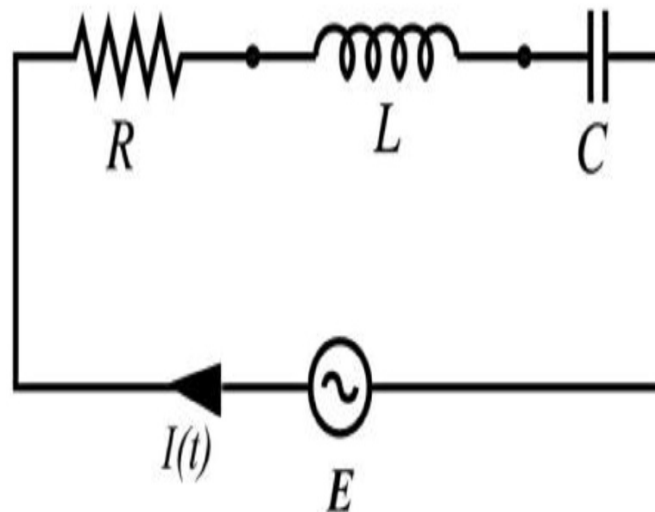
$$u_m(t) = \chi_m u_{m-1}(t) + h \int_0^t [H(\tau)R_m \{\vec{u}_{m-1}(\tau)\}] d\tau$$

3.7)

IV. MATHEMATICAL MODEL FOR RLC ELECTRICAL CIRCUIT

Various fractional model for electrical circuits such as RL, RC, RLC have already been proposed. Caputo derivatives and Numerical Laplace transform are considered to get the solution of RL and RC circuits. Further, RLC circuit is also analysed in time domain and the solution are found in terms of Mittag-Leffler function [23]. Analytic solution of RL electrical circuit described by a fractional differential equation of the order $0 < \alpha \leq 1$ has been obtained by using the Laplace transform of the fractional derivative in the Caputo sense[24]. RLC circuit of non-integer order is solved analytically by using the Laplace Transform method including convolution theorem[25]. In continuation to these developments this paper discusses a new and recent application of fractional calculus i.e. RLC electrical circuit considering an integro differential equation with parameter $\alpha \in (0,1]$ and the analytic approximate solution is obtained as an application of Homotopy analysis method.

Here, we present an oscillatory RLC electrical circuit in which resistor(R), inductor(L) and capacitor (C) are connected with voltage(E) in series. Here, the capacitance (C), the inductance (L) and the resistor (R) are considered as positive constants.



RLC Circuit

Nomenclature

$E(t)$ { The voltage of the power source (measured in volts = V)

$I(t)$ { The current in the circuit at time t (measured in amperes = A)

R { The resistance of the resistor (measured in ohms = V/A)

L { The inductance of the inductor (measured in henry = H)

C { The capacitance of the capacitor (measured in farads = F = C/V)

V. FORMULATION OF FRACTIONAL DIFFERENTIAL EQUATION MODEL AND ITS SOLUTION

If the voltage across R, L and C are $RI, L \frac{dI}{dt}$ and $\frac{1}{C} \int_0^t I(\varepsilon) d\varepsilon$ respectively, as per Kirchhoff's voltage law, around any loop in a circuit, the voltage rises must equal to the voltage drops. We have,

$$U_L(t) + U_R(t) + U_c(t) = E(t)$$

Or

$$L \frac{dI}{dt} + RI + \frac{1}{C} \int I dt = E(t)$$

Here we study the model for RLC circuit in the form of fractional differential equation as

$$LD^\alpha I(t) + RI(t) + \frac{1}{C} \int I(t) dt = E(t) \quad , \tag{5.1}$$

Where $D^\alpha I(t) = \frac{d^\alpha I}{dt^\alpha}$, $0 < \alpha \leq 1$.

For an AC voltage source, choosing the origin of time so that $V(0) = 0, V(t) = E_0 \sin(\omega t)$ and the differential equation becomes

$$D^\alpha I + \frac{R}{L} I + \frac{1}{LC} \int_0^t I(t) dt = \frac{E_0}{L} \sin \omega t$$

where $t = 0, I = 0$ or $I(0) = 0$.

(5.2)

Choosing the auxiliary linear operator $L[\phi(t, q)] = \frac{\partial \phi(t, q)}{\partial t}$ with the property $L(c) = 0$ and the auxiliary function $H(t) = 1$, we construct the homotopy as

$$R_m [\overrightarrow{I_{m-1}}(t)] = D^\alpha I_{m-1}(t) + \frac{R}{L} I_{m-1}(t) + \frac{1}{LC} \int_0^t I_{m-1}(x) dx - \frac{E_0}{L} \sin \omega t \tag{5.3}$$

$$I_m(t) = \chi_m I_{m-1}(t) + h \int_0^t R_m [\overrightarrow{I_{m-1}}(t)] dt \tag{5.4}$$

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1. \end{cases}$$

$$I_1(t) = h \int_0^t R_1 [\overrightarrow{I_0}(t)] dt$$

$$= h \int_0^t \left[D^\alpha I_0(t) + \frac{R}{L} I_0(t) + \frac{1}{LC} \int_0^t I_0(x) dx - \frac{E_0}{L} \sin \omega t \right] dt$$

Let $I_0(t) = \frac{t^\alpha}{\Gamma(1+\alpha)}$ then

(5.5)

$$\begin{aligned} I_1(t) &= h \int_0^t \left[D^\alpha \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{R}{L} \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{1}{LC} \int_0^t \frac{x^\alpha}{\Gamma(1+\alpha)} dx - \frac{E_0}{L} \sin \omega t \right] dt \\ &= h \int_0^t \left[\frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha)} + \frac{R}{L} \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{1}{LC} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} - \frac{E_0}{L} \sin \omega t \right] dt \\ &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{(1+\alpha)\Gamma(1+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] \\ &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] \tag{5.6} \end{aligned}$$

$$I_2(t) = \chi_2 I_1(t) + h \int_0^t R_2 [\overrightarrow{I_1}(t)] dt$$

(5.7)

$$\begin{aligned}
 I_2(t) &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] + \\
 &\quad h \int_0^t D^\alpha I_1(t) + \frac{R}{L} I_1(t) + \frac{1}{LC} \int_0^t I_1(x) dx - \frac{E_0}{L} \sin \omega t dt \\
 &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] + h \int_0^t \left[D^\alpha \left\{ h_1 t + \frac{Rh}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{h}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0 h}{\omega L} (\cos \omega t - 1) \right\} + \right. \\
 &\quad \left. RLht + RhLt1 + \alpha \Gamma(2+\alpha) + hLCt2 + \alpha \Gamma(3+\alpha) + E_0 h \omega L (\cos \omega t - 1) + hLC0tx + RLx1 + \alpha \Gamma(2+\alpha) + 1LCx2 + \alpha \Gamma(3+\alpha) + E_0 \omega L (\cos \omega x - 1) dx - E_0 L \sin \omega t dt \right] \\
 &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] + \\
 &\quad h \int_0^t \left[\left\{ h \frac{t^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{Rh}{L} \frac{t \Gamma(2+\alpha)}{\Gamma(2+\alpha)\Gamma(2)} + \frac{h}{LC} \frac{t^2 \Gamma(3+\alpha)}{\Gamma(3)\Gamma(3+\alpha)} + \frac{E_0 h}{\omega L} \omega^\alpha \cos \left(\frac{\alpha\pi}{2} + \omega t \right) \right\} + \frac{R}{L} \left\{ ht + \frac{Rh}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{h}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \right. \right. \\
 &\quad \left. \left. E_0 h \omega L (\cos \omega t - 1) + hLCt22 + RLt2 + \alpha \Gamma(3+\alpha) + 1LCt3 + \alpha \Gamma(4+\alpha) + E_0 \omega 2L \sin \omega t - E_0 \omega L t - E_0 L \sin \omega t dt \right\} \right] \\
 &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] + h \int_0^t \left[\left\{ h \frac{t^{1-\alpha}}{\Gamma(2-\alpha)} + \frac{Rh}{L} t + \frac{h}{2LC} t^2 + \frac{E_0 h}{\omega L} \omega^\alpha \cos \left(\frac{\alpha\pi}{2} + \omega t \right) \right\} + \right. \\
 &\quad \left. RLht + RhLt1 + \alpha \Gamma(2+\alpha) + hLCt2 + \alpha \Gamma(3+\alpha) + E_0 h \omega L (\cos \omega t - 1) + hLCt22 + RLt2 + \alpha \Gamma(3+\alpha) + 1LCt3 + \alpha \Gamma(4+\alpha) + E_0 \omega 2L \sin \omega t - E_0 \omega L t - E_0 L \sin \omega t dt \right] \\
 &= h \left[t + \frac{R}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{1}{LC} \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{E_0}{\omega L} (\cos \omega t - 1) \right] + h \left[\left\{ h \frac{t^{2-\alpha}}{\Gamma(3-\alpha)} + \frac{Rh}{L} \frac{t^2}{2} + \frac{h}{2LC} \frac{t^3}{3} + \frac{E_0 h}{\omega^2 L} \omega^\alpha \sin \left(\frac{\alpha\pi}{2} + \omega t \right) \right\} + \right. \\
 &\quad \left. RLht22 + RhLt2 + \alpha \Gamma(3+\alpha) + hLCt3 + \alpha \Gamma(4+\alpha) + E_0 h \omega L (\sin \omega t \omega - t) + hLCt36 + RLt3 + \alpha \Gamma(4+\alpha) + 1LCt4 + \alpha \Gamma(5+\alpha) + E_0 \omega 2L (1 - \cos \omega t \omega) - E_0 \omega L t22 - E_0 L (1 - \cos \omega t \omega) \right]
 \end{aligned}$$

Hence $I_2(t) = \frac{hE_0}{\omega L} \left(\frac{h}{\omega^2 LC} - \frac{h}{LC} - 1 \right) + ht \left(1 - \frac{Rh}{\omega LC} \right) + h^2 t^2 \left(\frac{R}{2L} + \frac{R}{2C} - \frac{h^2 E_0}{2\omega L^2 C} \right) + \frac{h^2 t^3}{3LC} + h^2 \frac{t^{2-\alpha}}{\Gamma(3-\alpha)} + \frac{Rh}{L} \frac{t^{1+\alpha}}{\Gamma(2+\alpha)} + \frac{h}{LC} (1 + R^2 h) \frac{t^{2+\alpha}}{\Gamma(3+\alpha)} + \frac{Rh^2}{L^2 C^2} (L + C) \frac{t^{3+\alpha}}{\Gamma(4+\alpha)} + \frac{h^2}{LC} \frac{t^{4+\alpha}}{\Gamma(5+\alpha)} + \frac{E_0 h}{\omega L} \left(1 - \frac{h}{\omega^2 LC} + \frac{h}{LC} \right) \cos \omega t + \frac{E_0 h^2}{L} \omega^{\alpha-2} \sin \left(\frac{\alpha\pi}{2} + \omega t \right) + \frac{RE_0 h^2}{\omega^2 LC} \sin \omega t.$

(5.8)
 ...and so on

Here only two terms of the HAM series solutions are used in evaluating the approximate solution shown in the following figures. The efficiency of the solution can be enhanced by calculating further terms of $I(t)$. Consider the electrical circuit RLC with $R = 250\Omega, L = 10^{-2} H, C = 10^{-4} F, E_0 = 1 V$ and $\omega = 100\pi$. Figures 1-2 show the solution for the current in the inductor, for different particular cases of α and h :

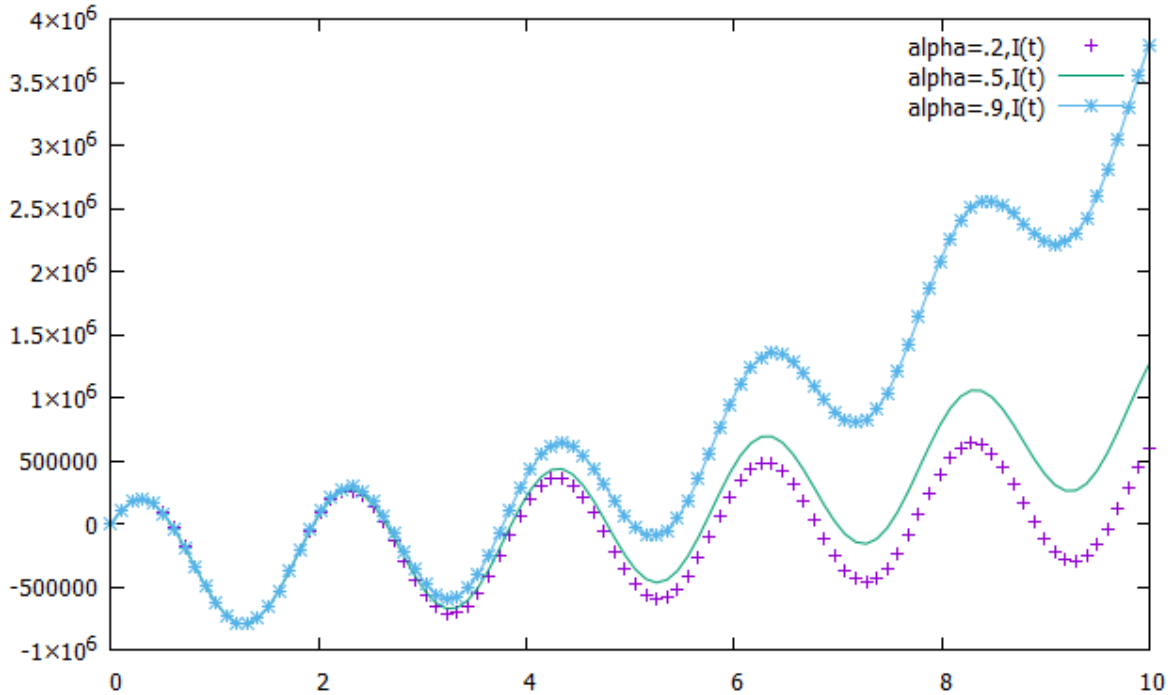


Fig. 1. plot for $h = 1$; $\alpha = 0.2, I(t)$ with points, $\alpha = 0.5, I(t)$ with lines, $\alpha = 0.9, I(t)$ with linespoint

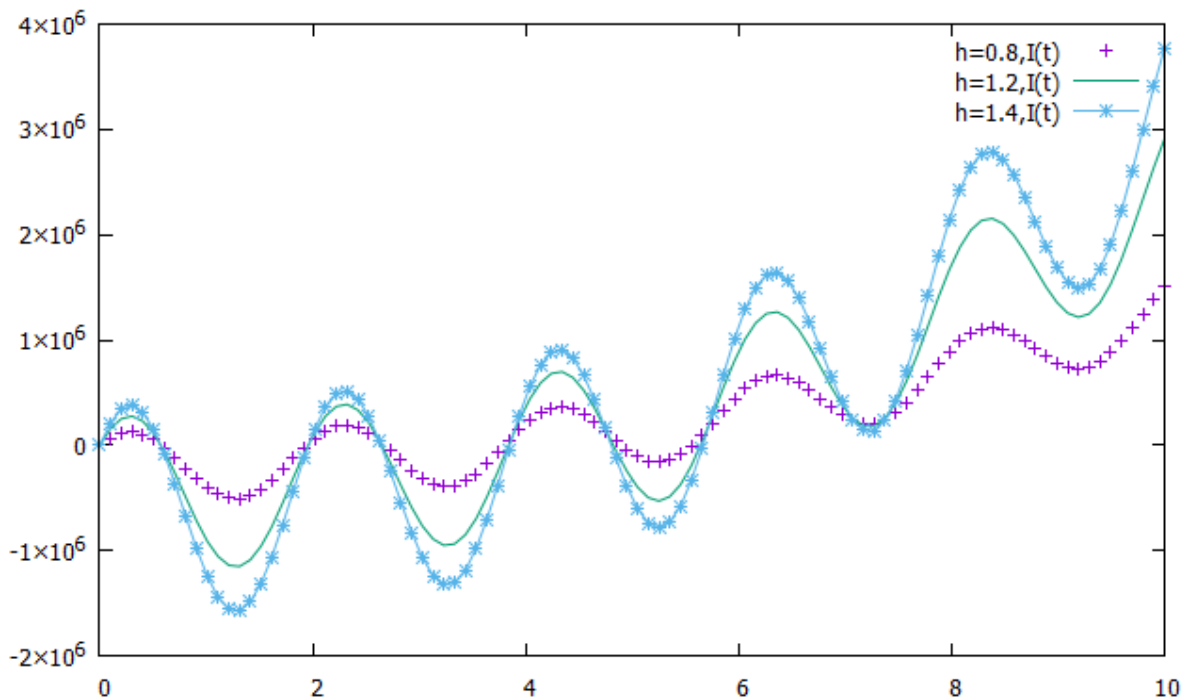


Fig. 2. plot for $\alpha = 0.7$, : $h = 0.8, I(t)$ with points, $h = 1.2, I(t)$ with lines, $h = 1.4, I(t)$ with linespoint .

VI. CONCLUSION

In the present article, We have presented an approximate solution for fractional integro-differential equation model associated with LCR circuit in time domain using the homotopy analysis method. The result obtained has been shown graphically for varying values of the exponent α of the fractional derivative and the auxiliary parameter h which shows that with the decrease in the values of α and h , the amplitude of the oscillation is decreased due to the increase in the damping capacity of the system and it leads the system to

exhibit an overdamped behavior. Hence the solution obtained depends on α and h and the electrical LCR circuit shows a non local effect of internal friction for α .

The homotopy analysis method used here has an advantage that the convergence of the solution region can be controlled by choosing the auxiliary parameter h , auxiliary function $H(t)$, auxiliary linear operator L and the initial guess. Hence it is a powerful tool to solve various problems in fractional calculus.

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